Graph Theory Word Maze

There are 20 words associated with Graph Theory hidden in the square below.
The Start and Finish squares are indicated and the words lie in a continuous path from one to the other. All moves are horizontal or vertical (not diagonal.).
The initial letter of each word and the order in which they appear are given to stop you getting lost!

Start	A	R	L	A	N	E	B	I	P	A	R	
	E	C	P	C	Y	C	T	O	R	E	T	
	D	G	E	L	E	E	N	U	T	T	I	
	D	E	E	R	T	C	A	W	E	G	R	
	E	A	I	D	J	A	L	K	D	I	A	
	G	R	L	A	L	P	M	O	C	H	P	
	R	T	P	O	E	T	E	I	M	P	L	
	E	E	L	O	R	E	V	S	A	H	E	
	N	N	X	E	T	O	D	E	M	N	I	
	E	O	C	E	U	N	N	A	I	O	A	
	C	T	E	D	L	E	R	I	L	T	N	Finish

1. A
2. E
3. P
4. C
5. T
6. D
7. L
8. T
9. A
10. B
11. R
12. W
13. D
14. C
15. V
16. C
17. E
18. N
19. S
20. H

Solution

Start

A	R	L	A	N	E	B	I	P	A	R
E	C	P	C	Y	C	T	O	R	E	T
D	G	E	L	E	E	N	U	T	T	I
D	E	E	R	T	C	A	W	E	G	R
E	A	I	D	J	A	L	K	D	I	A
G	R	L	A	L	P	M	O	C	H	P
R	T	P	O	E	T	E	I	M	P	L
E	E	L	O	R	E	V	S	A	H	E
N	N	X	E	T	O	D	E	M	N	I
E	O	C	E	U	N	N	A	I	O	A
C	T	E	D	L	E	R	I	L	T	N

Finish

1.	**A**	R	C								
2.	**E**	D	G	E							
3.	**P**	L	A	N	E						
4.	**C**	Y	C	L	E						
5.	**T**	R	E	E							
6.	**D**	E	G	R	E	E					
7.	**L**	O	O	P							
8.	**T**	R	A	I	L						
9.	**A**	D	J	A	C	E	N	T			
10.	**B**	I	P	A	R	T	I	T	E		
11.	**R**	O	U	T	E						
12.	**W**	A	L	K							
13.	**D**	I	G	R	A	P	H				
14.	**C**	O	M	P	L	E	T	E			
15.	**V**	E	R	T	E	X					
16.	**C**	O	N	N	E	C	T	E	D		
17.	**E**	U	L	E	R	I	A	N			
18.	**N**	O	D	E							
19.	**S**	I	M	P	L	E					
20.	**H**	A	M	I	L	T	O	N	I	A	N

