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**MEI Conference 2016**

# Using GeoGebra in Further Pure Maths

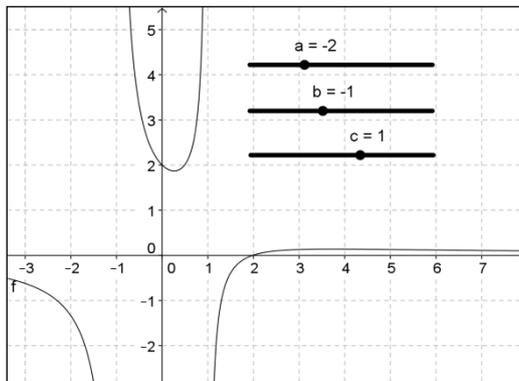
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## Task 1 – Curve sketching: Rational Functions

- Use  to add 3 sliders **a**, **b** and **c**.
- In the Input bar enter the function:  $f(x)=\frac{(x+a)}{(x+b)(x+c)}$ .

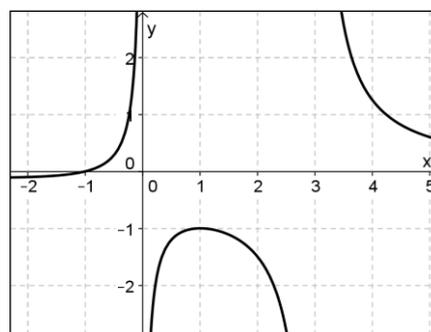
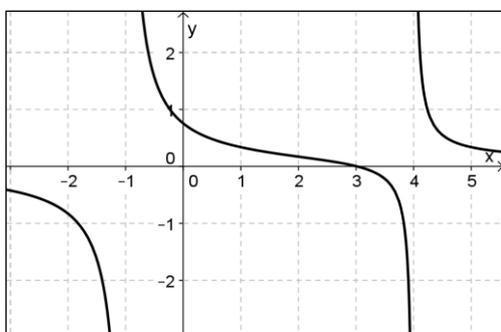


### Questions

- What point on the curve does the value of  $a$  give you?
- What is the relationship between the shape of the curve and the values of  $b$  and  $c$ ?

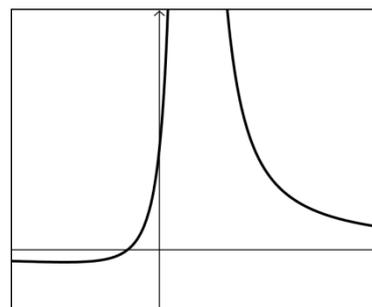
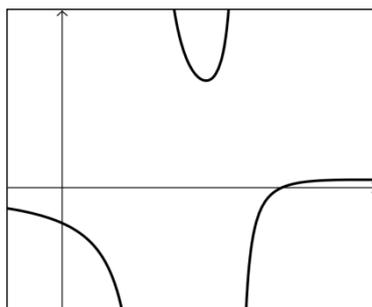
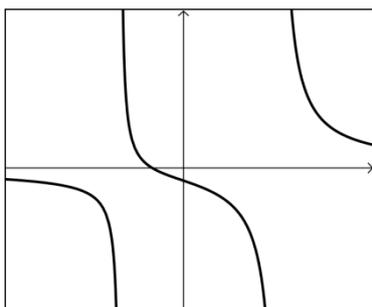
### Problem

Find the values of  $a$ ,  $b$  and  $c$  for the following curves:



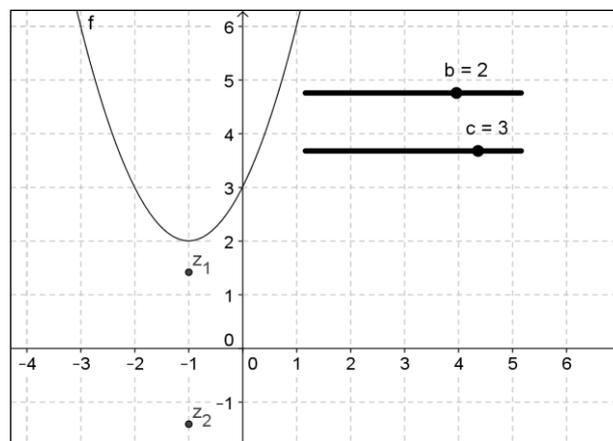
### Further Tasks

Can you find conditions on  $a$ ,  $b$  and  $c$  so that the curve will have one of these general shapes?



