



Innovators in  
Mathematics  
Education

**CASIO**®

## **MEI Conference 2016**

# **Teaching Further Pure using graphing technology**

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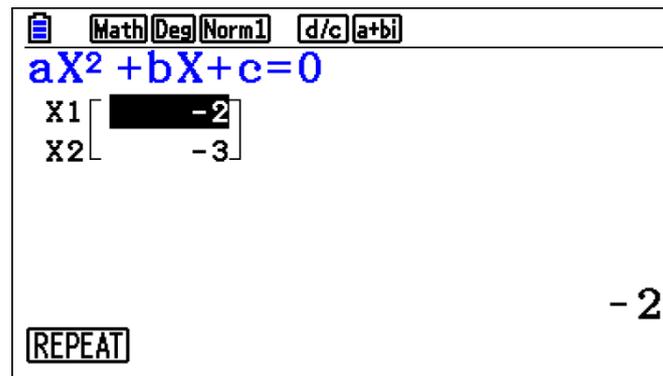
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## Task 1 – Complex Numbers: Roots of Quadratic Equations

1. Add a new Equation screen: **MENU** **ALPHA** **X,θ,T**
2. Change the Complex output to **a+bi**: **SHIFT** **MENU** **▼** **▼** **▼** **▼** **F2** **EXIT**
3. Select Polynomial and set the Degree to 2: **F2** **F1**
4. Set **a=1**, **b=5** and **c=6**: **1** **EXE** **5** **EXE** **6** **EXE**
5. Solve the equation: **F1**



Press **EXIT** to edit the values of **a** and **b**.

### Questions for discussion

- When are the roots of the quadratic real? When are the roots of the quadratic complex?
- Can you find values of  $b$  and  $c$  so that the roots are complex and the real part is 2? ... or 1? ... or  $-1$ ? ... or  $p$ ?
- Can you find a quadratic equation with roots  $2 \pm 3i$ ?
- Explain how you would find a quadratic equation with roots  $p \pm qi$  (for any  $p$  and  $q$ )?

**Problem** (Try the question with pen and paper first then check it on your calculator)

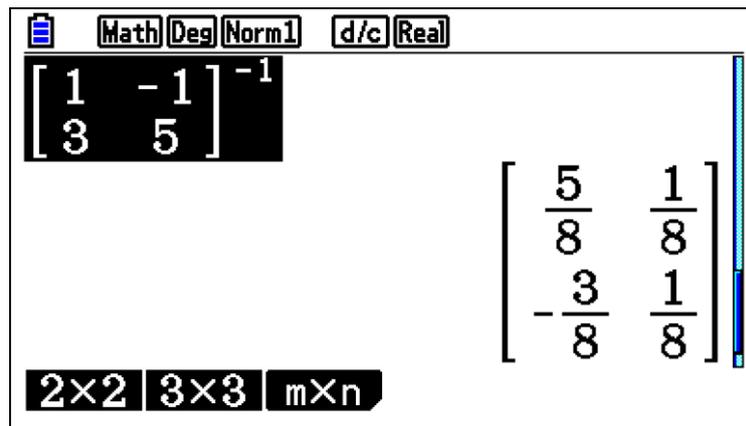
The function  $f(z) = z^3 - 10z^2 + 34z - 40$  has a root  $z = 3 + i$ . Hence find the other two roots.

### Further Tasks

- Investigate the relationship between the graphs of quadratic equations and their roots.
- Explain why the roots of a cubic with real coefficients will always form an isosceles triangle in the Argand diagram.

## Task 2 – Matrices: Determinants and inverse matrices

- Add a new Run-Matrix screen: **MENU** **1**
- Find the determinant of a 2x2 matrix.  
 Determinant: Option > Mat > Det: **OPTN** **F2** **F3**  
 Insert matrix: Math > Mat > 2x2: **EXIT** **EXIT** **F4** **F1** **F1**  
 Use the cursor keys to enter the values: **1** **▼** **3** **▲** **▶** **(←)** **1** **▼** **5** **EXE**
- Find the inverse of the same 2x2 matrix.  
**F1** **1** **▼** **3** **▲** **▶** **(←)** **1** **▼** **5** **▶** **^** **(←)** **1** **EXE**



### Questions for discussion

- What is the relationship between the matrix, the determinant and the inverse?
- What is the answer when a matrix is multiplied by its inverse?
- Are there any matrices that don't have an inverse?

**Problem** (Try the question with pen and paper first then check it on your calculator)

For the matrices  $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$  find  $A^{-1}$ ,  $B^{-1}$  and  $(AB)^{-1}$ .