Semi Eulerian Graph

Line from vertex to vertex

... can be drawn without edges crossing

More than one edge between the same pair of vertices

The number associated with an edge

... where path exists between any pair of vertices

Bipartite Graph

A subset of the vertices and edges of a larger graph

Minimum Spanning Tree

A walk with no repeated edges

Walk

Trail with no repeated vertices A

Point where edges meet

Node Value

Network

Simple Graph

K_5

Digraph

Graph with directional edges

Graph with no cycles

Trail

Weight

Connected graph with no odd nodes

Connected Graph with one pair of odd vertices

Tree

... connects all vertices with the least total edge length

Eulerian Cycle

Node

Vertex with odd degree

Cycle

Path

Odd node

Node with edge4 connections

Vertex

Graph where two distinct sets of vertices are not joined by any common edges

Complete graph with five vertices

Complete Graph

Arc

... traverses every edge exactly once

Edge

Edge

A sequence of consecutive edges

... where every vertex is joined to every other vertex

... has no loops or repeated edges

Subgraph

a closed path

Hamiltonian Cycle

Vertex of degree 4

Isomorphic Graphs

Graph consisting of arcs and nodes

Valency

Planar Graph

Connected Graph

Different shaped graphs with the same vertex connections

Eulerian Graph

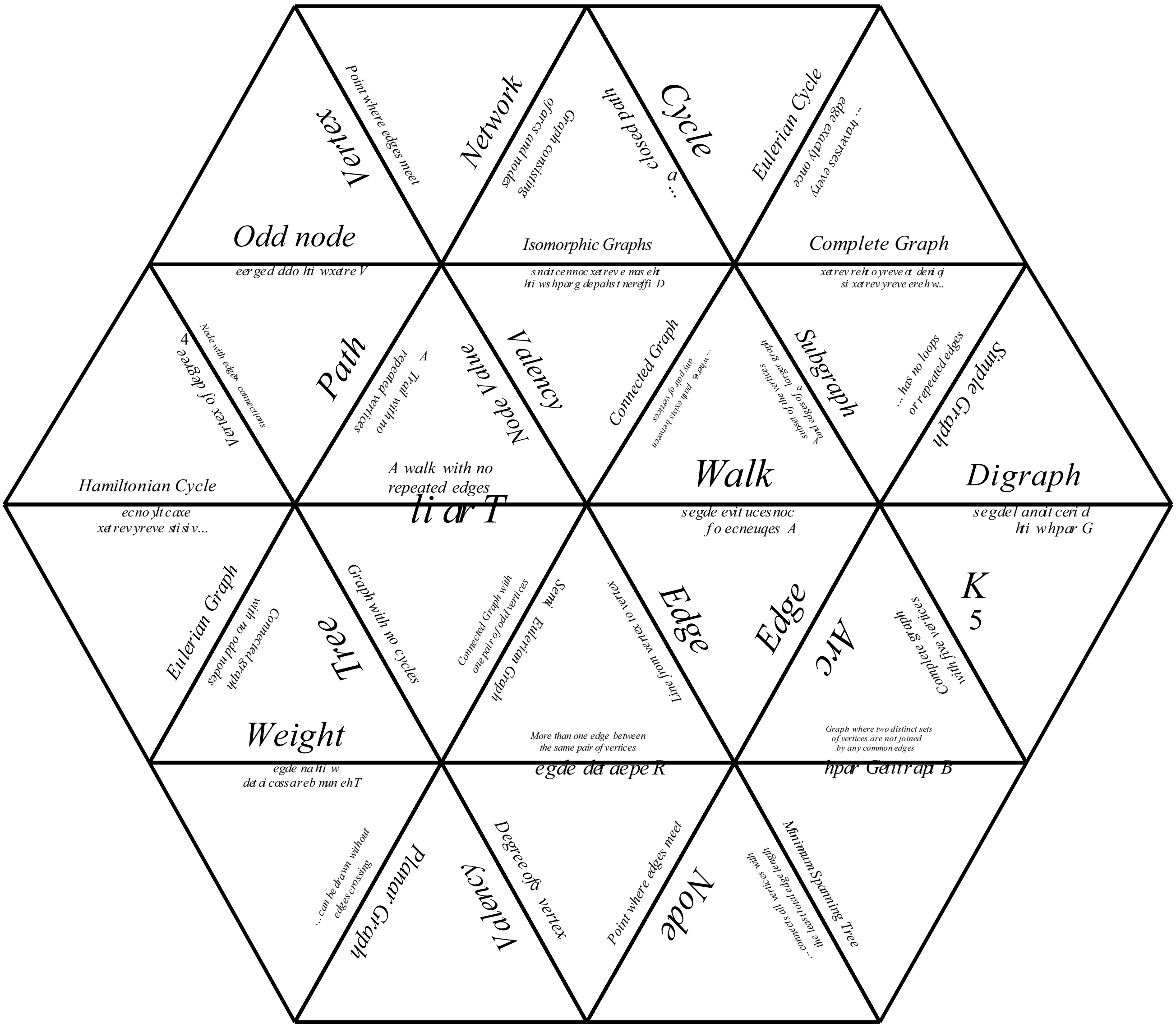
Valency

Degree of a vertex

Point where edges meet

Repeated edge

... visits every vertex exactly once


Vertex
Point where edges meet
Network
Graph consisting of arcs and nodes
Cycle
a ... closed path
Eulerian Cycle
... traverses every edge exactly once
Odd node
Vertex with odd degree
Isomorphic Graphs
Different shaped graphs with the same vertex connections
Complete Graph
... where every vertex is joined to every other vertex
Vertex of degree 4
Node with 4 edges connections
Path
A Trail with no repeated vertices
Node Value
Valency
Connected Graph
... where a path exists between any pair of vertices
Subgraph
A subset of the vertices and edges of a larger graph
... has no loops or repeated edges
Simple Graph
Hamiltonian Cycle
A walk with no repeated edges
Trail
Walk
Digraph
... visits every vertex exactly once
A sequence of connective edges
Graph with directed edges
Eulerian Graph
Connected graph with no odd nodes
Tree
Graph with no cycles
Connected Graph with one pair of odd vertices
Semi Eulerian Graph
Line from vertex to vertex
Edge
Edge
Arc
K
5
Complete graph with five vertices
Weight
More than one edge between the same pair of vertices
Graph where two distinct sets of vertices are not joined by any common edges
The number associated with an edge
Repeated edge
Bipartite Graph
... can be drawn without edges crossing
Planar Graph
Valency
Degree of a vertex
Point where edges meet
Node
Minimum Spanning Tree
... connects all vertices with the least total edge length

Bin Packing Exercise

Where objects of varying sizes must be placed into containers of a fixed capacity, the problem is described as 'bin packing'. The name is used for any problem of this general type, whether to do with objects, lengths, times or whatever the scenario.

