Mathematics in Education and Industry

Innovators in Mathematics Education

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Session B10

Curve sketching with the HP Calculator

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Investigating a family of curves

This investigation demonstrates the power of hand held technology to make the properties of a wide selection of curves readily accessible. This raises the question of what properties should mathematics educators now be regarding as important.

Exercise 1

(i) On your calculator draw circles, centre the origin, with radii 1 and 3. Set the scales so that they really are circles.

(ii) By considering $x^2 + y^2$, prove that the curve with parametric equations

$$x = 2\cos T + \sin nT,\ y = 2\sin T + \cos nT$$

will touch but not cross these circles for any positive integer values of *k*.

(iii) On your calculator draw the curve

$$x = 2\cos T + \sin 2T,\ y = 2\sin T + \cos 2T$$

along with the two circles. Find the co-ordinates of the cusps.

(iv) Now draw the curve

$$x = 2\cos T + \sin 4T,\ y = 2\sin T + \cos 4T$$

along with the two circles. Find the co-ordinates of the nodes (cross-over points).

(v) Investigate the properties of the family of curves

$$x = 2\cos T + \sin nT,\ y = 2\sin T + \cos nT.$$

Exercise 2

Now investigate the following families of curves.

(a) $$x = 2\cos T + k\sin nT,\ y = 2\sin T + k\cos nT$$

(b) $$x = 2\cos T + k\cos nT,\ y = 2\sin T + k\sin nT$$

CAS with the HP 40gs.

This investigation uses the power of the 40gs to discover some interesting numeric and algebraic concepts using CAS and the linear solver aplet.

Exercise 1

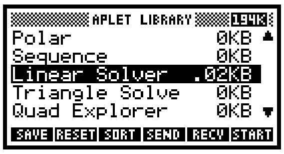
Consider the linear equations

$$3x+4y=5 \quad \text{and} \quad 11x+10y=9$$

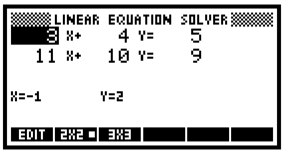
(i) What do you notice about the coefficients in each equation?

(ii) Using your knowledge of sequences what type of sequence do the coefficients make?

Using the linear solver applet and setting up a 2 × 2 matrix solve these two equations.



(iii) What are your solutions?



Now, make another set of equations with the coefficients increasing consecutively in the first and decreasing consecutively in the second.

(iv) What do you notice about the solutions?

(v) Does this happen with all equations set up in this fashion?

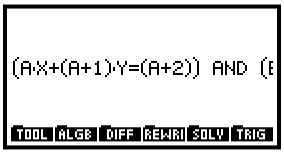
Lets see.

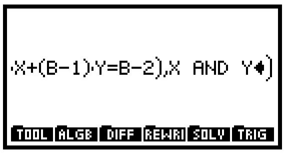
To prove this algebraically we will use CAS.

Press HOME and CAS. In the CAS screen we are going to solve the simultaneous equations $ax+(a+1)y=(a+2)$ and $bx+(b-1)y=(b-2)$.

The initial key strokes are ALPHA VARS X *x*+ (ALPHA VARS +1 highlight (A+1) X ALPHA 2 then up arrow to highlight SHIFT . (ALPHA VARS +3.
We have entered the first equation this must then be highlighted with the up arrow and press SHIFT (-)

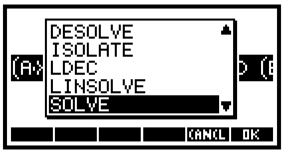
Enter the second equation type The key strokes are ALPHA MATH X x + (ALPHA MATH -1 X ALPHA 2 then up arrow to highlight SHIFT . (ALPHA MATH -3. You must then enter the unknowns that you wish to solve for. To do this press SOLV SOLVE and enter X and Y as below.

