

<h3>Algebra and functions</h3> <ul style="list-style-type: none"> <li>• expanding and factorising</li> <li>• surds</li> <li>• rationalising</li> <li>• laws of indices</li> <li>• dividing a polynomial</li> <li>• factor theorem</li> <li>• remainder theorem</li> </ul>	<h3>Sine and cosine rule</h3> <ul style="list-style-type: none"> <li>• using both to find missing sides and angles</li> <li>• finding two solutions for missing angles</li> <li>• area of a triangle = <math>\frac{1}{2} ab \sin\theta</math></li> </ul>
<h3>Quadratic functions</h3> <ul style="list-style-type: none"> <li>• graphs of quadratic functions</li> <li>• solving equations by factorising</li> <li>• completing the square</li> <li>• use of the formulae</li> <li>• sketching graphs of quadratic formulae</li> </ul>	<h3>Exponentials and logarithms</h3> <ul style="list-style-type: none"> <li>• function <math>y=a^x</math></li> <li>• writing expressions as logs</li> <li>• calculating using logs to base 10</li> <li>• laws of logs</li> <li>• solving equations of the form <math>a^x=b</math></li> <li>• changing base of logs</li> </ul>
<h3>Equations and inequalities</h3> <ul style="list-style-type: none"> <li>• simultaneous equations by elimination</li> <li>• simultaneous equations by substitution</li> <li>• linear and quadratic simultaneous equations</li> <li>• linear equalities</li> <li>• quadratic inequalities</li> </ul>	<h3>Binomial Expansion</h3> <ul style="list-style-type: none"> <li>• Pascal's triangle</li> <li>• Combinations and factorial rule</li> <li>• using <math>\binom{n}{r}</math> in the binomial; expansion</li> <li>• expanding <math>(a+bx)^n</math></li> </ul>
<h3>Sketching curves</h3> <ul style="list-style-type: none"> <li>• cubic functions</li> <li>• reciprocal graphs and functions</li> <li>• intersection points of graphs to solve equations</li> <li>• transformations <math>f(x+a)</math>, <math>f(x-a)</math>, <math>f(ax)</math>, <math>af(x)</math></li> </ul>	<h3>Radian measure and graphs of trig functions</h3> <ul style="list-style-type: none"> <li>• radians to measure angles</li> <li>• length of an arc</li> <li>• area of a sector</li> <li>• area of a segment</li> <li>• graphs of sine, cos and tan</li> <li>• values of functions in four quadrants</li> <li>• surd values for functions</li> <li>• transformations of sin, tan and cos curves</li> </ul>

## Coordinate geometry

- straight line in form  $ax+by+c=0$
- gradient of a straight line
- $y-y_1=m(x-x_1)$
- equation of a straight line
- parallel and perpendicular
- midpoint of a line
- distance between two points on a line
- equation of a circle

## Differentiation

- increasing and decreasing functions
- stationary points, max, min and inflexion
- using turning points to solve problems
- derivative of  $f(x)$  as the gradient of the tangent to the graph  $f(x)=y$
- formula for gradient of  $ax^n$
- expanding or simplifying functions to make them easier to differentiate
- second order derivatives
- rate of change at a particular point
- equation of tangent and or normal at a point

## Sequences and series

- nth term
- recurrence
- arithmetic sequences
- arithmetic series
- sum to  $n$  of an arithmetic series
- using  $\Sigma$  notation
- geometric sequences
- geometric series
- sum of a geometric series
- sum to infinity of geometric series

## Trig Identities

- simple trig identities
- solving simple trig equations
- solving equations in the form  $\sin(n\theta+a)$ ,  $\cos(n\theta+a)$  and  $\tan(n\theta+a) = k$
- solving quadratic trig equations

## Integration

- simple definite integration
- area under a curve
- area under a curve negative values
- area between a line and a curve
- the trapezium rule
- integrating  $x^n$
- integrating simple expressions
- using the integral sign
- simplifying expressions before integrating
- finding the constant of integration

