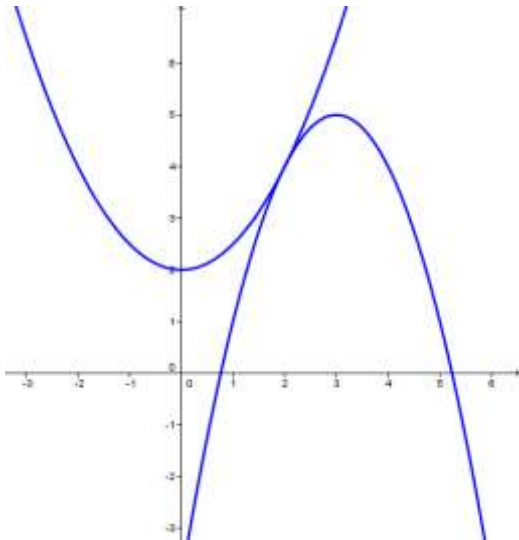


Mathematical Problem Solving

AS/A Level example

Solution to example 9



These two quadratic curves have a common tangent at $x = 2$.

Given that one curve has a vertex at $(0, 2)$ and the other has a vertex at $(3, 5)$, find the equation of each curve and the equation of the common tangent.

The equation of the parabola with the vertex at $(0, 2)$ can be written $y = ax^2 + 2$

The equation of the parabola with the vertex at $(3, 5)$ can be written $y = -b(x - 3)^2 + 5$

At $x = 2$ the curves have the same y coordinate so $a \cdot 2^2 + 2 = -b(2 - 3)^2 + 5$

$$\text{So } 4a + 2 = -b + 5 \Leftrightarrow 4a + b = 3 \quad (\text{A})$$

$$\text{For } y = ax^2 + 2, \frac{dy}{dx} = 2ax$$

$$\text{For } y = -b(x - 3)^2 + 5, \frac{dy}{dx} = -2b(x - 3)$$

At $x = 2$ the curves have the same gradient so $2a \cdot 2 = -2b(2 - 3)$

$$\text{So } 4a = 2b \Leftrightarrow 2a = b \quad (\text{B})$$

$$(\text{B}) \text{ in } (\text{A}) \quad 6a = 3 \Leftrightarrow a = \frac{1}{2}$$

$$\text{In } (\text{B}) \quad b = 1$$

The parabolas are $y = \frac{1}{2}x^2 + 2$ and $y = -(x - 3)^2 + 5$

The gradient at $x = 2$ for each parabola is 2

When $x = 2$, $y = \frac{1}{2} \cdot 2^2 + 2 = 4$ for each parabola

The equation of the common tangent is $y - 4 = 2(x - 2)$ which simplifies to $y = 2x$