Two algorithms are commonly used to attempt to solve these problems:

First-Fit Bin Packing:

> Number the bins, then always place the next item in the lowest numbered bin which can take that item.

First-Fit Decreasing Bin Packing:

> Reorder the items into decreasing order of size.
>
> Number the bins, then always place the next item in the lowest numbered bin which can take that item.

Activity

A builder uses piping of standard length 12 metres.
The following sections of varying lengths are required for a particular job:

Section	A	B	C	D	E	F	G	H	I	J	K	L
Length (metres)	2	2	3	3	3	3	4	4	4	6	7	7

Explore how many standard 12 m lengths of pipe will be required if each of the following methods is used:

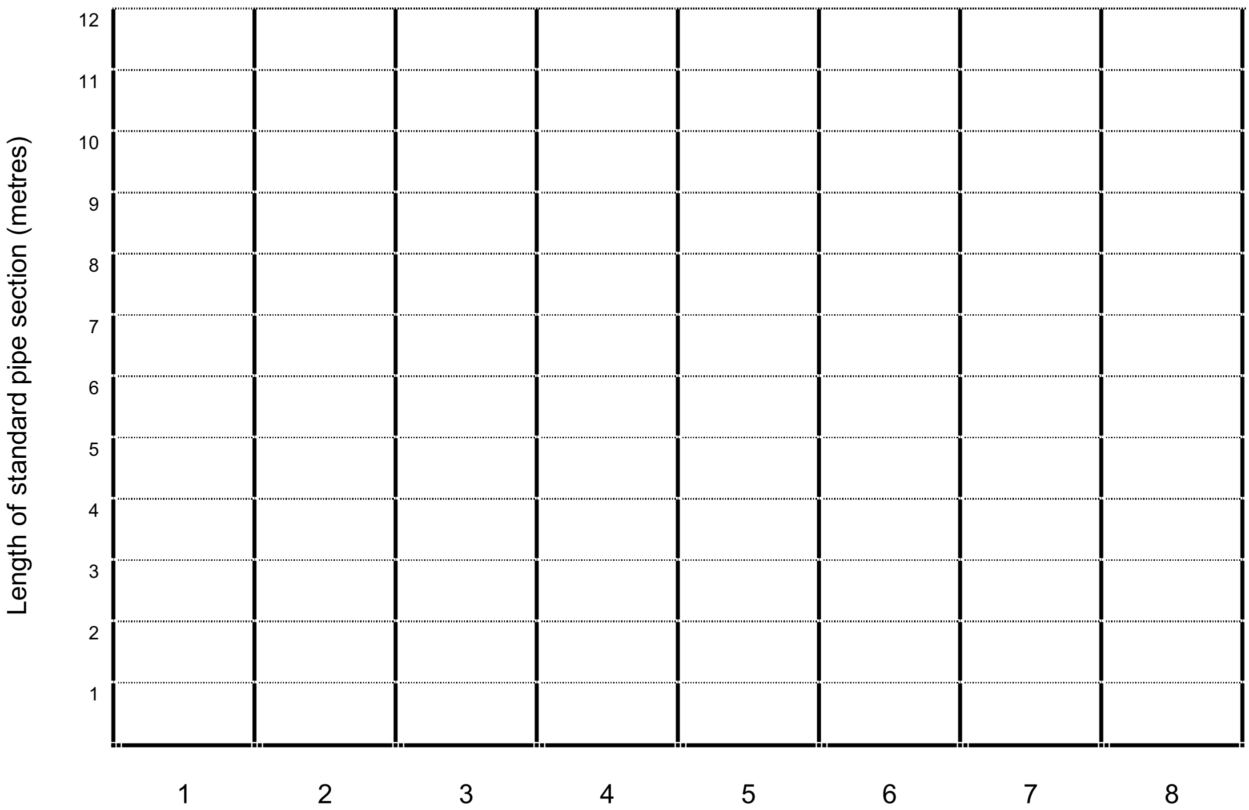
(a) First-fit bin packing

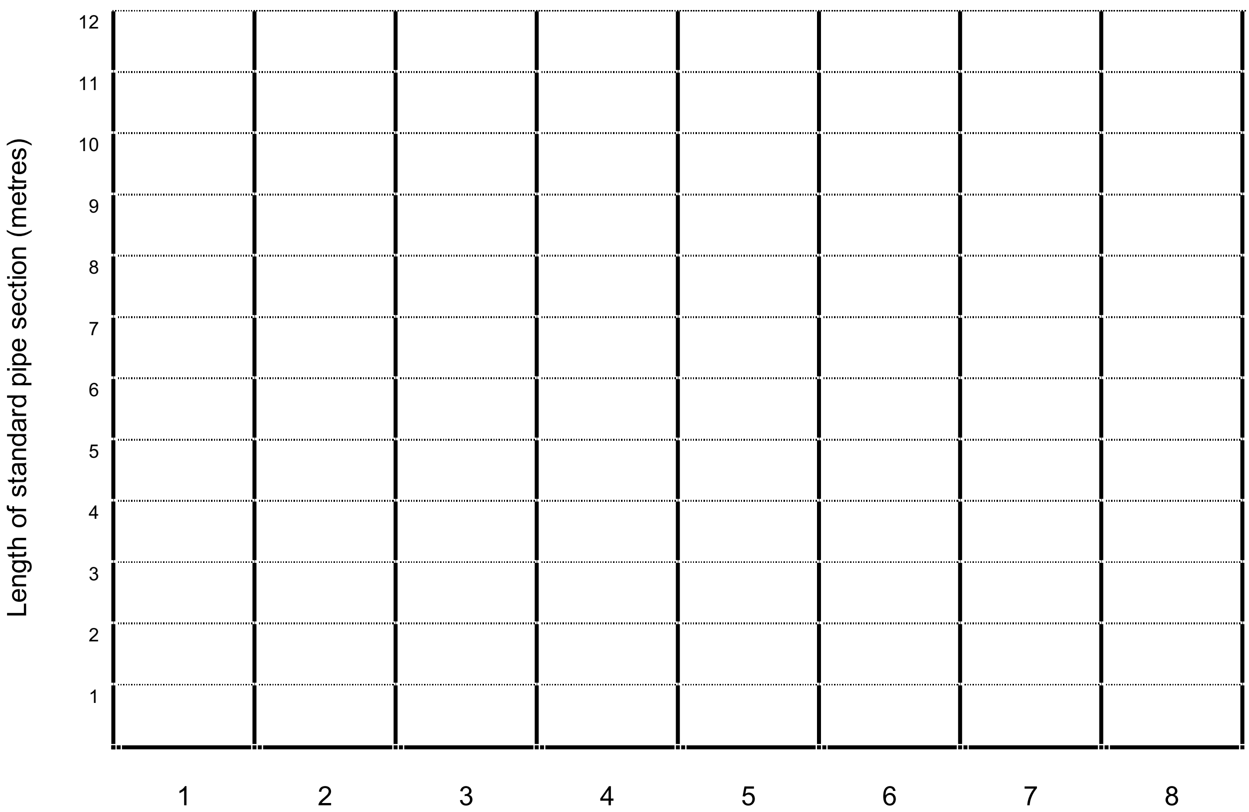
(b) first-fit decreasing bin packing

(c) trial and improvement

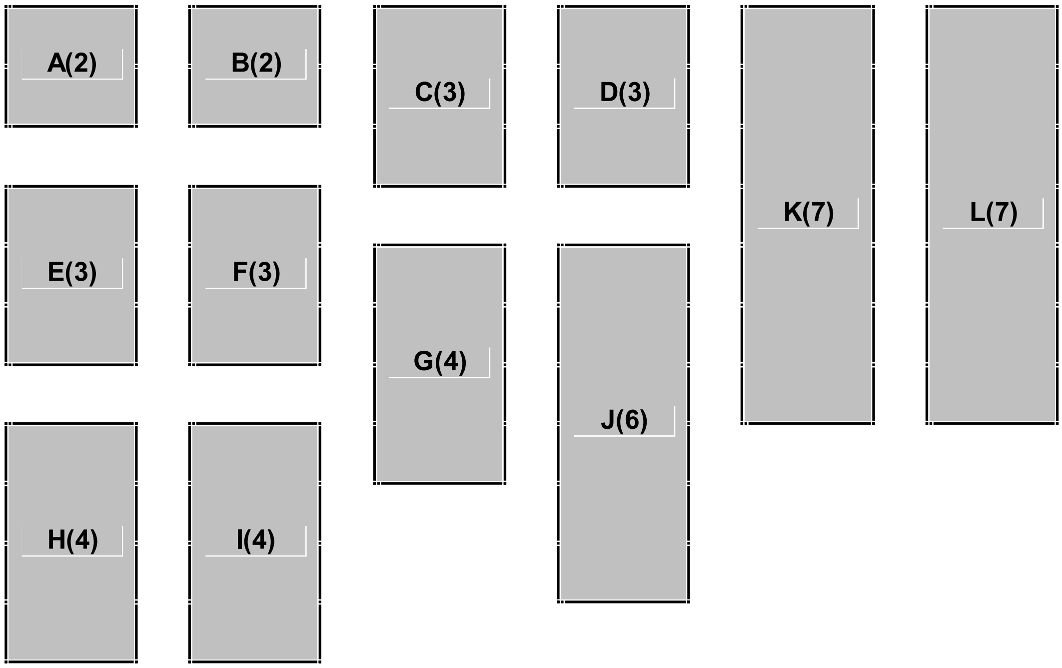
[Adapted from p. 209-210, AEB Discrete Mathematics, Heinemann, 1992]

