

Mathematical Paperfolding notes from MEI Conference 2010

Paper folding is a practical activity and trying to convey the joys and excitement I get from it via the written word is about as thrilling as listening to jugglers on the radio. You have been warned!

Geometric Constructions

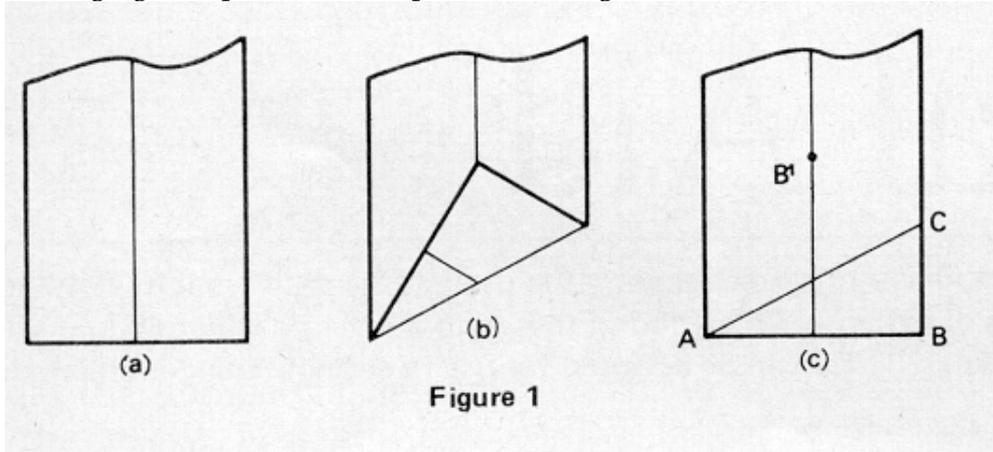
By its very nature, paper folding is a mathematical activity and I see it as a 3 dimensional way of producing the 2 dimensional Euclidian constructions; the ideas that follow are for constructing angles of 30° , 45° , 60° , 72° and 90° , some isometric paper and then a solid octahedron. You can fold paper into any shapes you like, curves with no particular properties are quite easy, but if we take a floppy sheet of paper and fold it carefully in three dimensions, this produces two planes which we know geometrically will intersect in a straight line. This is what everybody calls the "fold". We now have the basic constructional tool-a straight line.

If we take any two points on a sheet of paper and fold one onto the other, the fold (straight line) produced is the perpendicular bisector of the two points. Taking two points along the straight line and folding together again produces another perpendicular bisector and hence a right angle.

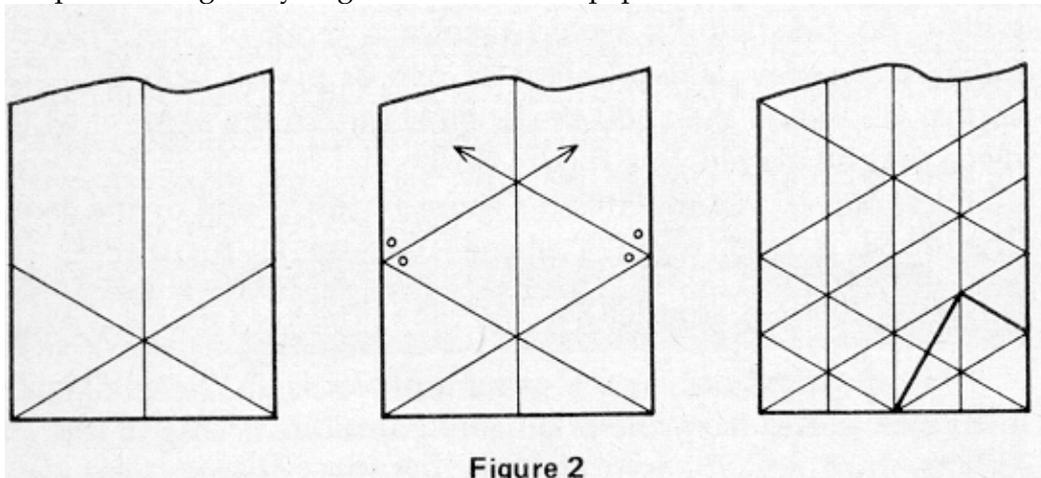
If we fold lines onto lines we will produce another bisector, this time it's the angle bisector between the lines. We now bisect our right angle to get a 45° angle.

