Mathematics in the diplomas

Richard Browne - MEI

Introduction of diplomas

- The Diploma is a new applied qualification developed with employers to provide learners with subject specific skills and knowledge alongside employability skills and a self managed project.
- The Diploma is available at 3 levels:
- Foundation: equivalent to 5 GCSE grades D-G
- Higher: equivalent to 7 GCSE grades A*-C
- Advanced: equivalent to 3.5 A levels

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Diploma components

- **Principal Learning** which is subject specific and represents approximately half the Diploma study time.
- **Generic Learning** which covers personal learning and thinking skills, functional skills in English, mathematics and ICT, and a project.
- **Additional Specialist Learning (ASL)** which allows students to broaden their learning experience, or focus on a specific area of their chosen subject.

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Additional Specialist Learning

- Generally, additional specialist learning can include almost any qualification – chosen freely from a large ‘catalogue’ –
- For Engineering students, additional specialist learning can include specially reconfigured BTEC units, newly created qualifications (eg the ABC Awards qualifications in Building Services Engineering), the new *Mathematics for Engineering* qualification from OCR, or GCSEs and A levels.

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Approach to teaching

- The emphasis of the Diploma in Engineering is on applying engineering concepts to real life examples.
- It is designed to enable students to understand engineering concepts and test their understanding in applied environments.
- Teachers should provide students with real world examples of application of the theory.

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Employer engagement

- Employers have helped define the content of curriculum, and have a key role to play in making the learning ‘come alive’.
- Significant numbers of employers have been and continue to be involved, providing projects, assisting with curriculum delivery, creating work experience placements, and supplying resources.

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Consortia

- Schools and colleges link with employers and sometimes HE institutions to form consortia to deliver diploma learning.

LONDON ENGINEERING PROJECT

The Southwark & Lambeth Partnership

- Seven schools.
- Two FE Colleges
- London South Bank University
- The Royal Academy of Engineering
- TEP (Gatsby Foundation)
- Transport for London
- Tubelines

What engineering employers want

- Engineering businesses now seek engineers with abilities and attributes in two broad areas - technical understanding and enabling skills.
- The first of these comprises: a sound knowledge of disciplinary fundamentals; a strong grasp of mathematics; creativity and innovation; together with the ability to apply theory in practice.
- The second is the set of abilities that enable engineers to work effectively in a business environment: communication skills; teamworking skills; and business awareness of the implications of engineering decisions and investments.

Educating Engineers for the 21st Century, RAEng, June 2007

Mathematics content

- The principal learning component of the L3 Engineering Diploma contains one unit of mathematics, rated at 60glh, *Mathematical Techniques and Applications for Engineers*.
- This is insufficient mathematical preparation for many university courses in engineering, for which A level Mathematics has usually been essential, and at least AS Further Mathematics is also desirable

The Engineering Professors' Council

- EPC concern over falling maths standards from the early 1990s
- EPC specific concerns:
  - algebraic manipulation,
  - basic geometry and trigonometry
  - general fluency in handling number concepts

EPC's concerns about the diploma

- EPC concerned when details were first published in 2007 re:
  - the mathematics content
  - teachers' ability to deliver
  - the level of real industrial support
- Concern expressed to the Chair of the Select Committee

Maths Task Group

- EPC and the Engineering Subject Centre formed a Maths Task Group to try to address these issues
- Membership includes NCETM, ECuk, RAEng, IMA, LMS, SSCs, Deans of Science, MEI
- The Task Group quickly reached a consensus on what was required

Mathematics for Engineering

- Based on a Loughborough University Foundation Year Course
- The Loughborough course was designed for students without A level mathematics who wished to study engineering at degree level
- The subsequent degree performance of students taking this course has often exceeded that of entrants with A level Mathematics

Unit size and coverage

- The unit is 180glh (in addition to the Principal Learning mathematics unit of 60glh)
- It has coverage similar to A level Mathematics
- It is now rated for UCAS points at the rate that would usually be allocated to a unit of 270glh

Applications orientation

- Teaching the mathematics in the context of applications is seen as critical
- Exemplars are being developed for each topic to illustrate real engineering applications
- Each exemplar will be supported by a relevant industrial company – JCB, Rolls-Royce….
- I have brought hard copies of some of these