## Task 2 – Complex Numbers: Roots of Quadratic Equations

1. Add a slider  and set its name to **b**.
2. Add a slider  and set its name to **c**.
3. In the Input bar enter the function:  **$f(x)=x^2+b*x+c$**
4. In the Input bar enter the function: **ComplexRoot[f]**



### Questions

- When are the roots of  $f(x) = 0$  real? When are the roots of  $f(x) = 0$  complex?
- Can you find values of  $b$  and  $c$  so that the roots are complex and the real part is 2? ... or 1? ... or  $-1$ ? ... or  $p$ ?
- Can you find a function  $f$  with roots  $2 \pm 3i$
- Explain how you would find a function  $f$  with roots  $p \pm qi$  (for some  $p$  and  $q$ )?

**Problem** (Try the problem with pen and paper first then check it on your software)

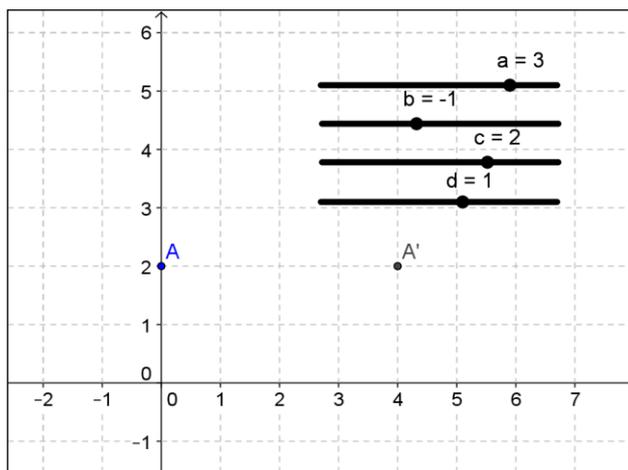
The function  $f(z) = z^3 - 10z^2 + 34z - 40$  has a root  $z = 3 + i$ . Find the other two roots.

### Further Tasks

- Explain why the roots of a cubic with real coefficients will always form an isosceles triangle in the Argand diagram.
- Find a cubic  $f(z)$  where the roots of  $f(z)$  form an equilateral triangle in the Argand diagram.

### Task 3 – Matrices: Transformation matrices

1. Use  to add 4 sliders **a**, **b**, **c** and **d**.
2. Create the matrix  $M = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$  by entering:  $M = \{\{a,c\},\{b,d\}\}$  in the input bar.
3. Use **New Point** (2<sup>nd</sup> menu)  to add a point, **A**.
4. Create the image of A under the transformation M by entering  $A' = M \cdot A$  in the input bar.



Answer the following questions for different matrices, **M**:

- What is the relationship between  $A'$  and  $A$  when  $A$  is on the x-axis?
- What is the relationship between  $A'$  and  $A$  when  $A$  is on the y-axis?
- How can you use these relationships to find the position of  $A'$  for any point  $A$ ?

**Problem** (Try the problem with pen and paper first then check it on your software)

The image of the quadrilateral  $OABC$  under the transformation  $M = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$  is a square.

$O=(0,0)$ ,  $A=(2,4)$  and  $B=(4,0)$ . Find the coordinates of  $C$ .

#### Further Tasks

- Under the matrix  $M = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$  the image of the point  $A=(-1,3)$  is  $A'=(2,4)$ .  
Find other matrices  $M$  and points  $A$  such that  $A'=(2,4)$ .
- $A=(2,1)$  is an invariant point under the matrix  $M = \begin{pmatrix} 3 & -4 \\ 2 & -3 \end{pmatrix}$  because the image of the point  $A=(2,1)$  is  $A'=(2,1)$ . Find the invariant points of other matrices  $M$ .

## Using GeoGebra in Further Pure

### Matrices

To enter the matrix  $M = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$  type **M={{a,c},{b,d}}** in the Input bar and press enter.

The operations **+**, **-** and **\*** will be applied in the standard way.

**M^-1** will find the inverse matrix.

To find the image of a point, **A**, under the transformation, **M**, type: **A'=M\*A**.

