The Band Brake

How hard do you need to press two objects together to generate a given braking force?
The Band Brake

Initial Problem Statement

In the design of many moving vehicles engineers not only have to design methods for efficient propulsion they also have to design efficient (and reliable!) methods for stopping. Most braking systems rely on friction to dissipate energy and slow the vehicle down. Usually this friction is generated by pressing one object against another.

How hard do you need to press two objects together to generate a given braking force?
Introduction

Discussion
Consider a motorised go kart as shown below. Discuss what methods could you use in a braking system?

Figure 1.
2. The band brake

The most common form of braking system uses something to rub against the wheels or an item that rotates with the wheels (e.g. a brake disc) as it is both effective and doesn’t damage the road. There are many different designs such as the band brake, the drum brake and the disc brake. This activity will concentrate on a design for a band brake.

**Multimedia**

The resource *Band Brake Animation* is available to demonstrate design and operation of a band-brake.

The band brake is based on the capstan principle and is shown schematically below.

![Diagram of the band brake](image)

*Figure 2.*

When the driver applies a force $T_s$ by pressing the brake pedal, the action of friction between the brake band and the spinning brake drum results in a tension force $T_l$ on the side of the band brake that is attached to the floor of the kart. The force $T_l$ is the “load force” The strength of the attachment of the band to the floor and the strength of the band material should be such that this force does not cause the band to break free of its fixing or snap. The force $T_s$ is the support force. It is the force required to generate the load force and it should be such that a person depressing the break pedal can easily generate it, either directly or through mechanical levers, etc.

**Discussion**

Given the direction of motion and wheel rotation as shown do you think that

(a) $T_l < T_s$?
(b) $T_l = T_s$?
(c) $T_l > T_s$?
The tensions $T_l$ and $T_s$ are related to each other through the equation:

$$T_l = T_s e^{\mu \theta}$$

where $\mu$ (pronounced mu) is the coefficient of friction between the band and the brake drum and $\theta$ is the contact angle made between the band and the brake drum. In this expression the contact angle is measured in radians. You can convert an angle in degrees to one in radians through the formula

$$\text{radians} = \frac{\text{degrees} \times \pi}{180}$$

**Activity 1**
Suppose you have $\theta = 1.658$ (radians), $\mu = 0.5$ and 10 Newtons of force is applied by the brake ($T_s$). What is the resulting load tension $T_l$?

**Activity 2**
Suppose you have $\theta = 1.658$ (radians), $\mu = 0.5$ and 30 Newtons of force is the measured load tension ($T_l$). What is the brake tension $T_s$?

**Activity 3**
Suppose you have $\theta = 1.658$ (radians), $T_s = 10$ and $T_l = 30$. What is the coefficient of friction, $\mu$?

**Multimedia**
The activity *Band Brake Interactive* is available to explore these calculations further. See appendix 1.
3. Calculating the braking force

**Discussion**
In the diagram below $T_i$ is 10 N and $T_l$ is 22.91 N. The forces on the band are not balanced so we would expect the band to move. However, the band does not move so there must be an additional force acting. What is providing the additional force?

**Figure 3.**
The frictional force, $F$, balances the forces acting on the system. This is the force that acts to slow down the kart. As it is applied tangentially to the rotating drum it creates a torque that slows down the rotation of the drum (which is connected to the wheel). A large torque will slow the drum down quicker than a small torque.

**Activity 4**
Remembering that $T_l > T_s$ write an expression relating $T_l$, $T_s$ and $F$ when all forces are balanced: Arrange your expression so that $F$ is the subject.

**Activity 5**
If the forces are $T_s = 10$ N and $T_l = 22.91$ N, what is the braking force, $F$?

**Activity 6**
The brake drum has a radius of 10 cm. What is the braking torque applied to the wheel in Nm?
4. Improving the design

In the previous activity you had $T_s = 10 \text{ N}$ and $T_l = 22.91 \text{ N}$ which resulted in a braking force, $F$, of 12.91 N. This applied a braking torque of 1.29 Nm.