Words now fail me, so I resort to diagrams. The ones below are reproduced from an excellent article by James Brunton in the July 1974 issue of "Mathematics in School".

Folding figure 1 produces an equilateral triangle ABB' , hence CAB is 30° and ACB is 60°

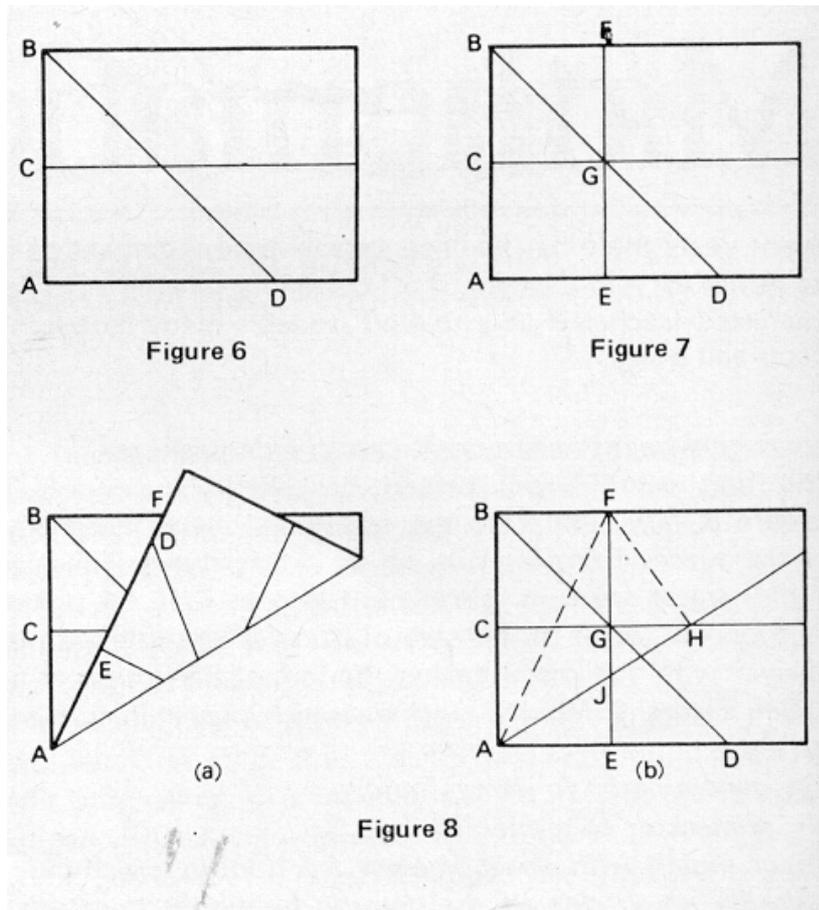


Keep on folding and you get some isometric paper.



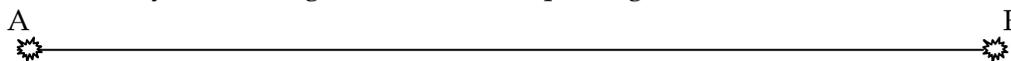
If you continue and make this paper 8 triangles wide can use this isometric paper for plaiting several of the octahedra page 3

Figure 6, 7 and 8 show how to generate the triangle AEF with short side in a ratio of 1:2 and so the lengths AF: AE are in a ratio of $\sqrt{5}: 1$, hence the angle FEH is 72° .



