Binomial and Normal distributions

Teaching Ideas

Which is more likely?

A. Roll 4 fair dice and get one 6.

B. Roll 5 fair dice and get no 6s.

By the end of this session, you will be able to work this out.

A gambling problem

In the 17th century, a French nobleman, the Chevalier de Mere, played two different games of chance.

•Rolling at least one 6 in four throws of a single die

•Rolling at least one double 6 in 24 throws of a pair of dice.

The Chevalier's reasoning

On one throw of a die,

$$\text{P(six)} = \frac{1}{6}$$

Average number of 6s in four throws =

$$4 \times \frac{1}{6} = \frac{2}{3}$$

Throwing two dice,

$$\text{P(double six)} = \frac{1}{36}$$

Average number of double 6s in 24 rolls =

$$24 \times \frac{1}{36} = \frac{2}{3}$$

Why did he lose more often on the second game?

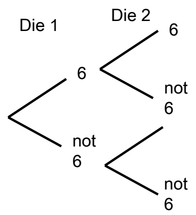
De Mere wrote to his friend Pascal

- Pascal and Fermat solved the problem between them, starting the study of probability
- For four throws of one die, find the probabilities

Number of 6s	0	1	2	3	4
probability					

Working out probabilities

The probabilities can be worked out using a tree diagram but the more dice there are the bigger it gets; this takes more time and makes it easier to go wrong.



Finding a pattern

Number of dice	No 6s	One 6	Two 6s	Three 6s	Four 6s
1	$1\times\left(\frac{5}{6}\right)$	$1\times\left(\frac{1}{6}\right)$			
2	$\left(\frac{5}{6}\right)^2$	$2\times\left(\frac{1}{6}\right)\times\left(\frac{5}{6}\right)$	$\left(\frac{1}{6}\right)^2$		
3	$\left(\frac{5}{6}\right)^3$	$3\times\left(\frac{1}{6}\right)\times\left(\frac{5}{6}\right)^2$	$3\times\left(\frac{1}{6}\right)^2\times\left(\frac{5}{6}\right)$	$\left(\frac{1}{6}\right)^3$	
4	$\left(\frac{5}{6}\right)^4$	$4\times\left(\frac{1}{6}\right)\times\left(\frac{5}{6}\right)^3$	$6\times\left(\frac{1}{6}\right)^2\times\left(\frac{5}{6}\right)^2$	$4\times\left(\frac{1}{6}\right)^3\times\left(\frac{5}{6}\right)$	$\left(\frac{1}{6}\right)^4$
5					

In general

$P(X = r) = {}^nC_r\, p^r\, q^{n-r}$

$r = 0, 1, 2, \ldots\ldots\ldots\, n$

where p is the probability of success

and q is the probability of failure

$X \sim B(n, p)$ shows that the random variable, X, has a binomial distribution with n trials and probability p of success each time.

What is success?

In this case, 6 on a die.

What is failure?

In this case “not 6” on a die.

p = P (success) = 1/6 in this case

q = P (failure) = 5/6 in this case

r = 0, 1, 2, 3 $\quad$ n = 3 $\quad$ for 3 dice

Back to the first question $P(X = r) = {}^nC_r\, p^r\, q^{n-r}$

A. Roll 4 fair dice and get one 6.

$n =$ $\quad$ $p =$ $\quad$ $r =$

B. Roll 5 fair dice and get no 6s.

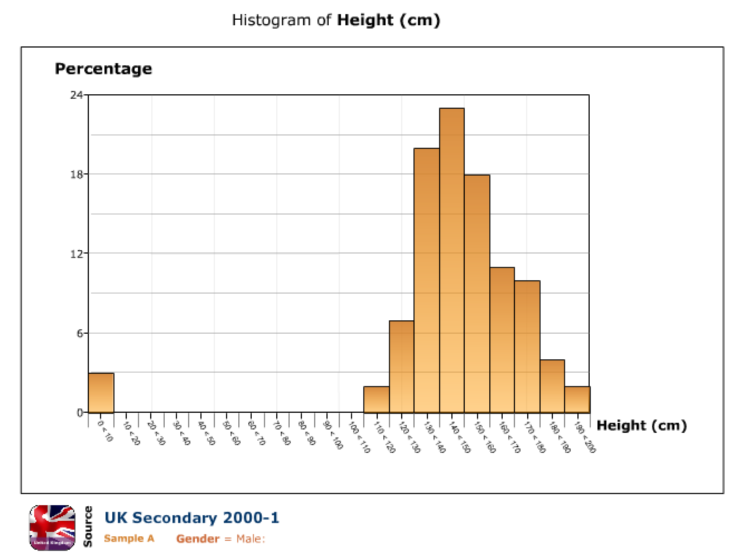
$n =$ $\quad$ $p =$ $\quad$ $r =$

Where does the Binomial distribution occur?

- You are conducting an experiment or trial n times
  - Eg tossing a fair coin 8 times
- There are 2 outcomes, which we can think of as “success” or “failure”
  - Eg heads and tails
- The probability of “success” is the same each time (symbol p). The probability of “failure” is 1-p
  - The probability of “success” on any trial is independent of what has happened in previous trials.
- The random variable we are interested in is “the number of successes”

Situation: Toss a biased coin 10 times **Random variable:** The number of heads	**Situation:** Throw a fair die 7 times **Random variable:** The number of 4s
Situation: Throw a fair die 12 times **Random variable:** The number of even scores	**Situation:** Throw a fair die **Random variable:** The score
Situation: Throw a fair die repeatedly **Random variable:** The number of times you have to throw it to get a 6	**Situation:** Choose 6 students from a class of 30 **Random variable:** The number of girls chosen

Situation: Choose 6 students from a large school **Random variable:** The number of boys chosen	**Situation:** Open a tube of “smarties” and count the number of each colour **Random variable:** The number of orange smarties
Situation: Choose a jury **Random variable:** The number of black people on the jury	**Situation:** Take a 40 question multiple choice test by guessing each answer **Random variable:** The number of questions wrong
Situation: Each student in the class tosses a coin **Random variable:** The number of heads	**Situation:** The probability of my suitcase getting to the right destination is 0.95. **Random variable:** The number of passengers on a flight with “lost” luggage



These 4 samples are taken from the Census at School database.

What population is the sample from? Do you think the sample is from a Normal population? Why?

http://datatool.censusatschool.org.uk/datatool.swf