**Discussion**

1.29 Nm is a reasonably small torque – how could you increase it and make the brake more effective?

**Hint**

Remember all the variables in the band brake equation $T_i = F \theta$.

**Hint**

Remember all the variables in the torque equation $T = Fd$. ($T$ is a torque not a tension!)
Notes

Torque

A torque is the turning moment of a force about a point of rotation.

The torque, $T$, of the force $F$ acting a perpendicular distance $d$ from the point of rotation, O, is given by:

$$T = Fd$$

The same torque can be achieved by two different sets of forces and distances. For example I can apply a large force at a short distance from the point of rotation or a small force at a larger distance. Two torques will be the same if the product $T = Fd$ is the same:

Case 1: $F = 100 \text{ N}, d = 3 \text{ cm} \Rightarrow T = 100 \times 3 = 300 \text{ Ncm}$
Case 2: $F = 10 \text{ N}, d = 30 \text{ cm} \Rightarrow T = 10 \times 30 = 300 \text{ Ncm}$

This is useful when, for example, a lid is stuck on a jar. If you cannot twist it off by hand (gripping the lid at its edge so the distance is the radius of the jar) you can use a device that gives you a longer lever - it lets you increase $d$, beyond the radius of the jar lid so that you can apply a higher torque and open the jar.

For a spinning object a high torque will lead to a high angular acceleration if applied in the direction or rotation or a high angular deceleration if applied against the direction of rotation. (A torque is a vector, like a linear force, and has both magnitude and direction. In the case of a torque the direction states whether it acts to turn clockwise or anticlockwise.)
Solutions

Introduction

Discussion solution
There are, of course, many methods that could be used. Some ideas might be:

- Using your feet. This might be bad for your shoes!
- Using something else to drag along the ground. What will this be?
- Will it wear out? Will it damage the track?
- Deploying a parachute. Effective but not re-usable.
- Using something to rub against the wheels or an item that rotates with the wheels (a brake disc).
2. The band brake

Discussion solution
In the diagram the drum is spinning clockwise. The friction between the drum and the belt will mean that the drum will drag the belt around with it. This will increase the tension in $T_l$ as the belt is stretched more due to this dragging effect. Conversely, the tension $T_s$ is reduced as the dragging effect will decrease the amount by which the band is stretched. This will mean that $T_l > T_s$.

Activity 1 solution
Finding $T_l$ when $\theta = 1.658$ (radians), $\mu = 0.5$ and $T_s = 10$ (Newtons)

$$T_l = T_s e^{\mu \theta}$$

$$= 10 e^{0.5 \times 1.658}$$

$$= 22.91 \text{ (N)} \quad (2 \text{ d.p.})$$

Activity 2 solution
Finding $T_s$ when $\theta = 1.658$ (radians), $\mu = 0.5$ and $T_l = 30$ (Newtons)

$$T_l = T_s e^{\mu \theta}$$

$$30 = T_s e^{0.5 \times 1.658}$$

$$T_s = \frac{30}{e^{0.5 \times 1.658}}$$

$$T_s = 30 \times e^{-0.5 \times 1.658}$$

$$= 13.09 \text{ (N)} \quad (2 \text{ d.p.})$$

Activity 3 solution
Finding $\mu$ when $\theta = 1.658$ (radians), $T_s = 10$ and $T_l = 30$.

$$T_l = T_s e^{\mu \theta}$$

$$30 = 10 e^{\mu \times 1.658}$$

$$e^{\mu \times 1.658} = \frac{30}{10} = 3$$

$$1.658 \mu = \ln 3$$

$$\mu = \frac{\ln 3}{1.658}$$

$$= 0.66 \quad (2 \text{ d.p.})$$
3. Calculating the braking force

**Discussion solution**

The additional force is as a result of the friction between the brake drum and the brake band. As the drum is rotating clockwise there is friction between the drum and the band acting in the same direction as $T_s$. This frictional force, $F$, balances the forces acting on the system.