Exemplars

- May be found at the following address
  http://www.raeng.org.uk/education/diploma/maths/default.htm

Progress

- The three initial EPC concerns are all being addressed, teacher support being the most challenging
- Work is still needed to convince some university Admissions Tutors to accept the Diploma

Take up of the level 3 Engineering Diploma

- DCSF data by region for 2008 and 2009 starts

	NW	NE	Y&H	WM	EM	EE	SW	SE	L	Total
2008	74	58	35	17	54	17	35	43	87	420
2009	203	60	52	23	111	94	51	106	76	776

Support for teachers and students

- Online learning resources for the compulsory level 3 mathematics unit are now available to any school or college that registers with the Further Mathematics Support Programme.
- MEI has also completed online resources to support the *Mathematics for Engineering* unit within the level 3 Engineering Diploma. These will be made available from September 2010

Support for teachers and students

- FMSP staff ran a stand at the SSAT Engineering conference on the 4th February, to promote the FMSP's support for teachers of the level 3 Engineering Diploma
- The RAEng has appointed two field workers to support the Engineering Diploma
- The RAEng obtained funding to support the diploma, and asked MEI to appoint a person to support related mathematics work in FE colleges. This has been done

Mathematical Techniques and Applications for Engineers

Learning outcomes
- Know how to use algebraic methods to solve engineering problems
- Be able to use trigonometric methods to solve engineering problems
- Be able to use statistical methods to solve engineering problems
- Know how to apply elementary calculus techniques to solve engineering problems

The qualifications emphasise using and applying mathematics in context

- 'The unit will not just concentrate on mathematical theory, as you will be introduced to a variety of useful techniques that will help you solve real engineering problems.' Edexcel spec re MTA

MEI's online learning resources

- 18 sections covering the content of the course
- Each section includes:
  - A pdf with Notes and Examples
  - A pdf giving Crucial Points
  - A number of interactive resources
  - A section test
  - An indication of what learners should be able to do after studying the section
- In some sections there are other learning resources or links to external websites

Mathematics for Engineering

- 'The aim of this qualification is to develop learners' mathematical understanding. The qualification should be delivered in the context of Engineering, so that it can be brought alive, de-mystifying the subject and making it accessible to and achievable by, all students.' OCR spec.
- The ASL unit was intended to assess problem solving in engineering contexts

Mathematics for Engineering – learning outcomes

The learner will:

- understand the idea of mathematical modelling
- be familiar with a range of models involving change, and growth and decay
- understand the use of trigonometry to model situations involving oscillations
- understand the mathematical structure of a range of functions and be familiar with their graphs

Mathematics for Engineering – learning outcomes (cont'd)

- know how 2-D and 3-D coordinate geometry is used to describe lines, planes and conic sections within engineering design and analysis
- know how to use differentiation and integration in the context of engineering analysis and problem solving
- understand the methods of linear algebra. Know how to use algebraic processes
- understand how to describe engineering situations using statistics and use probability as a measure of likelihood

Mathematics for Engineering – learning outcomes (cont'd)

- construct rigorous mathematical arguments and proofs in engineering contexts
- comprehend translations of common realistic engineering contexts into mathematics

Mathematics for Engineering Exam Structure

- Part 1: 2 hours
- 8 – 10 compulsory questions

- Part 2: 1.5 hours
- Context is pre-released
- 4 questions testing applications ability

Thanks for taking part

The Study of Root Mean Square (RMS) Value

Mechanical, Electrical, Electronics Engineering

Introduction

The root mean square value of a quantity is the square root of the mean value of the squared values of the quantity taken over an interval. The RMS value of any function $y = f(t)$ over the range $t = a$ to $t = b$ can be defined as:

$$\text{RMS value} = \sqrt{\frac{1}{b-a}\int_a^b y^2 dt}$$

One of the principal applications of RMS values is with alternating currents and voltages.

Root Mean Square (RMS) Value

The value of an AC voltage is continually changing from zero up to the positive peak, through zero to the negative peak and back to zero again.

