

Mathematical Problem Solving

GCSE example

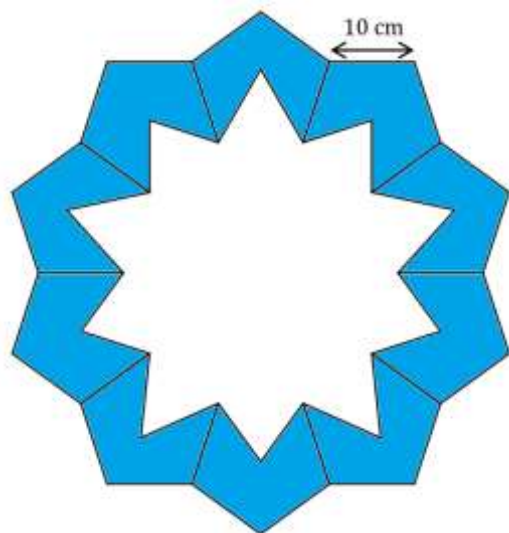
Example 2 – GCSE Higher Tier (calculator required)

The class have been studying trigonometry and have previously covered the formula for the area of a triangle.

- know and apply formulae to calculate: area of triangles
- know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$, $\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$ and $\tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$; apply them to find angles and lengths in right-angled triangles and, where possible, general triangles in two and three dimensional figures
- know and apply $\text{Area} = \frac{1}{2}ab \sin C$ to calculate the area, sides or angles of any triangle.

DfE: Mathematics GCSE subject content and assessment objectives 2013

In order to check the students' understanding of the methods used, the teacher presents the class with this problem on the board.



Find the shaded area.

It is important that the teacher makes it clear that this task will also develop the students' problem-solving skills. The students need to be made aware that there is information missing and that they will have to ask questions to find that information. The teacher should point out that some obvious questions such as "what is the area of one of the pentagons?" will not be answered.

The students should be given two minutes time to consider the problem without writing anything down. They should then spend two more minutes writing down as many questions as they can that would give key information for solving the problem. Students should then spend two more minutes comparing their questions with those of one or two other students. It is important at this stage for the students to say why they think their question will reveal some important information.

The students should hopefully come up with some questions like these:

- Is it symmetrical?
- Are all of the lengths 10 cm?
- Are all of the pentagons regular?
- Are the triangles all the same type?
- What type of triangles are they?
- Do we know any of the angles?
- How do you find the area of a pentagon?

These are all questions that the teacher should be prepared to answer. Some questions can be given definite answers, others should be answered by reminding the students about work they have covered in class. By answering these questions the teacher should be able to give the students enough information to be able to solve the problem.

The teacher should encourage and model the use of accurate mathematical language wherever possible.

Question

Answer

Is it symmetrical?

Yes it is. It has five lines of reflectional symmetry (but not more than 5) and a rotational symmetry of order five (but NOT one of order ten).

Does this tell you anything important about the problem?

Note: this should indicate that the “missing” triangles are of different types.

Question

Answer

Are all of the lengths 10 cm?

Most of the lengths are 10 cm.

Which lengths do you think aren't 10 cm?

Note: This line of questioning can be pursued until the students realise that there are two different types of triangle used.

Are all of the pentagons regular?

Yes, the design is based on regular pentagons.

Note: the students may have already ascertained this from the question above. It doesn't harm to repeat the information

These two questions should go together:

No, there are two types of triangle.

Are the triangles all the same type?

Do you know any useful features of the types of triangle we have here?

What type of triangles are they?

Do we know any of the angles?

You should know the internal angle of an equilateral triangle. I'm not going to give you any of the other angles as you should know enough to calculate them.

How do you find the area of a pentagon?

You don't know a formula for the area of a pentagon but you do know how to find the areas of some other shapes.

Could you split the pentagon up into some shapes that would help you to find its area.

Note: The teacher may wish to continue by looking at the triangles in question and asking the students how they could calculate the height of each triangle.

There may well be other sensible questions that the students ask and the teacher should be prepared to answer these.