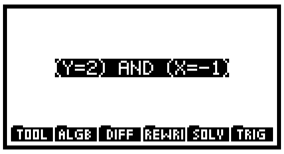




We have set up the equations in AP with the first increasing consecutively and the second decreasing consecutively. (Note there are also other algebraic combinations that can be used).

We will now solve these equations in CAS for x and y .





In the CAS screen choose SOLV menu and SOLVE and type x and y (as per the screen shots)

(vi) What are the solutions.

(vii) What can you conclude from this?

Extension 1

Consider the equations $2x+8y=14$ and $27x+20y=13$.

(i) Are the coefficients of the first equation ascending by a common difference? If so what is it?

(ii) Are the coefficients of the second equation descending by a common difference? If so what is it?

(iii) Use linear solver to find the point of intersection of the two equations.

(iv) What do you conclude?

(v) Use CAS to justify your conclusion. Hint use equations $ax+(a+d)y=a+2d$ and $bx+(b-e)y=b-2e$

Exercise 2

Consider the equations $3x+6y=12$ and $7x+21y=63$.

(i) What do you notice about the coefficients in each equation?

(ii) Using your knowledge of sequences what type of sequence does the coefficients make?

Use the linear solver aplet to solve the equations.

(iii) What are your solutions?

(iv) Would this be the same for all equations set up in this fashion? Try it with a couple of combinations of numbers.

Exercise 3

Consider the equations $2x+6y=18$ and $4x+8y=16$.

(i) How are these equations similar to the equations in exercise 2?

(ii) Record the solution when they are solved simultaneously.

(iii) What do you notice?

(iv) Prove your findings using CAS. (Hint: set up equations $ax+2ay=4a$ and $bx+3by=9b$)

So, does this work with other GP's? To do this, consider the equations $5x+20y=80$ and $7x+49y=343$. Then the equations $5x+35y=245$ and $x+4y=16$.

(v) What do these equations have in common?

(vi) Are their solutions consistent?

(vii) Prove using CAS

Extension 2

If two equations are set up with the coefficients increasing by a common ratio of $r=b$ in the first and $r=d$ in the second. Find, in terms of *b* and *d*, the point of intersection of the two equations. CAS would be a handy tool to use.

Answer key

Exercise 1

(i) The coefficients of the first equation increase by consecutive numbers and the coefficients of the second equation decrease numerically.

(ii) Arithmetic progression.

(iii) (-1,2)

(iv) Same (-1,2)

(v) Yes

(vi) When the coefficients of the first equation increase in arithmetic progression and the coefficients of the other equation decrease in arithmetic progression, the point of intersection is (1,2)

Extension 1

(i) Yes $d = 6$

(ii) Yes $d = -7$

(iii) (-1,2)

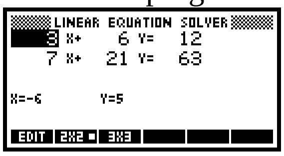
(iv) If one equation increases by a common difference of co-efficents and another equation decreases by a common difference of coefficients, the point of intersection of the equations is (-1,2).

(v) Using the equations in CAS and solving for x and y the solution is (-1,2)

Exercise 2

(vi) Increasing in by factor of 2 in the first and 3 in the second

(vii) Geometric progression

(viii) 

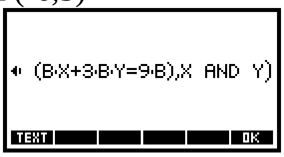
Exercise 3

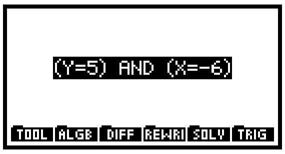
(i) They have the same common ratios as the previous equations

(ii) (-6,5)

(iii) Equations where the coefficients increase in consistent common ratio have the point of intersection (-6,5)

(iv) 





(v) Similar common ratios $r = 4$ and $r = 7$

(vi) Yes (-28,11)

Extension

Set the equation up in CAS as $ax+aby=b^2a$ and $cx+dcy=d^2c$ and solve for x and y. the solution is