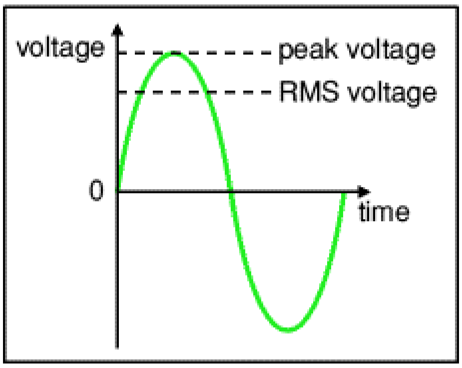


Figure-1: Difference between peak and RMS voltage

Clearly, for most of the time it is less than the peak voltage, so this is not a good measure of its real effect. Instead we use the *root mean square voltage* (V_{RMS}) which is $1/\sqrt{2} \approx 0.7$ of the *peak voltage* (V_{peak}):

$$V_{RMS} = 0.7 \times V_{peak} \text{ or } V_{peak} = 1.4 \times V_{RMS}$$

Similar equations also apply to the current. They are only true for sine waves (the most common type of AC) because the factors (here 0.7 and 1.4) take different values for other shapes. We can calculate these values as follows:

Take a sine wave representing either current or voltage with peak value *A*:

$$y(t) = A\sin\omega t \ \ldots (1)$$

where $\omega = 2\pi f$ (rads^{-1}) and f = frequency (Hz). The time average $\bar{y}(T)$ over period T (seconds) of the signal $y(t)$ is given by:

$$\bar{y}(T) = \frac{1}{T}\int_0^T y(t)dt \ \ldots (2)$$

So the RMS value y_{RMS} is given by:

$$y_{RMS} = \sqrt{\frac{1}{T}\int_0^T y^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T A^2\sin^2\omega t dt} \ \ldots (3)$$

We know that $\cos 2\theta = 1 - 2\sin^2\theta$, hence:

$$\sin^2\omega t = \frac{1}{2}(1-\cos 2\omega t) \ldots (4)$$

Substituting (4) into (3), we get, in sequence

$$y_{RMS} = \sqrt{\frac{1}{T}\int_0^T \frac{A^2}{2}(1-\cos 2\omega t)dt}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2T}\int_0^T (1-\cos 2\omega t)\ dt}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2T}\left[T - \frac{\sin 2\omega T}{2\omega}\right]}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2} - \frac{A^2\sin(2\omega T)}{4\omega T}} \ \ldots (5)$$

As $T \to \infty$, the second (oscillatory) term in equation (5) tends to zero. Hence, we obtain:

$$y_{RMS} = \frac{A}{\sqrt{2}}$$

$$\Rightarrow \text{RMS Value of y} = \frac{1}{\sqrt{2}} \times \text{Peak Value of } y$$

$$\therefore \ y_{RMS} = 0.7 \times y_{peak}$$

The *RMS value* is the effective value of a varying voltage or current. It is the equivalent steady DC (constant) value which gives the same effect. For example, a lamp connected to a *6V RMS AC* supply will shine with the same brightness when connected to a *steady 6V DC* supply. However, the lamp will be dimmer if connected to a *6V peak AC* supply because the RMS value of this is only 4.2V (it is equivalent to a steady 4.2V DC).

What do AC meters show? Is it the RMS or peak voltage?

AC voltmeters and ammeters show the *RMS value* of the voltage or current. DC meters also show the RMS value when connected to varying DC provided that the DC is varying quickly; if the frequency is less than about 10Hz you will see the meter reading fluctuating.

What does '6V AC' really mean? Is it the RMS or peak voltage?

If the peak value is meant it should be clearly stated, otherwise assume it is the *RMS value*. In everyday use, AC voltages (and currents) are always given as *RMS values* because this allows a sensible comparison to be made with steady DC voltages (and currents), such as from a battery. For example, a *6V AC supply* means 6V RMS with the peak voltage about 8.6V. The UK mains supply is 230V AC; this means 230V RMS so the peak voltage of the mains is about 320V!

So what does root mean square (RMS) really mean?

First square all the values, then find the average (mean) of these square values over a complete cycle, and finally find the square root of this average. That is the RMS value. Confused? Then just accept that RMS values for voltage and current are much more useful quantities than peak values.