<h3>Parametric equations</h3> <ul style="list-style-type: none"> <li>• Using parametric equations</li> <li>• Conversion to Cartesian</li> <li>• Finding the area under a curve given by parametric equations</li> </ul>	<h3>Functions</h3> <ul style="list-style-type: none"> <li>• mapping diagrams</li> <li>• range, mapping diagrams and graphs</li> <li>• composite functions</li> <li>• Inverse functions</li> </ul>
<h3>The binomial expansion</h3> <ul style="list-style-type: none"> <li>• Expanding <math>(a+bx)^n</math></li> <li>• Use of partial fractions</li> </ul>	<h3>Exponential and log functions</h3> <ul style="list-style-type: none"> <li>• <math>y=a^x</math></li> <li>• <math>y=e^x</math></li> <li>• Using <math>e^x</math> and the inverse of the exponential function <math>\log_e x</math></li> </ul>
<h3>Differentiation</h3> <ul style="list-style-type: none"> <li>• Parametric differentiation</li> <li>• Implicit relations</li> <li>• <math>a^x</math></li> <li>• Rates of change</li> <li>• Simple differential equations</li> </ul>	<h3>Numerical methods</h3> <ul style="list-style-type: none"> <li>• finding approximate roots of <math>f(x) = 0</math> graphically</li> <li>• Iterative and algebraic methods to find approximate roots of <math>f(x)=0</math></li> </ul>
<h3>Vectors</h3> <ul style="list-style-type: none"> <li>• Diagrams</li> <li>• Unit vectors</li> <li>• 2D &amp; 3D</li> <li>• Scalar product</li> <li>• Vector equation of a straight line</li> <li>• Angle between two lines</li> </ul>	<h3>Transforming graphs of functions</h3> <ul style="list-style-type: none"> <li>• Modulus function <math> f(x) </math> and <math>f( x )</math></li> <li>• Solving equations involving a modulus</li> <li>• Applying combinations of transformations to curves</li> </ul>
<h3>Integration</h3> <ul style="list-style-type: none"> <li>• Standard functions</li> <li>• Reverse chain rule</li> <li>• Trigi identities</li> <li>• Partial fractions</li> <li>• Substitution</li> <li>• By parts</li> <li>• Numerical</li> <li>• Areas and volumes</li> <li>• Differential equations</li> </ul>	<h3>Trigonometry</h3> <ul style="list-style-type: none"> <li>• functions of secant, cosecant and cotangent</li> <li>• graphs of secant, cosecant and cotangent</li> <li>• Simplifying expressions, proving identities and solving equations using sec, cosec and cot</li> <li>• using inverse trig functions and their graphs</li> </ul>

Partial fractions	Algebraic fractions
Further trig identities <ul style="list-style-type: none"><li>• Addition trig formulae</li><li>• double angle formulae</li><li>• solving equations and proving identities using double angle</li><li>• using a <math>a\cos\theta + b\sin\theta</math></li><li>• factor formula</li></ul>	Differentiation <ul style="list-style-type: none"><li>• chain rule</li><li>• product rule</li><li>• quotient rule</li><li>• exponential function</li><li>• logarithmic function</li><li>• differentiating <math>\sin x</math>, <math>\cos x</math>, <math>\tan x</math></li><li>• further trig functions</li></ul>

## Functions challenge

The function  $f(x) = 2x + 4$  can be written using a function machine like this:



You can reverse this by undoing the operations like this. This is called finding the inverse.