**Activity 4 solution**

The forces are balanced so they must sum to a zero net force. As $T_l > T_s$ this is achieved when

$$T_l = T_s + F$$

Rearranging this gives

$$F = T_l - T_s$$

**Activity 5 solution**

In the case where $T_s = 10$ N and $T_l = 22.91$ N, the net braking force due to friction, $F$ is

$$F = T_l - T_s = 22.91 - 10 = 12.91$$

**Activity 6 solution**

The torque of a force about a point is given by the force multiplied by the perpendicular distance to the point.

Perpendicular distance = 0.1 m

Braking torque = $12.91 \times 0.1 = 1.29$ Nm
4. Improving the design

**Discussion solution**

You could apply more force.

This can be amplified by using a brake cylinder. A doubling of the applied force $T_s$ will double the load tension $T_l$ so that the net force available for braking is doubled:

Net braking force due to friction $= 45.82 - 20 = 25.82$ (N) (2 d.p.)

You could use a higher friction material.

A higher friction material will increase the value of the coefficient of friction $\mu$. Consider a 50% increase in $\mu$ so that $\mu = 0.75$.

$$T_i = T_s \mu \theta = 10e^{0.5 \times 2.487} = 34.68 \text{ (N) (2 d.p.)}$$

As expected!

It may sound easier to improve efficiency by increasing the contact angle to large values than find very high friction materials. For example, consider the case where the band is wrapped all the way around the drum and then goes to the brake. In this case the angle of contact is increased by $2\pi$ to $\theta = 1.658 + 2\pi = 7.941\ldots$

In this case

$$T_i = T_s \mu \theta = 10e^{0.5 \times 7.941\ldots} = 53.02 \text{ (N) (2 d.p.)}$$

Net braking force due to friction $= 53.02 - 10 = 43.68$ (N). However, how would you keep the band off the drum when it is not being used? One solution would be to let the band rest very lightly on the drum. This would still lead to some friction and would quickly wear the belt. The engineering of such large contact angle solutions may be difficult!

You could increase the radius of the drum.

The radius of the brake drum does not appear in expression (1) so an increase in its value would not affect the calculation of $T_s$ and $T_l$ so how can this change the effectiveness of the brake? Recall the force applies a torque which slows down the wheels so that while the applied force is unchanged, the distance at which it is applied does change. A doubling of the radius will therefore lead to a doubling of the applied torque.
Appendix 1
using the interactives

Band Brake Interactive
This resource explores the connections between the angle of contact, the coefficient of friction and the load and support tensions associated with the band brake. It requires you to calculate one of these variables given values for the other three.

The buttons at the bottom of the screen allow you to choose which variable you would like to test. For example, the ‘T_l’ choice will ask you to find the load tension. You are given the values for the coefficient of friction (μ), the angle (θ), and the support tension (T_s) and are required calculate the load tension using the band brake formula.

![Figure 6.](image)

Enter your result by typing into the empty box next the variable you have been asked to find. The answer for the coefficient of friction (μ) should be rounded to 2 decimal places, while the answers for the tension and angle should be answered to the nearest newton and degree respectively.

You can check your answer by pressing the green “Check” button in the bottom right hand corner of the screen. Remember, that the formula for the band brake uses an angle given in radians not degrees. You will therefore need to make the conversion from degrees to radians or radians to degrees depending on which variable you are finding.
Appendix 2

mathematical coverage

PL objectives
Use and apply mathematical modelling to solve engineering problems
• The engineering problem is quantified using mathematical expressions

Use trigonometry and coordinate geometry to solve engineering problems
• Using degrees and radians and converting between them

Use algebra to solve engineering problems
• Solve problems involving exponential growth