IMPORTANCE OF RMS VALUE

Since a digital voltmeter measures the RMS value of a fluctuating voltage, care must be taken when finding the RMS values of very low frequency signals otherwise inaccuracies can arise. The second term in equation (5) is an oscillation of maximum amplitude $\frac{A^2}{4\omega T}$ that decreases with increasing period T. For a signal measured over a time period $T \ll \infty$, there is obvious uncertainty over how much the true RMS deviates from $\frac{A}{\sqrt{2}}$.

The maximum uncertainty is the maximum error over the assumed RMS and is given by:

$$\text{Maximum uncertainty} = \frac{\frac{A^2}{4\omega T}}{\frac{A^2}{2}} = \frac{1}{2\omega T} \quad \ldots (6)$$

So, at lower frequencies (smaller values of ω), longer averaging times are required to reduce the uncertainty of the measurements to an acceptable level.

Equivalent uncertainties may be derived for other non-sinusoidal waveforms.

WORKED EXAMPLE – 1:

A sinusoidal voltage has a maximum value of 100V. Calculate its RMS value.

Solution: A sinusoidal voltage v having a maximum value of 100V may be written as:

$$v(\theta) = 100\sin\theta$$

Over the range $\theta = 0$ to $\theta = 2\pi$ (a complete cycle), the RMS value is given by:

$$\text{RMS value} = \sqrt{\frac{1}{2\pi - 0}\int_0^{2\pi} v^2 d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (100\sin\theta)^2 d\theta}$$

$$= \sqrt{\frac{10000}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta}$$

Using equation (4), we can write:

$$\text{RMS value} = \sqrt{\frac{10000}{2\pi}\int_0^{2\pi} \frac{1}{2}(1 - \cos 2\theta)\, d\theta}$$

$$= \sqrt{\frac{10000}{2\pi}\frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{10000}{2\pi}\frac{1}{2}\left[\left(2\pi - \frac{\sin 4\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]}$$

$$= \sqrt{\frac{10000}{2\pi}\frac{1}{2}[2\pi]}$$

$$= \frac{100}{\sqrt{2}} = 70.71V$$

Please note: Being a sinusoidal wave, we can calculate the RMS value for this voltage using the relation derived on page-1, i.e.

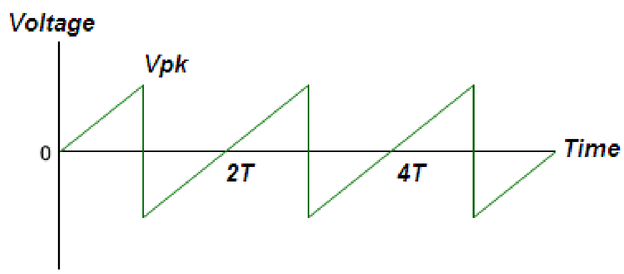
$$v_{RMS} = 0.7 \times v_{peak}$$

$$\Rightarrow v_{RMS} = 0.7 \times 100 \approx 70.71V$$

But it is not so straight-forward in case of a non-sinusoidal function as shown in the next worked example.

WORKED EXAMPLE – 2:

Find the RMS value for a voltage defined as the following saw-tooth function over an interval [0, 2T]:



$$V(t) = \begin{cases} \frac{V_{pk}}{T}t, & \text{for } 0 < t < T \\ -V_{pk} + \frac{V_{pk}}{T}(t - T), & \text{for } T < t < 2T \end{cases}$$

Solution: Since the function is symmetrical, we can find the required RMS value by finding it only in the interval [0, *T*] as follows:

$$V_{RMS} = \sqrt{\frac{1}{T-0}\int_0^T \left(\frac{V_{pk}}{T}t\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\frac{V_{pk}^2}{T^2}\int_0^T (t)^2 dt}$$

$$= \sqrt{\frac{V_{pk}^2}{T^3}\left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{V_{pk}^2}{T^3}\frac{T^3}{3}}$$

$$= \frac{V_{pk}}{\sqrt{3}}$$

Hence, if the peak value of this voltage is 200V, then its RMS value will be approximately 115V.

Extension Activity – 1:

A current, $i(t) = 30\sin 100\pi\, t$ amperes is applied across an electric circuit. Determine its mean and RMS values, each correct to 4 significant figures, over the range $t = 0$ to $t = 10$ milliseconds.