To find the image of a shape, **poly1**, under the transformation, **M**, type:

**ApplyMatrix[M,poly1]**

### Complex Numbers

To enter a complex number such as  $z_1 = 2 + 3i$  enter **z\_1=2+3i** in the Input Bar.

The operations **+**, **-**, **\*** and **/** will be applied in the standard way. To find the conjugate of  $z_1$  use: **conjugate(z\_1)**.

The function **ComplexRoot[f]** will plot the roots of the polynomial, **f**, as a complex number.

### Polar Curves

Polar curve can be plotted using either of the following forms:

**r = cos(2θ) + 3** (NB you can use **Alt-t** for **θ**) or **(cos(2t)+3; t)**

Options > Advanced > Angle Unit: Radians (computer software only).

To display a Polar grid right-click in the graphics area and select the last option: Graphics. On the grid option select: Grid type: Polar.

### Vectors

To enable the 3D view: View > 3D Graphics.

Entering a set of coordinates with a lower-case name will create the object as a position vector, e.g. **v=(2,3,-1)**

**u\*v** will find the scalar product of **u** and **v**. **u⊗v** will find the vector product of **u** and **v**.

**Line[A,v]** will create a line through point **A** in the direction **v**. Planes can be entered directly in Cartesian form: e.g. **2x+3y-z=2**.

### Maclaurin series

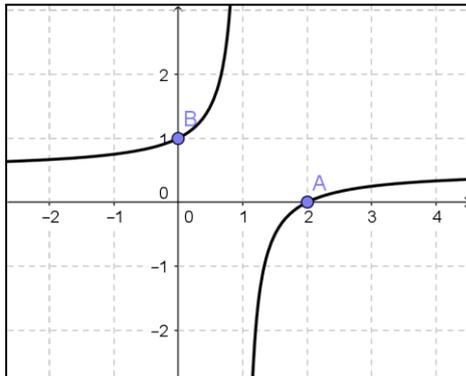
Enter a function **f(x)** such as **f(x)=sin(x)**

Add a **Slider** and set the type to *Integer*

In the input bar enter: **TaylorPolynomial[f, 0, n]**

## Construction Problems for Further Pure

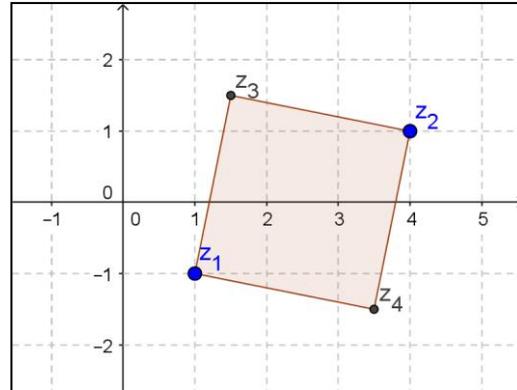
1.



Create a point A fixed to the x-axis and a point B fixed to the y-axis.

Construct a rational function that passes through A and B.

2.



Create two complex numbers  $z_1$  and  $z_2$ .

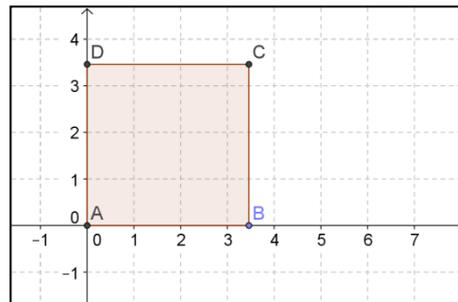
Construct complex numbers  $z_3$  and  $z_4$  such that the 4 points form a square in the Argand diagram with  $z_1$  and  $z_2$  as a diagonal.

3. Add a complex number  $z_1$ .

Construct a cubic with real coefficients such that all it has a zero at  $z_1$  and all three zeros of the cubic lie on the line  $x = \text{Re}(z_1)$ .

You might find the following GeoGebra commands useful:  
`real(z_1)`, `imaginary(z_1)`,  
`ComplexRoot[f]`

4.



Given a square with variable side (named **poly1**) find a matrix M such that the command **ApplyMatrix[M,poly1]** will construct a rectangle with the same area as the square whose sides are in the ratio 2:1.