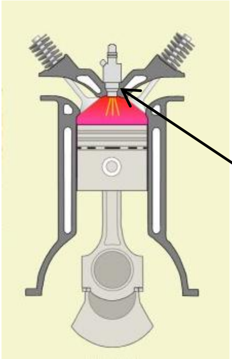
The application of Integration to measure volumes

Introduction:

For my year in industry, I am placed with Delphi Diesel systems. Delphi design, develop and manufacture fuel injectors for many of the world's leading truck engine manufacturers.

Fuel injectors expel fuel into the cylinder at high pressures. When compressed, the fuel and air combust, forcing the piston down.



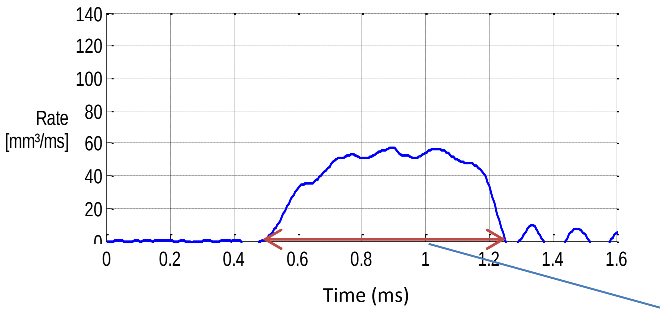
This product delivers multiple volumes of fuel every second.

Consequently, it is essential to know how much fuel is being expelled per injection. This information is needed in order to meet the energy and emissions requirements of the engine.

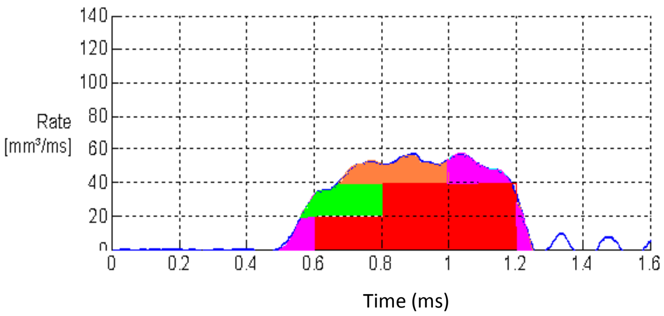
Calculating the fuel delivered per injection:

To measure the volume of fuel expelled, the injector injects into a cylinder of known diameter, displacing a piston. Due to a restriction imposed by the custom setup of the measurement system, we are unable to measure the injected volume from the final position of the piston. Instead, we have to use other measurements to ***calculate*** the total fuel injected.

From the piston's speed we are able to measure the ***rate of injection***. This is the amount of fuel delivered by the product, per unit time- i.e. mm³/s. We are also able to measure the ***time period*** for which the injector is injecting. Using this data, we can plot a rate trace graph.



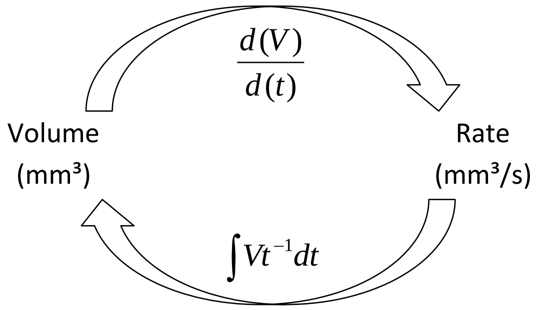
This makes it clear to see that the area under the graph (for the time period) is equal to the total volume of fuel injected. In its most crude form, we could estimate the area by counting the squares.



Total number of squares ≈ 8
Volume per square= 20 x 0.2= 4mm³

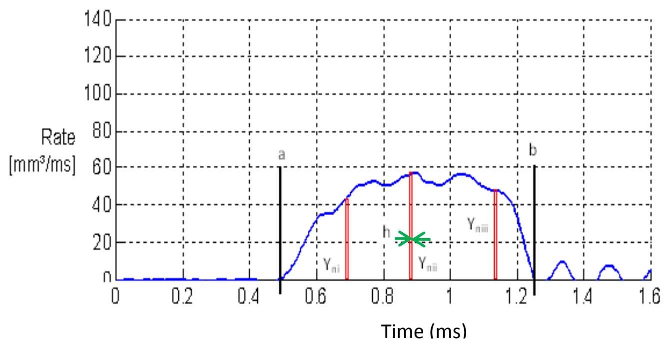
Total volume= 8 x 4= **32mm³**

Remembering that the rate is the change in volume, per unit time, we can use ***calculus*** to find our answer.



As shown, ***integration*** is the method to use to find the volume from the rate. It is, however, not possible to simply integrate, as this curve is a series of connected data points - it has no equation. Instead we must use one of the rules of integration to find the area under the curve - ***The Trapezium Rule***, given as:

$$\int_a^b y\,dx \approx \frac{1}{2}h\left\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\right\} \qquad h = \frac{b-a}{n}$$



The area under the curve can be thought of much like a bar chart. It is divided into lots of individual rectangles of the same width but differing length. The combined area of these rectangles is the total volume of fuel ejected by this particular injection.

In this particular example, there are 215 invisible trapeziums, in the thresholds of 1.3ms and 0.44ms, each 4µs wide:

$$\int_{0.44}^{1.3} Vt^{-1}dt \approx \frac{1}{2} \text{x } 215 \{(y_0 + y_{215}) + (y_1 + y_2 + \ldots + y_{214})\}$$

$$h = \frac{1.3 - 0.44}{0.004} = 215$$

This equation is written and used on computer software. The software, Matlab, has calculated the volume of fluid injected in this injection to be 32.02mm³.

This is very close to our initial guess of 32mm³.

Note: For a typical injector test, the Matlab software integrates 1675 rate traces. Each rate trace containing 1000 data points at 4µs intervals, a total 1,675,000 points per test are evaluated!

Integration is an essential mathematical tool in our test work.