

# Mathematical Problem Solving

## AS/A Level example

### Solution to example 16

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Write down the first five terms of

$$\sum_{r=1}^{10} r^2$$

Write down the first five terms of

$$\sum_{r=1}^{10} (r-1)^2$$

Use your answers to help you find the value of

$$\sum_{r=1}^{10} r^2 - \sum_{r=1}^{10} (r-1)^2$$

Find an expression in terms of  $n$  for the value of

$$\sum_{r=1}^n r^2 - \sum_{r=1}^n (r-1)^2$$

Find an expression in terms of  $m$  and  $n$  (where  $n > m$ ) for the value of

$$\sum_{r=m}^n r^2 - \sum_{r=m}^n (r-1)^2$$

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$$\sum_{r=1}^{10} r^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \dots + 10^2$$

The first 5 terms are  $1 + 4 + 9 + 16 + 25$

The first 5 terms of

$$\sum_{r=1}^{10} (r-1)^2$$

are  $0 + 1 + 4 + 9 + 16$

For

$$\sum_{r=1}^{10} r^2 - \sum_{r=1}^{10} (r-1)^2$$

all of the terms in the first series will cancel with terms in the second except for  $10^2$  so the answer is 100.

For

$$\sum_{r=1}^n r^2 - \sum_{r=1}^n (r-1)^2$$

all of the terms will cancel except for  $n^2$  so the answer is  $n^2$ .

For

$$\sum_{r=m}^n r^2 - \sum_{r=m}^n (r-1)^2$$

all of the terms will cancel except for  $n^2$  from the first series and  $-(m-1)^2$  from the second so the answer is  $n^2 - (m-1)^2$ .