### Further Tasks

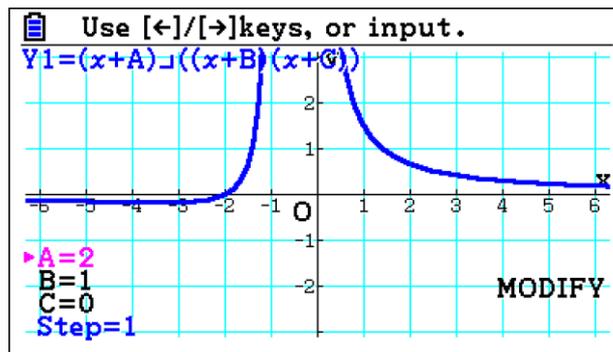
- For other 2x2 matrices, A and B, investigate the relationship between  $A^{-1}$ ,  $B^{-1}$  and  $(AB)^{-1}$ .
- Investigate the determinants and inverse of matrices for standard transformations: reflection, rotation and stretches.

## Task 3 – Rational Functions

1. Add a new Graphs screen: **MENU** **5**

2. Enter the function  $Y1=(x+A)/((x+B)(x+C))$  : **(** **X,θ,T** **+** **ALPHA** **X,θ,T** **)** **÷** **(** **(** **X,θ,T** **+** **ALPHA** **log** **)** **(** **X,θ,T** **+** **ALPHA** **In** **)** **)** **EXE**

3. Plot the curve using Modify: **F5**



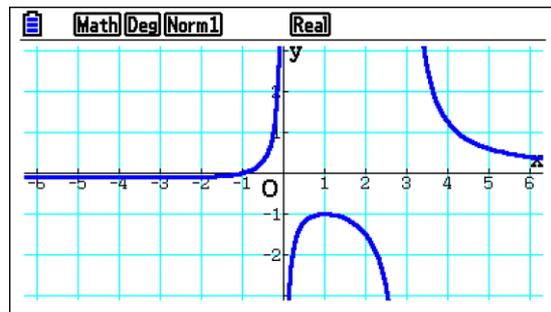
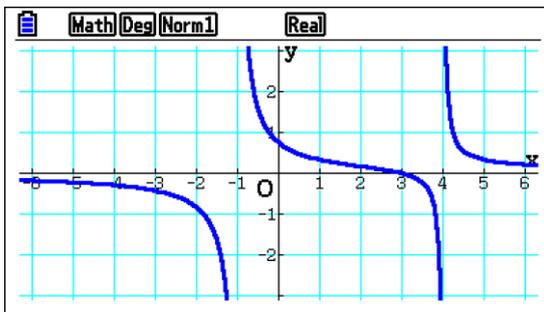
Use **◀**/**▶** to vary the parameters and **▼**/**▲** to change which one you are varying.

### Questions for discussion

- What point on the curve does the value of **A** give you?
- What is the relationship between the shape of the curve and the values of **B** and **C**?

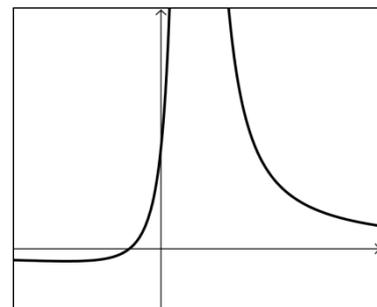
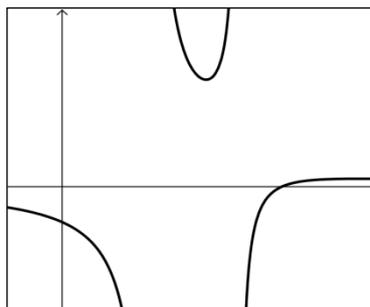
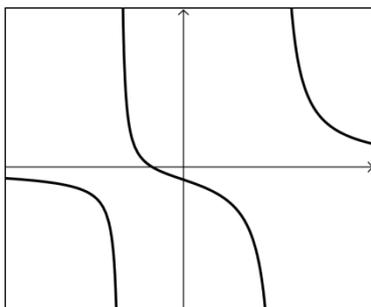
**Problem** (Try the question with pen and paper first then check it on your calculator)

Find the values of **A**, **B** and **C** for the following curves:



