*CAS and the Curriculum

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*Who am I?

*Some History

*'Technology' usage in mathematics has been common place throughout history. For example:
  *Abacus
  *Tables of known results (log, trig, stats etc)
  *Slide Rule
  *Napier's Bones
  *Calculators (First 4 function built around 1623!)
  *Leibniz's Step Reckoner
  *Babbage's Difference Engine

*The purpose of each of these 'technologies' is to speed up the process of solving a problem by passing some of the work to the 'technology'

*Technology

*The first electron calculators (rather than the earlier mechanical ones) was invented in 1961, and by 1967 Texas Instruments had invented the first handheld four-function calculator.

*By 1971 HP had produced the first electronic scientific calculator capable of computing logs and trigonometric ratios.

*Modern Calculators

*"However, by the end of the nineteenth century, with much of the credit due to Gauss, mathematics had changed substantially; it had become much more qualitative and less computational. Mathematicians concerned themselves more with questions about the structure of mathematics than they did with the computation of solutions to particular problems. There were many reasons for this change, but one reason seems to be that it was becoming increasingly difficult to solve significant problems by computational means." (Caviness 1986 p218)

*Caviness 1986

Purpose of Technology

- If you consider the purpose of computational aids to be to enable their users to be able to do more mathematics, more quickly and more accurately then you can see that the many devices we looked at above, as well as the large collections of books and tables of results which have been used throughout recorded history, achieve this aim well.
- But these only help with numerical problem… What about algebra?

Computer Algebra Systems

- CAS is an attempt to extend the use of technology beyond the numerical and into the algebraic.
- CAS in its modern form was really invented in the late 1970's and early 1980's and was restricted to running on large main-frame computers.
- By 1986 it was possible to run one brand of CAS in the form of mU-MATH on an IBM PC. mU-MATH formed the basis of a later product called DERIVE, which in turn is the basis of the CAS engine present in all the Texas Instruments calculators.

Examples of CAS

- Handheld
  - TI-Nspire CAS
  - Casio ClassPad
  - HP 50G
- Computer:
  - Mathematica
  - WolframAlpha.com
  - Microsoft Mathematics
  - Maple

What does it do?

- Algebraic Manipulation
- Simplification
- Solving
- Factoring
- Calculus
- Limits

- Let's look at some examples…

CAS is to …

- So we can see that in many ways CAS is an attempt to do for algebra what the scientific calculator did for number…
- Whether you see this as a good or a bad thing will probably depend on your opinion of the scientific calculator however!

Investigative Questions

*Factor x^n-1

*For n=1 to 10, all the coefficients of the polynomial factors are 1, is this true for all values of n?

**Factoring*

**What's happening in the rest of the world?*

*Whilst here in the UK we are still shying away from the use of technology in Mathematics (particularly in formal high stakes assessment) other parts of the world are exploring a technology known as CAS (Computer Algebra System) which allows the computer/calculator to perform symbolic algebraic manipulation.

*There are several countries in the world that already require the use of CAS as part of their formal examinations including Norway, Denmark, Australia… Many other parts of the world also permit (but don't require) the use of CAS.

**Computer Algebra Systems*

*Let's have a look at some examples of questions from CAS enabled curricular, focusing particularly on the VCAA Mathematical Methods course from Australia.

**Exam Questions*

CAS Solution

VCAA Mathematical Methods CAS
May 2010 Paper 2 Question 4
http://www.vcaa.vic.edu.au/vcaa/vce/studies/mathematics/cas/pastexams/2010/2010mmcas2-w.pdf

**Example Question*

*It is important to remember that ALL questions that can be answered with CAS could equally be answered without it…

… the difference is how long it would take to do it!

*CAS enabled questions typically express some of the following properties:

- *Greater use of parameters in questions
- *More complicated Algebraic expressions
- *Higher Order Polynomials
- *More contextualized problems
- *The algebra doesn't always factorise nicely!!
- *Fewer 'mechanical' questions such as 'integrate ….'

**How have the questions changed?*

*Why use CAS?

*For me, the main reason to use a symbolic CAS calculator over a numerical calculator is the ability to quickly and easily generalise results.

*However other advantages include the ability focus on 'key concepts' of a topic without students getting bogged down in the algebraic manipulation

*The ability to use CAS to check answers

*Can be used to change the focus in the classroom from answering questions to asking the right questions

*Allows for the potential restructuring of the order of the curriculum based on conceptual difficulty rather than on computational difficulty as it is at the moment.

*Why NOT use CAS?

***There are many reasons why some people do not like CAS:**

*Lesser emphasis on traditional 'paper and pencil' methods

*Requires little understanding (button pushing)

*How will students understand the maths without learning to do it by hand?

*Can we trust a mathematical proof which is based on logic done by a machine? (4 Colour Theorem)

*How does the use of CAS change the role of the teacher?

*What is Mathematics?

***Ultimately the debate over the use of Computer Algebra (and even Graphical Calculators) comes down to the question of what you see Mathematics as being about…**

*Is Maths about learning and applying skills and algorithms?

*Is Maths about problem solving?

*Is Maths about logical reasoning?

*Is Maths about framing problems in a Mathematical form?

*Is Maths about interpretation of models and results?

*Can you do Maths on a computer?

How we answer this will likely determine the role technology plays in our schools…

*Kennedy 1995

*Look around you in the tree of Mathematics today, and you will see some new kids playing around in the branches. They're exploring parts of the tree that have not seen this kind of action in centuries, and they didn't even climb the trunk to get there. You know how they got there? They cheated: they used a ladder. They climbed directly into the branches using a prosthetic extension of their brains known in the Ed Biz as technology. They got up there with graphing calculators. You can argue all you want about whether they deserve to be there, and about whether or not they might fall, but that won't change the fact that they are there, straddled alongside the best trunk-climbers in the tree – and most of them are glad to be there. Now I ask you: Is that beautiful, or is that bad?

*This presentation is available for download from:

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*Thanks!