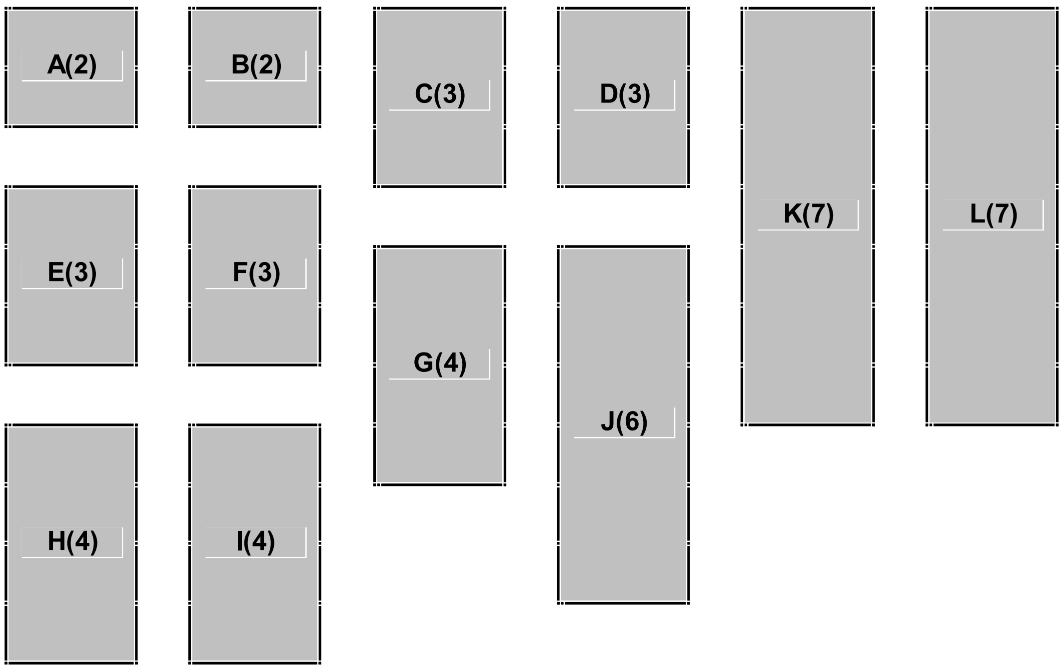
Bins


Length of standard pipe section (metres)
12
11
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8


Length of standard pipe section (metres)
12
11
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8