(Answers: 19.10 A, 21.21 A)

Extension Activity – 2:

A company responsible for drying the timber used in engineering applications has found that the percentage moisture in timber, *y*, is related to the number of days of drying, *x*, needed to become dry by the equation $y(x) = -x^{0.05} - 0.25x + 100$. Determine, correct up to 3 significant figures, the RMS value of moisture present in the timber between days $x = 0$ to $x = 100$. Also plot a graph of this function using any available software and estimate after how many days, the timber will be completely dry, i.e. $y = 0$.

(Answer: 86.6%, Approximately 392 days)

Extension Activity – 3:

The tidal height *H* at a particular location varies with time *t* over a fortnight as:

$$H = (4 + 3.5\cos\frac{\pi t}{7})\cos 4\pi t$$

where *t* is measured in days. To assess the location's potential as a site for generating electricity through tidal power, an engineer wants to know the RMS value of *H* over a fortnight, i.e. for $0 \le t \le 14$ days. Plot *H* as a function of *t* and find its RMS value over this period.

Where to find more

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.
3. http://www.kpsec.freeuk.com/acdc.htm

Information for Teachers

The teachers should have some knowledge of

- What RMS value is
- Trigonometrical identities
- Integration
- Plotting graphs of any function using ICT

Topics covered from "Mathematics for Engineering"

- Topic 1: Mathematical Models in Engineering
- Topic 3: Models of Oscillations
- Topic 4: Functions
- Topic 5: Geometry
- Topic 6: Differentiation and Integration

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 03: Understand the use of trigonometry to model situations involving oscillations
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 3.1: Solve problems in engineering requiring knowledge of trigonometric functions
- AC 3.2: Relate trigonometrical expressions to situations involving oscillations
- AC 4.1: Identify and describe functions and their graphs
- AC 4.2: Analyse functions represented by polynomial equations
- AC 6.3: Find definite and indefinite integrals of functions
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other Units of The Advanced Diploma in Engineering

- Unit-4: Instrumentation and Control Engineering
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

Mathematics behind the Water Network in Rural Mountain Areas

Public Health Engineering

Introduction

Some of the greatest improvements in public health in the UK in the last two hundred years were brought about by engineers. Sir Joseph William Bazalgette for instance created the sewer system in central London in the 19th century which helped to reduce the significant problems of water borne infections, such as cholera.

Things we take for granted today, such as running water in our home and flushing toilets are all down to the design vision of engineers past and present. But what about people living in areas where running water is not freely available, such as parts of the remote, rural UK or in developing countries?

Look carefully at the photograph: the tap has been left running, clothes are being washed and drainage is poor. Is this a problem? If it is, what might the solution be?

Water borne diseases remain a major problem in many parts of the world. 1.1 billion people have no access to safe water supplies and the number who do not have access to improved sanitation is even higher; 6000 children a day die because of poor water and sanitation. The World Health Organisation recommends that, in order to maintain good health, each person should have access to at least 20 and preferably 50 litres of clean water a day.

Water is a precious resource which many of us take for granted. Each year the demand for water rises as our standard of living improves. We now use 70% more than we did 30 years ago and it is estimated that the average person uses 150 litres of water every day. Disputes over water are an ever growing cause of conflict in the world.

Although water is one of the world's natural resources and falls freely to the ground, the job of making it clean and safe to drink involves sophisticated technology.

Did you know?

- On average a person in the UK uses more water per day in their home than someone living in Africa.
- Ten litres of tap water costs around 1p and can be as much as 1,000 times cheaper than soft drinks, caffeinated drinks and bottled water.

Scenario

A development organisation has been asked to develop a water supply system for a village in a remote mountain area.

The organisation has sent a team of technicians led by an engineer to do the initial survey and prepare a report. Here are some of the questions they needed to answer:

1. Is there a suitable water source to supply the village?
2. How many people live in the village?
3. How much water needs to be provided each day and what is the peak demand?
4. Should the final water outlets be public taps in the village or would it be possible to provide individual house connections?
5. How well informed are villagers about basic hygiene and sanitation so that they can make best use of a safer water supply to improve their health?
6. How can the system be managed and maintained over the longer term?

Survey Report

The team found one spring above the village which could meet at least some of the villagers' water needs. Although it was the dry season, the flow from the spring was 0.35 litres/sec giving a total of 25,920 litres/day. The villagers reported that the flow does not vary with the seasons.

