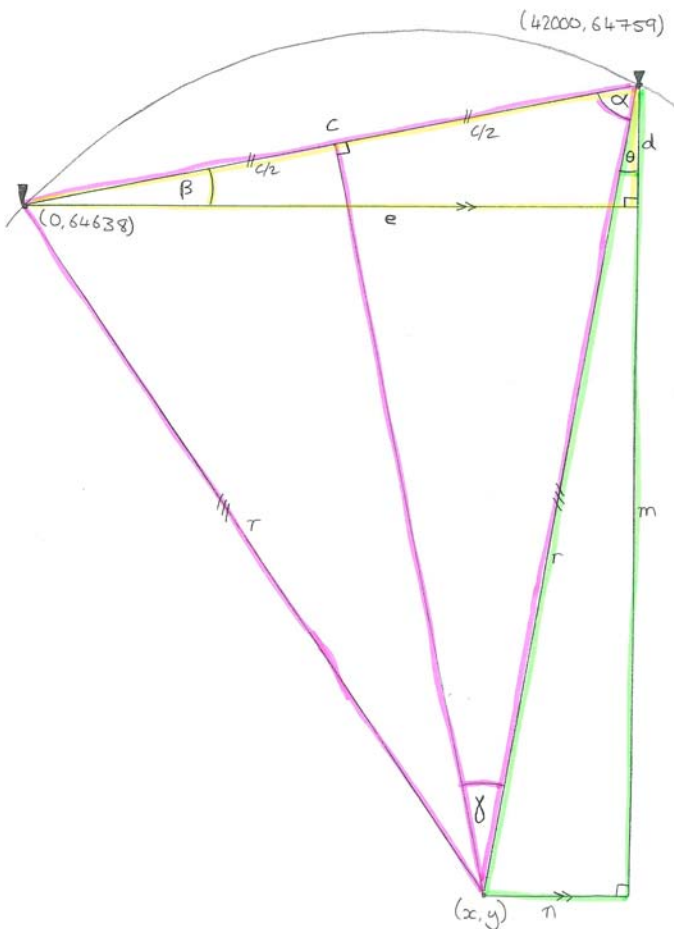


## Checking computer generated data

The task was to check computer generated levels of a curved bridge deck over the motorway, at 11 horizontal 2000mm spacings. The only levels to be assumed as certain were those at the abutments (bridge supports).

Information given;

- Level at north abutment; 64.638 m above sea level
- Level at south abutment; 64.759 m above sea level
- Distance between abutments; 42000mm
- Radius of bridge curve; 144499mm



Solution;

Consider it to be a task to find the y-coordinates of a circle at set x values. To do this the centre point of the circle must first be established.

Centre of circle;

Assume the north abutment to be at co-ordinate (0, 64638) and the south abutment at (42000, 64759). Draw the following diagram and work out the values.

We know:

$$r=144499$$

$$e=42000$$

We can calculate the rest:

$$d = 64759 - 64638 = 121$$

$$c = \sqrt{(d^2 + e^2)} = \sqrt{(121^2 + 42000^2)} = 42000.1743$$

$$\tan \beta = d/e$$

$$\beta = \tan^{-1}(121/42000) = 0.1650672544^\circ$$

$$\sin \gamma = (c/2)/r$$

$$\gamma = \sin^{-1}[(42000.1743/2)/144499] = 8.356408248^\circ$$

$$\alpha = 180^\circ - 90^\circ - \gamma = 180^\circ - 90^\circ - 8.356408248^\circ = 81.64359175^\circ$$

$$\theta = 180^\circ - 90^\circ - \beta - \alpha = 180^\circ - 90^\circ - 0.1650672544^\circ - 81.64359175^\circ = 8.191340994^\circ$$

$$\cos \theta = (m+d)/r \quad (m+d) = r \cos \theta = 144499 \cdot \cos(8.191340994) = 143024.7887$$

$$\sin \theta = n/r \quad n = r \sin \theta = 144499 \cdot \sin(8.191340994) = 20588.12345$$

$$y = 64759 - (m+d) = 64759 - 143024.7887 = -78265.7887$$

$$x = 42000 - n = 42000 - 20588.12345 = 21411.8766$$

Therefore **circle centre** = (21411.8766, -78265.7887)

**Check** for known points; (0, 64638) & (42000, 64759)

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(0 - 21411.8766)^2 + (64638 - [-78265.7887])^2 = 144499^2$$

$$2.0879961 \times 10^{10} = 2.0879961 \times 10^{10} \quad \text{-OK-}$$

$$(42000 - 21411.8766)^2 + (64759 - [-78265.7887])^2 = 144499^2$$

$$2.0879961 \times 10^{10} = 2.0879961 \times 10^{10} \quad \text{-OK-}$$

As we now know the centre of the circle we can easily establish the y-value for any given x-value. This is done by re-arranging the formula of a circle;

$$(x - a)^2 + (y - b)^2 = r^2$$

to

$$y = b + \sqrt{[r^2 - (x - a)^2]}$$

This is then used to find the relevant y-values, which are in turn compared to the computer generated levels;

Point	x	Height (y)	Height on Drawing
A	0	64638	64638
B	2000	64923	64923
C	4000	65180	65180
D	6000	65409	65409
E	8000	65609	65609
F	10000	65782	65782
G	12000	65926	65926
H	14000	66043	66043
I	16000	66132	66132
J	18000	66193	66193
K	20000	66226	66226

Point	x	Height (y)	Height on Drawing
L	22000	66232	66232
M	24000	66210	66210
N	26000	66160	66160
P	28000	66083	66083
Q	30000	65978	65978
R	32000	65845	65845
S	34000	65684	65684
T	36000	65495	65495
U	38000	65278	65278
V	40000	65033	65033
W	42000	64759	64759

All values are found to be correct. Success!