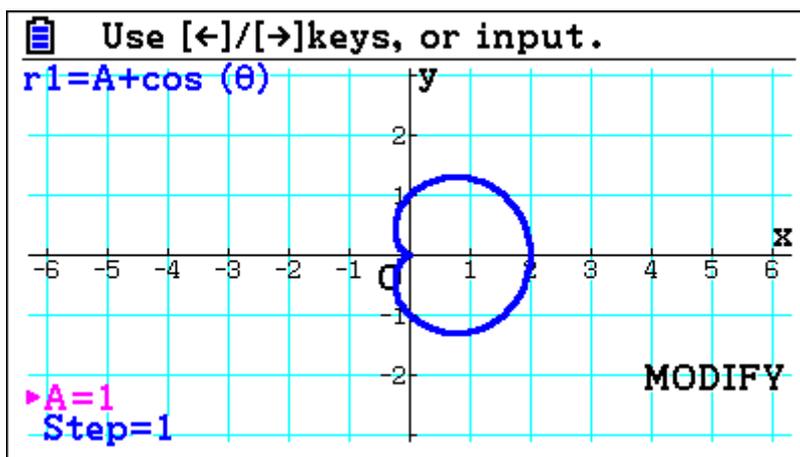
### Further Tasks

Find conditions on **A**, **B** and **C** so that the curve will have one of these general shapes.



## Task 4 – Polar curves

1. Add a new Graphs screen and check the angle is set to radians:  
 $\boxed{\text{MENU}}$   $\boxed{5}$   $\boxed{\text{SHIFT}}$   $\boxed{\text{MENU}}$   $\blacktriangledown$   $\boxed{\text{F2}}$   $\boxed{\text{EXIT}}$
2. Set the type to Polar (r=):  $\boxed{\text{F3}}$   $\boxed{\text{F2}}$
3. Enter the function  $r1=A + \cos(\theta)$  :  $\boxed{\text{ALPHA}}$   $\boxed{X,\theta,T}$   $\boxed{+}$   $\boxed{\text{COS}}$   $\boxed{(}$   $\boxed{X,\theta,T}$   $\boxed{)}$   $\boxed{\text{EXE}}$
4. Plot the curve using Modify:  $\boxed{\text{F5}}$



Use  $\blacktriangleleft$ / $\blacktriangleright$  to vary A.

### Questions for discussion

- What is the maximum/minimum distance from the pole and for what values of  $\theta$  does this occur?
- How is this polar curve related to the Cartesian curve  $y = A + \cos x$ ?

**Problem** (Try the question with pen and paper first then check it on your calculator)

Plot the following curves:

$$r = 2 + \cos \theta$$

$$r = 2$$

$$r = 1 + \sin 2\theta$$

$$r = 3 + \cos \frac{\theta}{2}$$

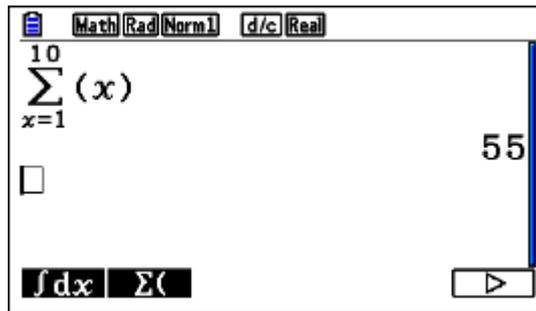
$$r = 3 - 2 \sin \theta$$

### Further Tasks

- For what values of  $\theta$  does  $r$  take its maximum and minimum values? How can these be deduced from the polar equation?
- For which parts of the graph does  $r$  take negative values? What are the conditions such that  $r = a + b \cos \theta$  and  $r = a + b \sin \theta$  doesn't take negative values?
- The default setting is to plot values of  $\theta$  from 0 to  $2\pi$  (this can be changed with V-Window). Are there any curves for which this results in the same graph being traced over again? Are there any graphs for which the graph is incomplete using this range?

## Task 5 – Summation of simple finite series

- Go into Run-Matrix mode: **MENU** **1**
- Add a summation sign: **F4** **F6** **F2** **Math** **▶** **Σ(**
- Calculate  $\sum_{x=1}^{10} x$ : **X,θ,T** **▶** **X,θ,T** **▶** **1** **▶** **1** **0** **EXE**



Investigate  $\sum_{x=1}^n x$  for different values of  $n$ .

### Questions for discussion

- Can you find an expression in terms of  $n$  for  $\sum_{x=1}^n x$ ?
- Can you find an expression in terms of  $n$  for  $\sum_{x=1}^n x^2$ ?
- How would you find an expression in terms of  $n$  for  $\sum_{x=1}^n x(x+2)$ ?

**Problem** (Try the question with pen and paper first then check it on your calculator)

Find an expression for  $\sum_{x=1}^n (x-1)(x+3)$  in terms of  $n$  and hence find  $\sum_{x=1}^{10} (x-1)(x+3)$ .

### Further Tasks

- Investigate sums of the form  $\sum_{x=m}^n$ , i.e. sums that don't start from 1.
- Investigate  $\sum_{x=1}^n x^3$  and find a relationship between this and  $\sum_{x=1}^n x$ .