The spring is 1.2km from the village and is 115m higher than the village.

They also counted 115 houses with 6 people on average in each household, giving a population of this small mountain village of 690 people. As per the recommendation from World Health Organisation, they should get at least 20 litres of clean water each day. This shows a minimum requirement of 13,800 litres of safe water each day for household purposes.

Only a very few houses have toilets of any sort, but the villagers insist that their priority is water supply and show very little interest in discussing simple toilets and their possible health impact.

The villagers currently collect water from this spring and a number of other springs further away. They collect water early in the morning and in the later afternoon.

The team did some preliminary calculations and decided that in a first phase they would propose a pipe leading from the spring directly to five public taps in the village with each tap receiving one fifth of the spring flow. In a second phase, they would recommend the construction of a storage reservoir in the village which would be filled by the pipeline with the taps connected to and therefore supplied from the storage tank.

EXTENSION ACTIVITY – 1:

- Why would this storage reservoir be so important?
- Why do you think they did not propose house connections?

APPLICATION OF MATRICES

The team did some quick calculations for the size of pipe required between the spring and the village (1.2km = 1200m as calculated above). They found that the cheapest (=smallest) pipe available was just a bit too small if they used it for the whole pipeline, but the next larger pipe which is nearly twice as expensive would be larger than required if they used it for the whole pipeline. Therefore, they decided to divide the pipeline into two lengths L_1 of the smaller pipe, and L_2 of the larger pipe such that:

$$L_1 + L_2 = 1{,}200 \quad \ldots (1)$$

When the water is flowing through long pipes, there is some friction between the flowing water and the walls of the pipe which causes pressure loss. If the water network is along the plain area, we need good pressure from the source of water to reach its destination in order to overcome this friction. But in a gravity-fed water network system, this could be achieved due to the difference in elevation between the inlet and the outlet of the water.

A commonly used formula for pressure loss in a pipe for such water networks is the Hazen-Williams formula:

$$p_{loss} = \frac{10.4 \times L \times Q^{1.85}}{C^{1.85} \times d^{4.87}}$$

where

p_{loss} = pressure loss (m)

L = length of pipe (m)

Q = water flow (m^3/sec)

C = Hazen Williams coefficient (for PVC pipes, $C = 150$)

d = internal diameter of pipe (m)

In our case, the internal diameter of smaller pipe is 15mm and external diameter is 20mm; whereas the internal diameter of the larger pipe is 25mm and external diameter is 32mm. Also, the water is flowing at the rate of 0.35 litres/sec or 0.00035 m^3/sec.

Thus, pressure loss in smaller pipe per 100m will be:

$$p_{loss} = \frac{10.4 \times 100 \times (0.00035)^{1.85}}{(150)^{1.85} \times (0.015)^{4.87}} \approx 30.22 \text{ m}$$

And the pressure loss in larger pipe per 100m will be:

$$p_{loss} = \frac{10.4 \times 100 \times (0.00035)^{1.85}}{(150)^{1.85} \times (0.025)^{4.87}} \approx 2.52 \text{ m}$$

The most economic solution can be obtained by using the smallest pipe sizes possible (in terms of diameter) since there is no point in using a larger diameter pipe which does not even run full. On the other hand, if the pipe is too small in diameter, the flow will be too less. In order to calculate the smallest pipe sizes possible, we assume that the difference in elevation between the inlet and the outlet is used to push the water through the pipe with a margin of safety of 5m (say).

Hence, the total pressure loss should be the same as the difference in level between the spring and the village with this margin of safety of 5m. This gives us a second equation relating L_1 and L_2, namely:

$$\frac{L_1 \times 30.22}{100} + \frac{L_2 \times 2.52}{100} = 115 - 5 = 110$$

or,

$$30.22L_1 + 2.52L_2 = 11{,}000 \quad \ldots (2)$$

This information can be represented as the following matrix equation:

$$\begin{bmatrix} 1 & 1 \\ 30.22 & 27.52 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 1200 \\ 11000 \end{bmatrix}$$

or, $AX = B$

We can find the unknown lengths in vector X by inverting the matrix A and then multiplying it with vector B:

$$X = A^{-1}B$$

Hence,

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \mathit{Inverse\ of} \begin{bmatrix} 1 & 1 \\ 30.22 & 2.52 \end{bmatrix} \times \begin{bmatrix} 1200 \\ 11000 \end{bmatrix}$$