Iteration with paper folding

Now for a completely different idea. It's easy to fold paper in halves, quarters, eights etc. but here's a way of dividing a line into five equal segments. Let the line be A to F



Take a guess at a point one fifth of the way along the line call this B. There will be an error of some sort, lets assume you over estimated and call the error ϵ



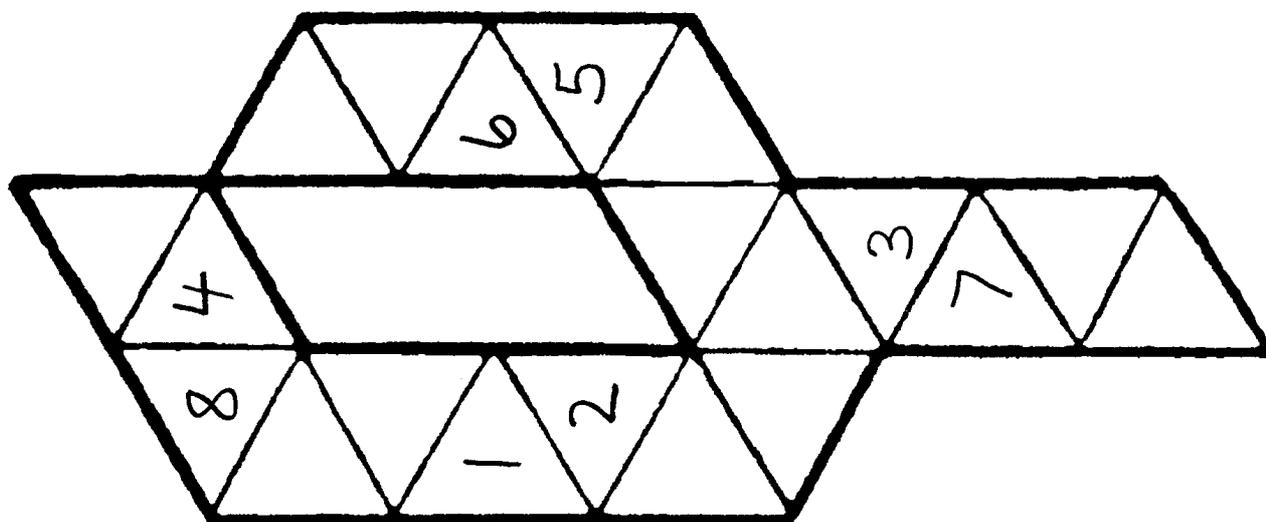
The line from B to F is now $(4/5^{\text{th}} - \epsilon)$ of the full length AF. Folding F to B gives the point D, and folding F to D gives the point E. The length EF is $1/4$ of BF and the error is also $1/4 \epsilon$. So EF is a much better estimate of $1/5$ of the line length.



Now repeat the iteration process with A and E. Very soon you'll have divided the line into 5 (almost) equal intervals

Using the isometric paper we folded you should be able to get three of these nets from a single sheet.

As a folding purist I wouldn't like to use scissors, but you can cut out this net and start plaiting to make an 8-sided octagonal die



Major sources of information

Apart from James himself, the articles he wrote are the best source of material I've found. These were written from July 1973 until September 1974 in "Mathematics in School" published by the Mathematical Association office@m-a.org.uk and are still available at £4.80 each.

There is an excellent book "**Geometric Exercises in Paperfolding**" by **T. Sundara Row** published by Open Court Books in 1905. I'm sure you won't find the original, but rather surprisingly it is still available as a reprint from Dover Publication (my copy is ISBN0-486-21594-6).

Other books of interest

The classic "Mathematical Models" by Cundy and Rollett has bits on plaiting polyhedron

"Mathematics through Paper Folding" Alton T. Olson published by NCTM in the USA.

"Paper Magic- Folding Polygons" a Channel 4 publication from ETV

Tarquin books "Mathematical Origami" by David Mitchell covers polyhedra