What mathematics do students study in A level Mathematics courses?

Since the structure of A level Mathematics (and Further Mathematics) was changed in September 2004, students with only a single A level in Mathematics will have studied only two applied modules (in addition to the four core modules, Core 1 to Core 4, which cover the compulsory ‘pure’ content of the A level).

Please see http://www.meidistance.co.uk/pdf/revised_gce_maths_criteria_20040105.pdf for the official ‘pure’ core for AS/A level Mathematics, which must be covered by ALL AS/A level specifications.

Possible combinations of modules studied for A level Mathematics are:

| Core 1, Core 2, Core 3, Core 4 + one of the combinations of 2 applied modules shown below |
|-----------------------------------------------|--------------------------------|---------------------------------|--------------------------------|-------------------|-------------------|
| Statistics 1                                  | Mechanics 1                   | Decision 1                      | Statistics 1                   | Decision 1        | Mechanics 1       |
| Mechanics 1                                   | Decision 1                    | Statistics 1                    | Statistics 2                   | Decision 2        | Mechanics 2       |

There are no prescribed applied modules that are required to be studied, hence students could study any one of these combinations in order to gain an A level in Mathematics.

The following pages summarise the approximate content of the core modules in A level Mathematics, in AS/A level Further Mathematics and in the first two modules of each applied strand. However, there are differences between the content of such modules for the different A level specifications (and additionally a few other applied modules may be available from specific boards, e.g. Numerical Methods by MEI).

Those students who have studied an AS or A level in Further Mathematics will have had the opportunity to study more applied mathematics modules than those with just the single A level Mathematics. This highlights the worth of the further mathematics qualification for those students who wish to study for mathematics-related degrees at university. Please see www.furthermaths.org.uk to find out more about Further Mathematics.

There is considerable scope for MEI to work with universities to help support the learning and teaching of undergraduates, for both first year Mathematics courses and for mathematics service courses in mathematics-related degrees. For more information, please see www.mei.org.uk and navigate to ‘University’ under the ‘What we do’ menu.

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## PURE MATHEMATICS

### Algebra
- Simultaneous equations
- Solving quadratics, completion of square
- Surds/indices
- Logarithms
- Inequalities (only involving linear and quadratic expressions, and the modulus function)
- Polynomials (factor/remainder theorems)
- Binomial expansion
- Partial Fractions

### Trigonometry
- Sine rule, cosine rule
- Radians, arc length, sector area
- Exact values of sin, cos, tan of standard angles
- Sec, cosec, cot, arcsin, arccos, arctan
- Compound/double angle formulae

### Sequences and Series
- Arithmetic/geometric sequences/series
- Sigma notation
- Sequences defined recursively

### Curve Sketching
- Graphs of quadratics, polynomials (from the factorised form)
- Relationships between graphs of $y = f(x)$, $y = f(x + a)$, $y = f(ax)$

### Functions
- Composition
- Inverses, calculating inverses
- Even, odd, periodic functions
- Modulus function
- Inverse trig functions

### Exponential and Log
- Their graphs
- Standard properties
- Use in solving properties

### Coordinate Geometry
- Equations of straight lines, gradient
- Parallel and perpendicular lines
- Equation of a circle
- Circle theorems

### Parametric Equations
- Finding gradients
- Conversion from cartesian to parametric equations

### Calculus
- Differentiation of powers of $x$, $e^x$, $\ln x$, $\sin x$, $\cos x$, $\tan x$
- Product rule, quotient rule
- Chain rule
- Integration by inspection
- Integration by substitution (simple cases only)
- Integration by parts
- Differential equations (to include only variables which are separable)
- Implicit differentiation
- Volumes of revolution

### Vectors
- Scalar product
- Equations of lines
- Intersection of lines

### Numerical Methods
- Roots by sign change
- Fixed point iteration
Topics that students will not have met unless they have done AS/ A Level Further Mathematics include:

<table>
<thead>
<tr>
<th>Complex Numbers</th>
<th>Matrices</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Definitions, basic arithmetic</td>
<td>✓ Definitions, basic arithmetic</td>
</tr>
<tr>
<td>✓ Argand diagram</td>
<td>✓ Matrices as linear transformation, composition</td>
</tr>
<tr>
<td>✓ Polynomial equations with complex roots</td>
<td>✓ Determinant, inverse</td>
</tr>
<tr>
<td>✓ Polar form*</td>
<td>✓ Use in solving linear simultaneous equations, equations of planes and geometric interpretation</td>
</tr>
<tr>
<td>✓ De Moivre's theorem*</td>
<td>✓ Characteristic polynomial*</td>
</tr>
<tr>
<td>✓ Exponential notation*</td>
<td>✓ Eigenvalues and eigenvectors*</td>
</tr>
<tr>
<td>✓ n(^{th}) roots of complex numbers*</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Proof</th>
<th>Curve Sketching</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Language with proof</td>
<td>✓ Graphs of rational functions</td>
</tr>
<tr>
<td>✓ Proof by induction</td>
<td>✓ Conic sections*</td>
</tr>
<tr>
<td>✓ Proving hyperbolic trig identities</td>
<td></td>
</tr>
<tr>
<td>✓ Uniqueness of inverse matrix</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coordinate Systems</th>
<th>Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Polar coordinates</td>
<td>✓ Telescoping</td>
</tr>
<tr>
<td>✓ Intrinsic coordinates*</td>
<td>✓ Limits of series (beyond GPs)</td>
</tr>
<tr>
<td></td>
<td>✓ Maclaurin/taylor series and approximations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculus</th>
<th>Hyperbolic Trig Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Using inverse trig functions,</td>
<td>✓ Definition</td>
</tr>
<tr>
<td>hyperbolic trig functions</td>
<td>✓ Identities</td>
</tr>
<tr>
<td>✓ More advanced substitution</td>
<td>✓ Their use in calculus</td>
</tr>
<tr>
<td>✓ Integrating factor*</td>
<td></td>
</tr>
</tbody>
</table>