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \frac{1}{-27.7}\begin{bmatrix} 2.52 & -1 \\ -30.22 & 1 \end{bmatrix} \times \begin{bmatrix} 1200 \\ 11000 \end{bmatrix}$$

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \frac{1}{-27.7}\begin{bmatrix} -7976 \\ -25264 \end{bmatrix} = \begin{bmatrix} 287.94 \\ 912.06 \end{bmatrix}$$

Hence, we get length of smaller pipe, L_1= 287.94m and the length of the larger pipe, L_2 =912.06m.

Conclusion

The calculation done above shows the most economic way to buy the pipes of two different sizes for the network under discussion. Once we have done these calculations, we can discuss the way forward for network improvement and extension possibilities. For the data under consideration, one may suggest to use the larger sized pipe (32 mm diameter) throughout the network as it will provide an opportunity for future expansion (in case, there could exist an another spring near the first one which can be added later). But others may have an opinion to fulfil the needs of the people living in the surrounding area at the optimum cost rather than investing much money at the first stage.

Extension Activity – 2:

Calculate the pressure loss per 100m for the pipes under consideration if the spring flow is 0.4 litres/sec. What would be the lengths of both types of pipe in that case?

Extension Activity – 3:

The villagers only take water for two hours in the morning from 0600 to 0800, and two hours in the evening from 1600 to 1800. In Phase-2, a reservoir will be built in the village at the end of the pipeline and close to the taps. What capacity in litres should the reservoir have to avoid wasting any water from the spring? If the reservoir is circular and has a height of 2m, what should be its diameter?

Extension Activity – 4:

In Phase-1, if the end of pipeline network serves five taps for water-collection, calculate the pressure at each tap assuming that to be same for all taps.

Compare this tap-pressure with the one in Phase-2 assuming the same number of taps serving at the end. What did you observe?

Where to find more

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.
3. *A handbook of gravity flow systems*, Thomas D. Jordan Jr, 1984, published by ITDG Publishing.
4. http://en.wikipedia.org/wiki/Hazen-Williams_equation

Tim Foster, Independent Consultant, Switzerland

Tim Foster is a qualified civil engineer who has specialised in emergency humanitarian response. He has worked for civil engineering consultants, the United Nations High Commissioner for Refugees, Oxfam, MSF, CARE and IFRC in Europe, Africa and Asia. He has been involved with RedR since the early eighties; he was the first Director of RedR International's Secretariat and is currently a RedR UK trustee.

Tim has regularly acted as a resource person, trainer and convener for RedR's emergency relief training programme and contributed to both editions of Engineering in Emergencies. More recently he has co-facilitated training workshops for Emergency Shelter Cluster Coordinators for IFRC/UNHCR. He is co-author of *Financial management for emergencies*, the lead author for *Managing people in emergencies* and helped develop and edit the *Emergency Personnel Network* website.

Recent and ongoing projects include the WASH cluster emergency materials project, the Emergency Shelter Cluster NFI project, and a review of IFRC's Emergency Shelter Cluster Coordinator deployment in Tajikistan.

Tim's key competencies are in needs assessment, programme design, management and evaluation, organisational development, training and learning.

Information for Teachers

The teachers should have some knowledge of

- Using empirical formula for calculations
- Solving simultaneous equations by matrix inversion method

Topics covered from "Mathematics for Engineering"

- Topic 1: Mathematical Models in Engineering
- Topic 7: Linear Algebra and Algebraic Processes

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 07: Understand the methods of linear algebra and know how to use algebraic processes
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 7.2: Use matrices to solve two simultaneous equations in two unknowns
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other Units of The Advanced Diploma in Engineering

- Unit-1: Investigating Engineering Business and the Environment
- Unit-3: Selection and Application of Engineering Materials
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

Answers to Extension Activities

EA1: Please discuss in class and obtain the view-points from students.

EA2: Pressure loses is 36.2m/100m and 3.36m/100m along the smaller and the larger pipes, respectively. The lengths will be xxx m for smaller pipe and xxx m for the larger pipe.

EA3: To be updated

EA4: To be updated