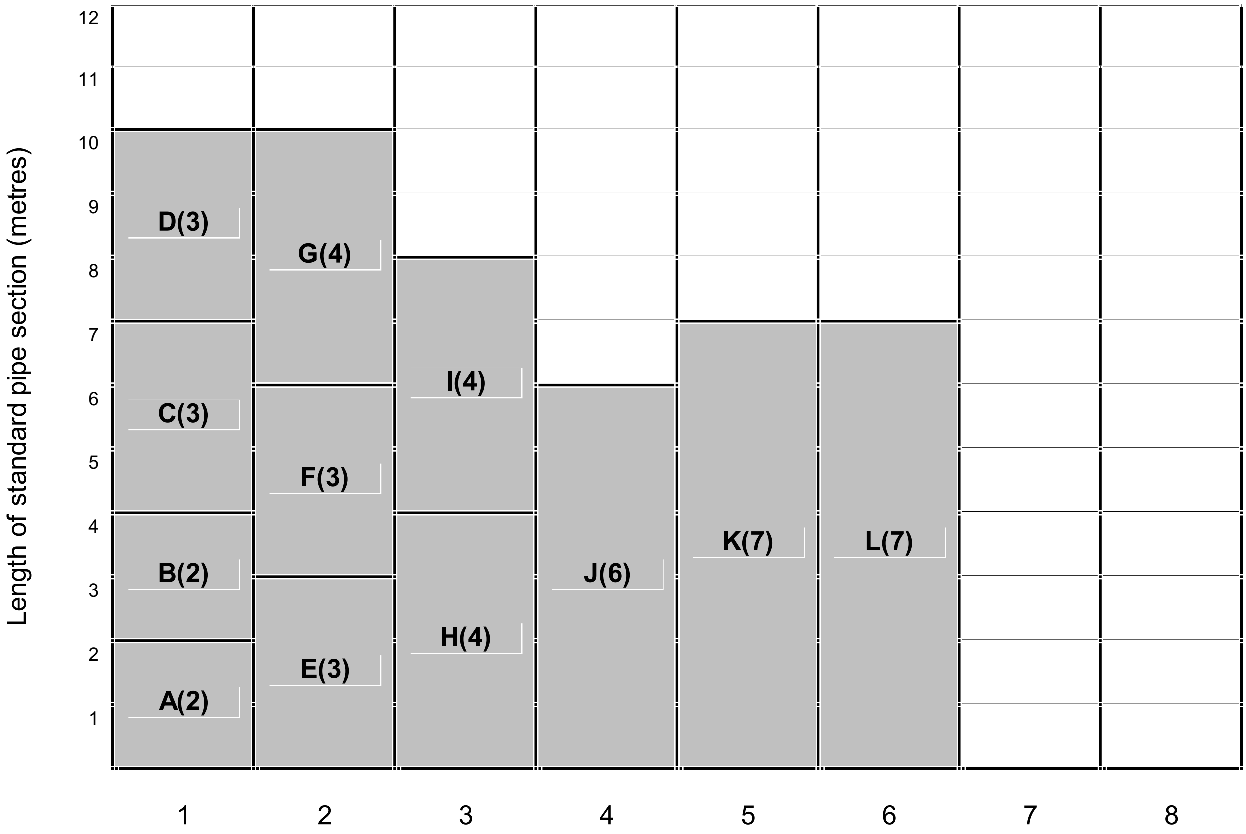
Pipe Sections



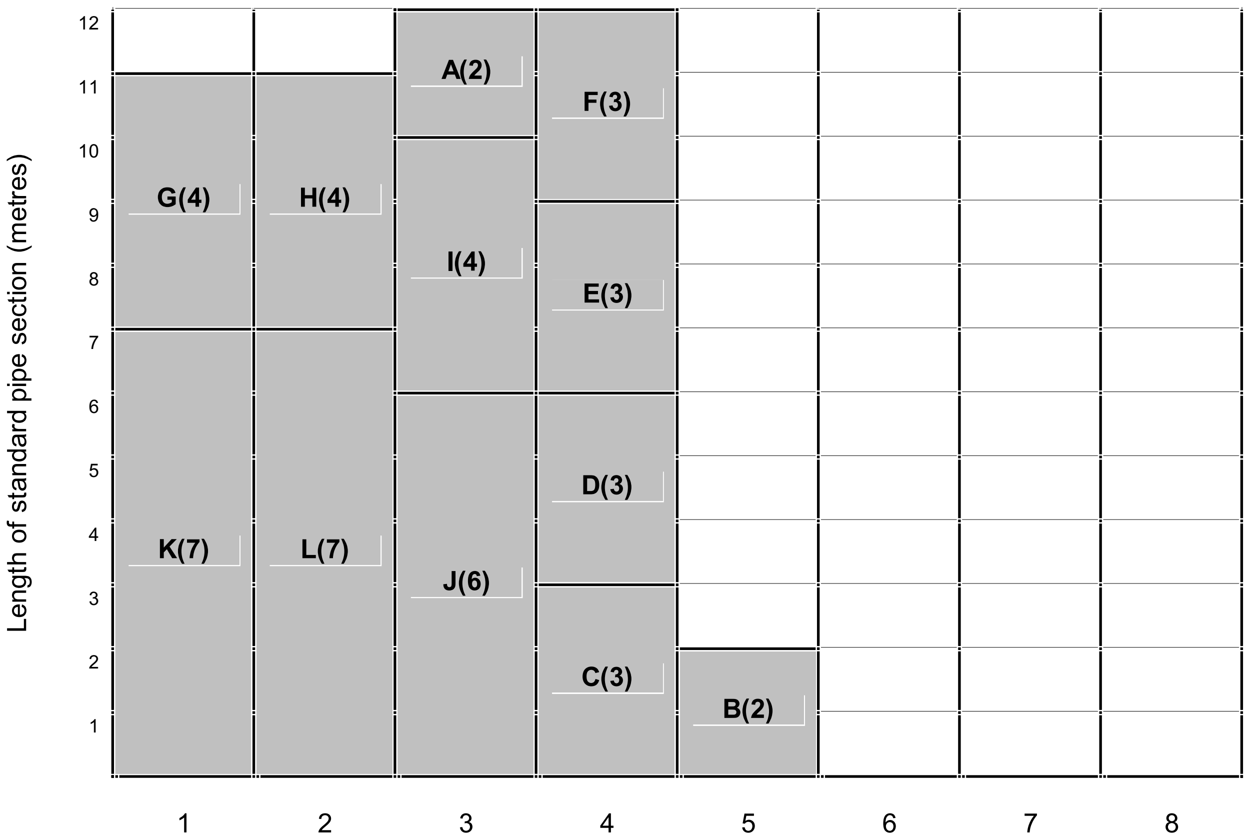


Solutions

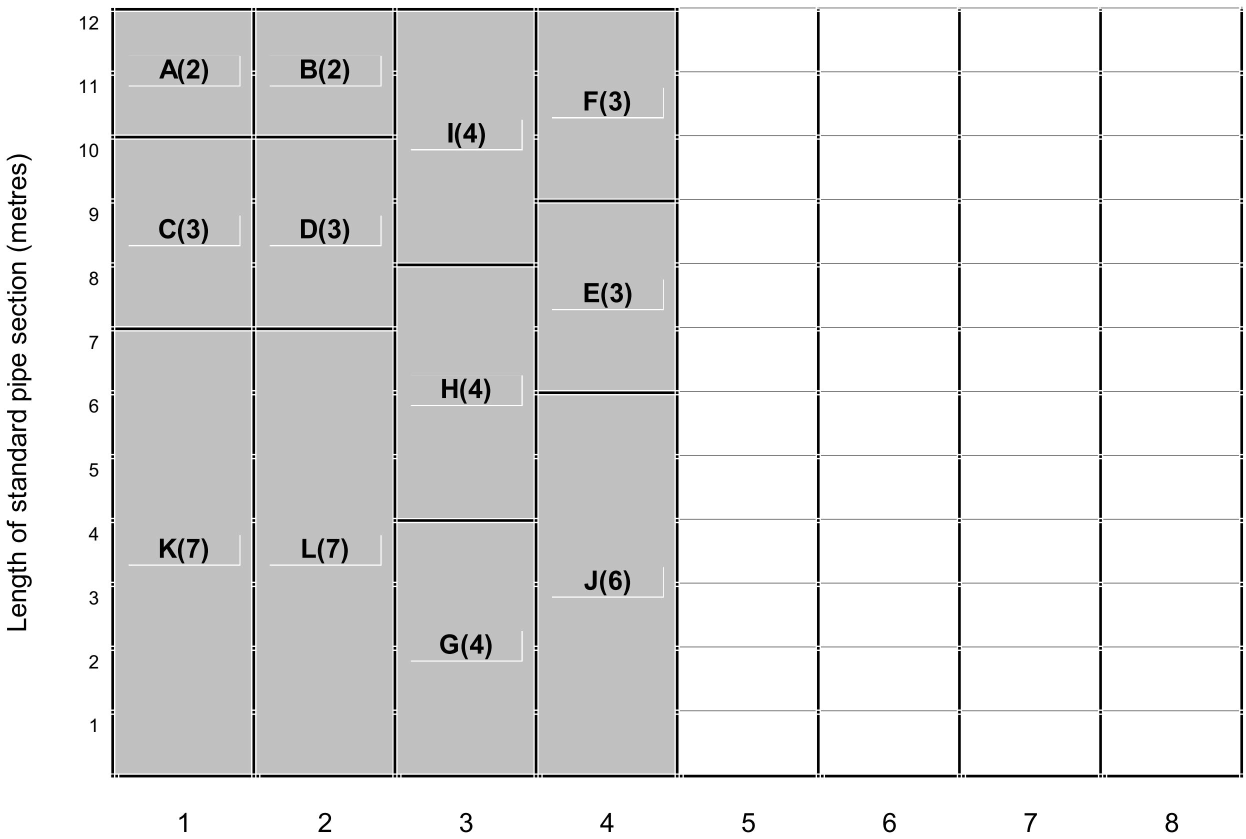
(a) First-fit bin packing



(b) First-fit decreasing bin packing



(c) Trial and improvement



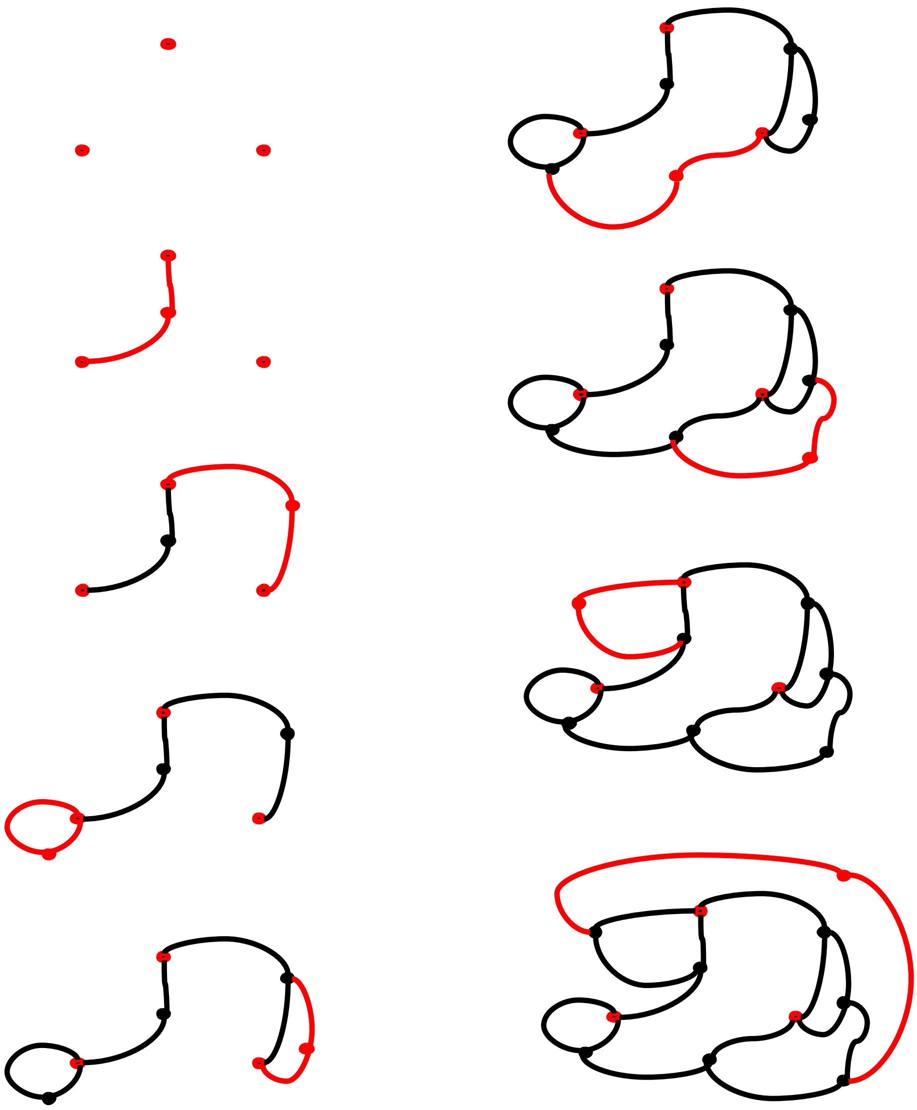
Sprouts

This is a simple pen-and-paper game for two players which involves arcs and nodes. It was invented in 1967 by Professor John H Conway and Michael S Paterson and has been described and analysed in books and on the internet. (Enter the words sprouts and Conway into a search engine for more information.)

Sprouts with three spots:

Draw three spots anywhere on the paper.
Each player in turn draws a line joining one spot to another spot (or itself) and places a new spot somewhere on this line.
No lines may cross,