(*topics which are dependent on the material in the full A Level Further Mathematics having been studied, as opposed to AS Level Further Mathematics)

Depending on the specification followed and modules studied some students who have A Level Further Mathematics will also know some: Group Theory, Vector Spaces, Fields, Differential Geometry, Mutivariable Calculus, Differential Equations (including second order), Numerical Methods.

Some students, particularly those who have not studied Further Mathematics, may not have had the opportunity to have studied certain applied modules, e.g. M2, D2 to name but two, during their time in the sixth form. The following pages give information on the approximate content of the first two modules of each applied strand.
# MECHANICS

## Motion Graphs
- Displacement-time, distance-time, velocity-time
- Interpreting the graphs
- Using differentiation and integration

## Newton’s Laws Applied Along a Line
- Motion in a horizontal plane
- Motion in a vertical plane
- Pulleys
- Connected bodies

## Constant Acceleration and “SUVAIT” Equations
- Introduction to the variables
- Using the variables
- Standard properties
- Use in solving equations

## Vectors and Newton’s Laws in 2 Dimensions
- Resolving forces into components
- Motion on a slope (excluding and including friction)

## Projectiles
- Finding the maximum height, range and path of a projectile

## Collisions
- Coefficient of restitution
- Conservation of linear momentum
- Impulse
- Calculations involving a loss of energy

## Centre of Mass
- Uniform bodies (symmetry)
- Composite bodies
- Centres of mass when suspended

## Equilibrium of a Rigid Body
- Moment about a point
- Coplanar forces
- Toppling/sliding

## Variable Acceleration
- Using differentiation in 1 and 2-D
- Using integration in 1 and 2-D

## Energy, Work and Power
- Work done
- GPE, KE
- Conservation of energy
- Power (force does work)

## Uniform Motion in a Circle
- Angular speed
- Acceleration
- Horizontal circle, conical pendulum
STATISTICS

Correlation and Regression
- Product moment correlation
- Spearman coefficient rank correlation
- Independent and dependant variables
- Least squares regression
- Scatter diagrams

Data Presentation
- Bar charts, pie charts
- Vertical line graphs, histograms
- Cumulative frequency

The Binomial Distribution and Probability
- Probability based on selecting or arranging
- Probability based on binomial distribution
- Expected value of a binomial distribution
- Expected frequencies from a series of trials

Discrete Random Variables
- Expectation and variance of discrete random variables
- Formulae extensions E(aX+b)

Probability
- Measuring, estimating and expectation
- Combined probability
- Two trials
- Conditional probability
- Simple applications of laws

Exploring Data
- Types of data
- Stem and Leaf diagrams
- Measures of central tendency and of spread
- Linear coding
- Skewness and outliers

Hypothesis Testing
- Establishing the null and alternate hypothesis
- Conducting the test and interpreting the results
- Use of the binomial or normal distribution
- Type 1 and type 2 errors

Normal Distribution
- Properties (including use of tables)
- Mean and variance
- Cumulative distribution function
- Continuous random variables (probability density function and mean/variance)
- As an approximation to binomial or Poisson distributions
- t-distribution

Poisson Distribution
- Properties (including use of tables)
- Mean and variance
- Use as an approximation to binomial distribution

Sampling/ Estimation
- Randomness in choosing
- Sample means and standard errors
- Unbiased estimates of population means
- Use of central limit theorem
- Confidence intervals

Chi-squared
- Introduction
- Conditions
DECISION

**Graphs**
- Graphs

**Networks**
- Prim
- Kruskal
- Dijkstra
- TSP
- Route inspection
- Network flows

**Algorithms**
- Communicating
- Sorting
- Packing
- Efficiency and complexity

**Linear Programming**
- LP graphical
- LP simplex
- Two stage simplex

**Critical Path Analysis**
- Activity networks
- Cascade charts

**Simulation**
- Monte Carlo methods
- Uniformly distributed random variables
- Non-uniformly distributed random variables
- Simulation models

**Game Theory**
- Game theory
- Using simplex

**Optimisation**
- Matchings
- Hungarian algorithm
- Transportation problem
- Dynamic programming

**Logic and Boolean Algebra**
- Logical propositions and truth tables
- Laws of Boolean algebra
- Combinatorial circuits and switching circuits