We say the inverse function is  $f^{-1}(x) = \frac{x-4}{2}$

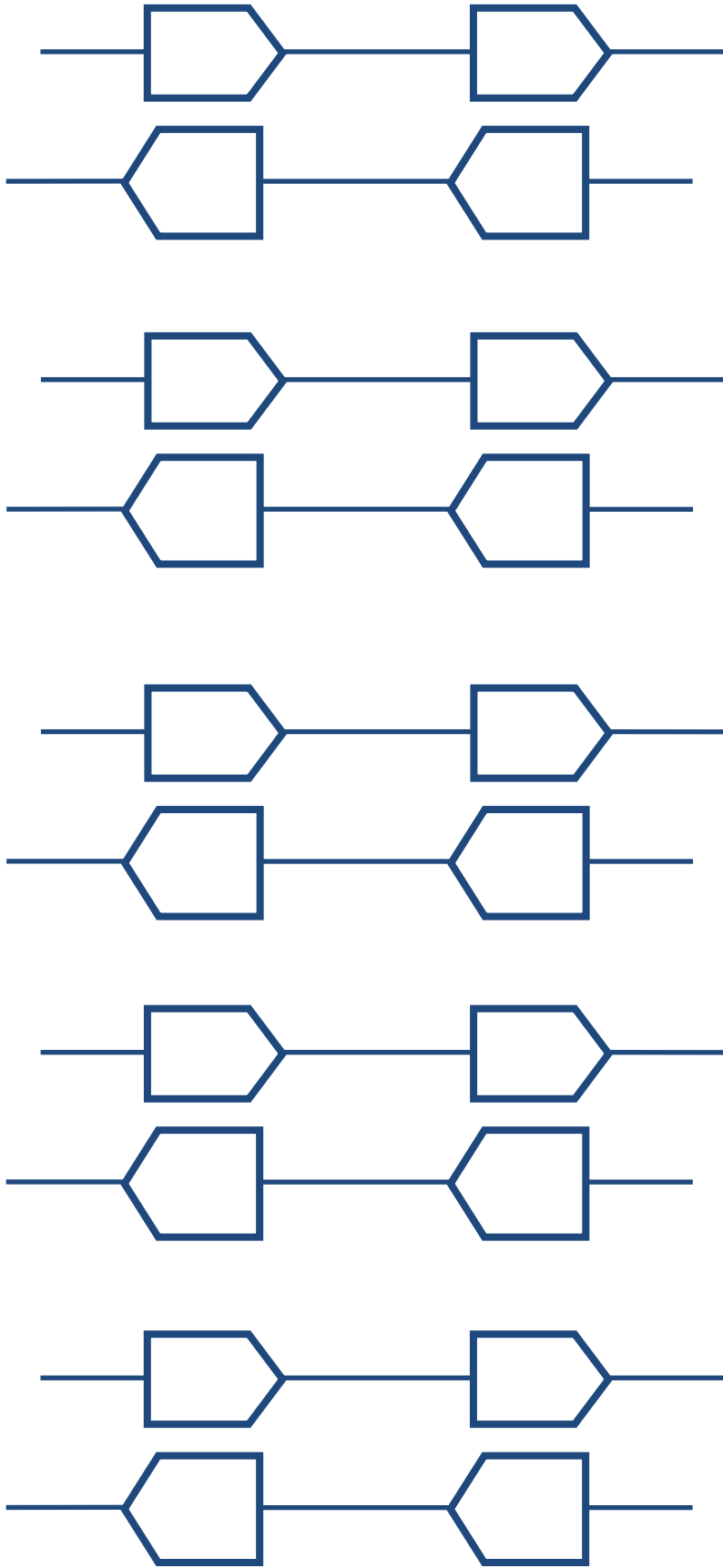
Can you find the inverses of these functions? Use the function machines if you want.

1.  $f(x) = 3x+5$
2.  $f(x) = 4x+7$
3.  $f(x) = \frac{x}{2} + 1$
4.  $f(x) = \frac{x+2}{3}$
5.  $f(x) = \frac{2}{3}x + 3$
6.  $f(x) = 3 - 2x$
7.  $f(x) = x^2$
8.  $f(x) = \sin x$

Draw a graph of a function and its inverse – on the same pair of axes – what do you notice?