No spot may have more than three lines coming out of it (i.e. we would say that the degree of any vertex cannot exceed three).
The game continues until no further moves are possible and the player to make the final move is the winner.

A possible three spot game:



Analysis:

Providing students have met the fact that for any graph the node-sum is twice the number of edges (from the hand-shake lemma), then it is quite easy to prove that a three spot game of sprouts can never exceed 8 moves.

The game begins with 3 vertices and no edges.
Every move adds to the network 2 edges and one vertex.
Therefore after m moves, the network will have gained 2m edges and m vertices.
So the final graph contains 2m edges and m + 3 vertices.

Since the winning move will leave at least one vertex of degree 2, the maximum number of 3-nodes is m + 2.

Therefore	node sum	$(m + 2) \times 3 + 2$
and since	node sum =	$2 \times$ (no. of edges)
we have	$2 \times (2m)$	$(m + 2) \times 3 + 2$
so	$4m$	$3m + 8$
and so	m	8

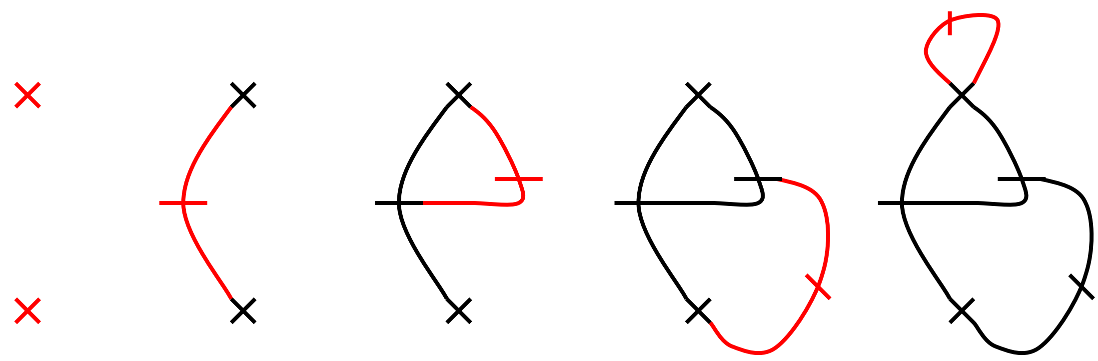
Thus no game of three spot sprouts can exceed 8 moves.

Sprouts can be played with any number of spots and it can be shown that an n spot game will never exceed 3n – 1 moves.

Brussels Sprouts:

A similar game can be played with crosses instead of spots, where each cross represents a vertex of maximum degree 4. A move joins two 'branches' of existing crosses (or a cross to itself) and a cross is placed on the new line.

The start of a possible two cross game:



In fact in Brussels Sprouts the first player always wins if the number of crosses is odd and the second player always wins where the number of crosses is even!