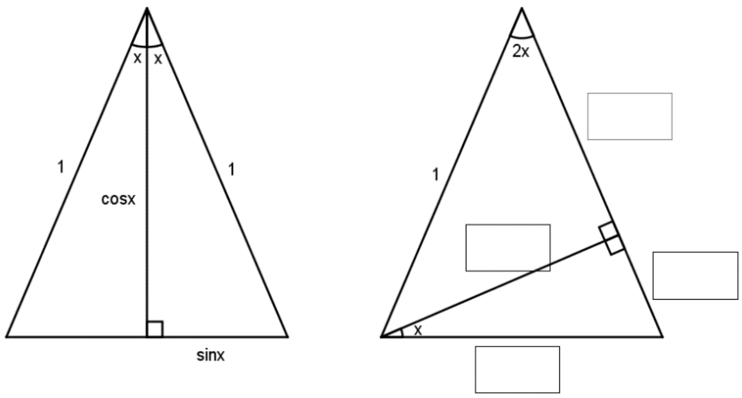
Trigonometric Identities

E6	Understand and use double angle formulae; use of formulae for $\sin(A \pm B), \cos(A \pm B)$ and $\tan(A \pm B)$; understand geometrical proofs of these formulae. Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$
E8	Construct proofs involving trigonometric functions and identities

Commentary

If the values of $\sin 13^\circ$ and $\cos 13^\circ$ are known, how could the values of $\sin 26^\circ$ and $\cos 26^\circ$ be evaluated? How can the diagram below, which shows two copies of the same isosceles triangle, help in answering this question?



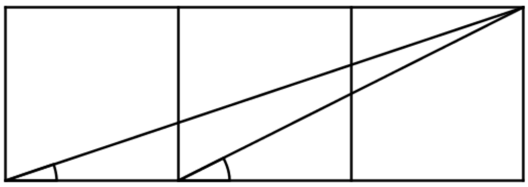
The double angle formulae can be derived 'for free' from the compound angle formulae by setting $A = B$. However, is there an argument for tackling the double angle formulae first? And then how can the idea above be adapted to prove the compound angle formulae?

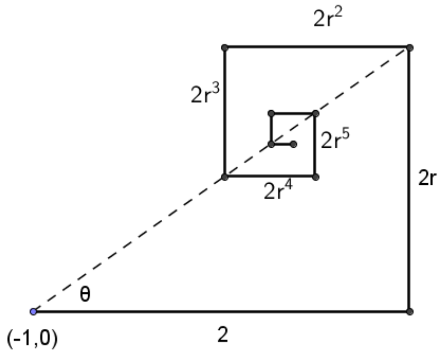
Often the natural reading of $\sin(A+B)$ is to immediately think of the compound angle formulae. Similarly, the natural reading of $\sin\left(x+\frac{\pi}{6}\right)$ is to think of transformations of graphs. What insights into graph transformations can students get from the compound angle formulae?

The process of completing the square expresses a general quadratic in the helpful form in which the variable appears only once and consequently the quadratic curve can be seen to be a transformation of $y = x^2$. In this a similar idea is met with expressions of the form $a\cos\theta + b\sin\theta$ where expressing them as $r\sin(\theta \pm \alpha)$ shows that they are simply transformations of $\sin\theta$. The ways in which sine curves interact like this is at the heart of many natural phenomena related to waves.

Sample MEI resource

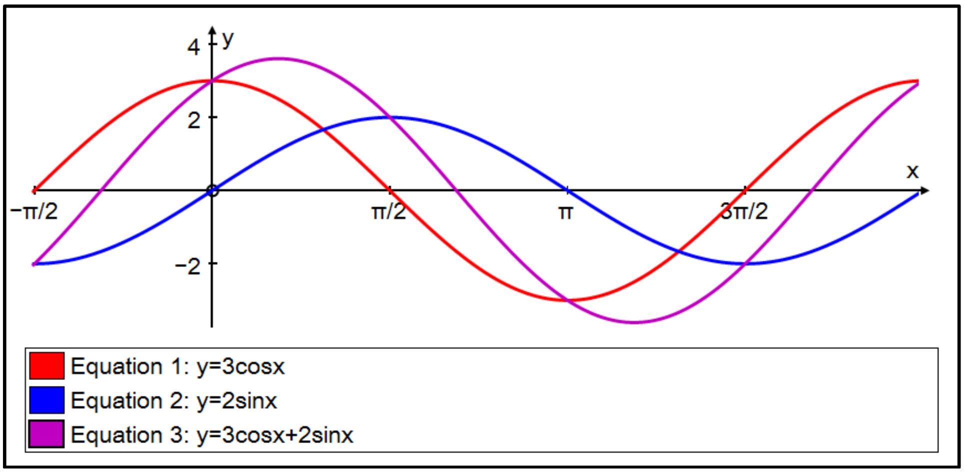
'Three challenge questions' (which can be found at https://my.integralmaths.org/integral/sow-resources.php) uses the compound angle formulae in a problem solving context.





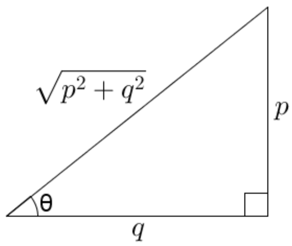
Effective use of technology

Using a graph plotter, investigate the curves of the form $y = p\cos x + q\sin x$. Are these transformations of $y = \sin x$?



Technology allows students to cojecture that the amplitude is $\sqrt{p^2+q^2}$ and this then leads to the result:

$$
\begin{aligned}
y &= p\cos x + q\sin x \\
&= \sqrt{p^2+q^2}\left(\frac{p}{\sqrt{p^2+q^2}}\cos x + \frac{q}{\sqrt{p^2+q^2}}\sin x\right) \\
&= \sqrt{p^2+q^2}\left(\sin\theta\cos x + \cos\theta\sin x\right) \\
&= \sqrt{p^2+q^2}\sin\left(x+\theta\right)
\end{aligned}
$$



# Trigonometric Identities	**Time allocation:**

Pre-requisites

- Trigonometry: the earlier Trigonometry (AS & A level) and Trigonometric Functions units
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Links with other topics

- Transformation of graphs: $y = \sin x \cos x$ is a transformation of $y = \sin x$ (since it is the same as $y = \frac{1}{2}\sin 2x$)
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Questions and prompts for mathematical thinking

- Find two ways of solving the equation $\sin 2\theta = \sin\theta$
- Explain connections between the graph of $y = \cos^2 x$ and the graph of $y = \cos 2x$.
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Opportunities for proof

- Prove $\sin(A+B) = \sin A\cos B + \cos A\sin B$ from a diagram
- A, B and C are the angles of a non right-angled triangle. Prove that $\tan A + \tan B + \tan C = \tan A\tan B\tan C$.
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Common errors

- Not knowing the formulae: an over-reliance on the formulae booklet and so not appreciating that, for example, $\sin x\cos x$ can be written in a more helpful form.
- Mis-use of the principle angle e.g. $\arcsin\left(\frac{-2}{3}\right) \approx -41.8^\circ$ then not proceeding to give the angles in the correct range.
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