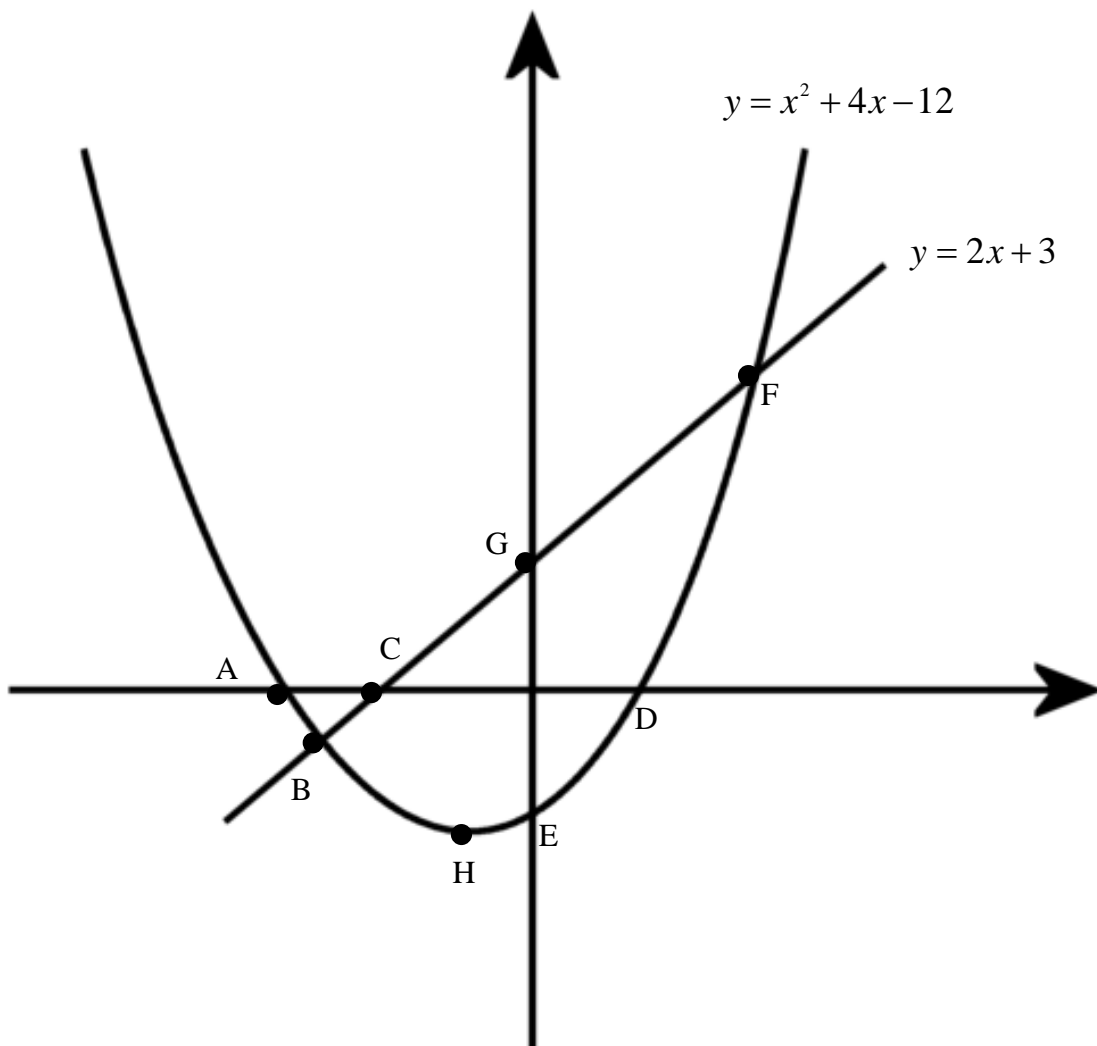
What is different about numbers 7 and 8 – why do you think this happens?

# Function machines



## Quadratic Challenge I

Can you work out the coordinates of all the points named with letters below?



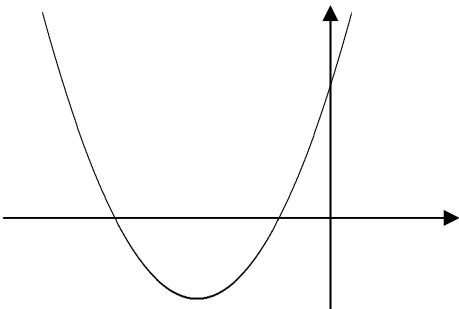
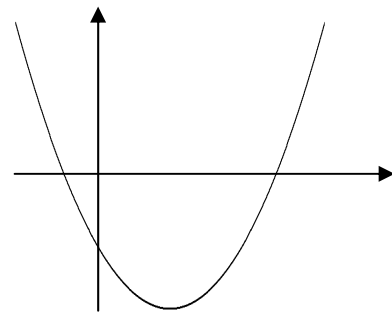
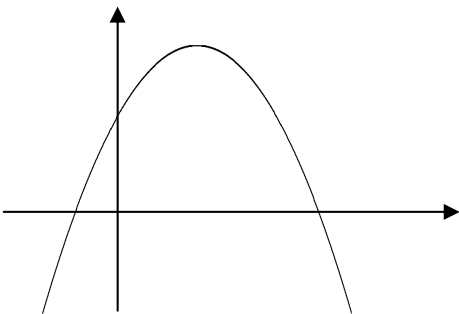
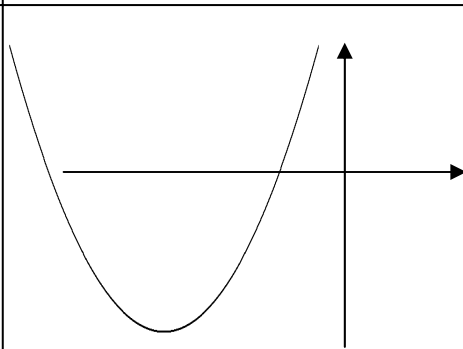
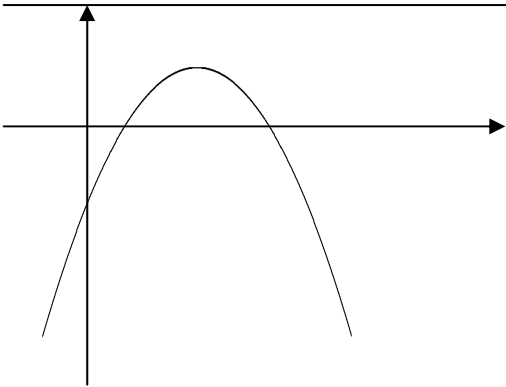
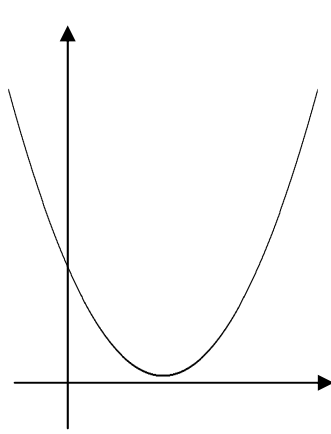
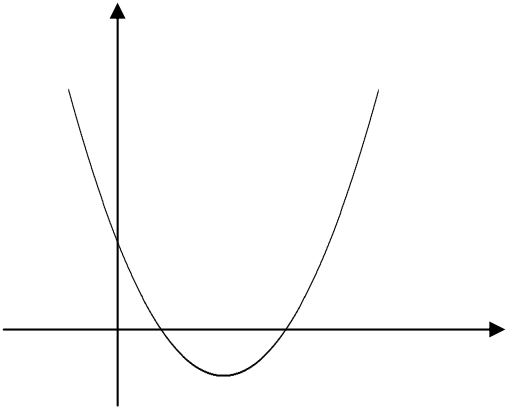
## Quadratic Challenge II

Cut out all the cards. Can you match all the cards?

$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	$y = (x - 8)(x + 2)$
$y = (x + 4)(x + 2)$	$y = (x + 2)(4 - x)$
$y = (x - 4)(2 - x)$	$y = (x - 8)(x - 2)$
$y = (x - 4)(x - 4)$	$y = (x + 8)(x - 2)$
$y = (x + 3)^2 - 25$	$y = (x - 4)^2$
$y = (x - 5)^2 - 9$	$y = -(x - 3)^2 + 1$
$y = -(x - 1)^2 + 9$	$y = (x + 3)^2 - 1$



$y = (x - 3)^2 - 25$	<b>Minimum at (3, -25)</b>
<b>Minimum at (-3, -1)</b>	<b>Maximum at (1, 9)</b>
<b>Maximum at (3, 1)</b>	<b>Minimum at (5, -9)</b>
<b>Minimum at (4, 0)</b>	<b>Minimum at (-3, -25)</b>
$x = 0, y = -16$	$x = 0, y = 16$
$x = 0, y = 16$	$x = 0, y = -8$
$x = 0, y = 8$	$x = 0, y = 8$
$x = 0, y = -16$	$y = 0, x = 8$ or $-2$
$y = 0, x = -4$ or $-2$	$y = 0, x = -2$ or $4$
$y = 0, x = 4$ or $2$	$y = 0, x = 8$ or $2$
$y = 0, x = 4$	$y = 0, x = -8$ or $2$



## Simultaneous Equations Challenge

You can now solve pairs of simultaneous equations. What happens if there are more than two? Like these:

$$3x+4y+z=3$$

$$x+y+z=2$$

$$2x+y-z=2$$

The solution to this problem is on worksheet A but it is in the wrong order. Can you sort it out so that it is in the right order? Then try worksheet B which uses a different method.

Which method seems to work best for you?

Now try these – can you solve them? Use one of the two methods that you explored on sheets A and B.

$$x-y+zx=10$$

$$3x+y+2z=34$$

$$-5x+2y-z=-14$$

$$x+y+2z=11$$

$$2x-3y-z=-9$$

$$2x-y+3z=7$$

Type the equations above into a graph plotting programme like Autograph or Omnigraph.

- What can you see?
- Is it what you expected?
- Why is there only one solution to the set of equations?
- Can you think of a situation where there might be a different number of solutions?
- Can you describe a situation when there are no solutions – what would it look like?

## Worksheet A

$$3x+4y+z=3 \text{ – equation 1}$$

$$x+y+z=2 \text{ – equation 2}$$

$$2x+y-z=2 \text{ – equation 3}$$

Solve to get

$$x=2 \text{ and } y=-1$$

Take away equation 2  
from equation 1

$$\begin{array}{r} 2x+y-z=2 \\ + \quad x+y+z=2 \\ \hline \rightarrow 3x+2y=4 \end{array}$$

You now have a pair of simultaneous equations and you know how to solve these.

$$2x+3y=1 \quad \& \quad 3x+2y=4$$

Add equation 3 to  
equation 2

Solution is  $x=2$ ,  $y=-1$  and  $z=1$

$$3x+4y+z=3$$

$$6 + -4 + z = 3$$

$$z = 1$$

Pick any of the three  
equations – for example  
equation 1 :  $3x+4y+z=3$

Substitute in the values  
of  $x$  and  $y$

$$3x+4y+z=3$$

$$- \quad x+y+z=2$$

$$\rightarrow 2x+3y=1$$

## Worksheet B

$$3x+4y+z=3 \text{ – equation 1}$$

$$x+y+z=2 \text{ – equation 2}$$

$$2x+y-z=2 \text{ – equation 3}$$

*Now substitute into equation 2*

$$x+y+z=2 \text{ – equation 2}$$

$$x+y+(3 - 3x - 4y)=2$$

$$-2x-3y+3=2$$

$$-2x-3y=-1$$

*We know*

$$z = 3 - 3x - 4y$$

*Put in values of  $x=2$  and  $y=-1$   
and get  $z=1$*

$$3x+4y+z=3 \text{ – equation 1}$$

$$z = 3 - 3x - 4y$$

*Solution is  $x=2$ ,  $y=-1$  and  $z=1$*

*Solve the two new equations you have made*

$$-2x-3y=-1 \text{ \& } x+y=1 \text{ to get}$$

$$x=2 \text{ and } y= -1$$

*Now substitute into equation  $2x+y-z=2$*

*– equation 3*

$$2x+y - (3 - 3x - 4y )=2$$

$$5x+5y-3=2$$

$$5x+5y=5$$

$$x+y=1$$

*Take equation 1 and rearrange  
it to make  $z$  the subject of the  
formula*