MEI Conference 2017

Using GeoGebra to support problem solving in A level mathematics

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Session description

This hands-on session is aimed at exploring how GeoGebra can support problem-solving in A level mathematics. The session will focus on pure mathematics topics. Some familiarity with basic features of GeoGebra will be assumed.

Delegates are requested to bring a laptop with GeoGebra installed to this session. GeoGebra can be installed from www.geogebra.org/download

MEI GeoGebra Institute

MEI is a GeoGebra Institute. We support teachers and students through:

- face-to-face and online workshops for teachers;
- the design of free teaching and professional development materials for A level and GCSE Maths;
- online support for GeoGebra users;
- presentations at conferences.

There is more information available on the MEI GeoGebra Institute page: tube.geogebra.org/institute-mei

As a GeoGebra Institute we have created some self-study "How-to" guides in order to support learning how to use GeoGebra. These are available as GeoGebraBooks via our GeoGebra Institute page and also on the MEI Use of IT support page (see below for links). Currently available are the following four guides:

- Graphics/Algebra views and Graphics Style Bar
- Functions, Dynamic Text and Graphics 2
- Commands, Spreadsheets and Statistics
- Vectors, CAS and 3D views

Useful links

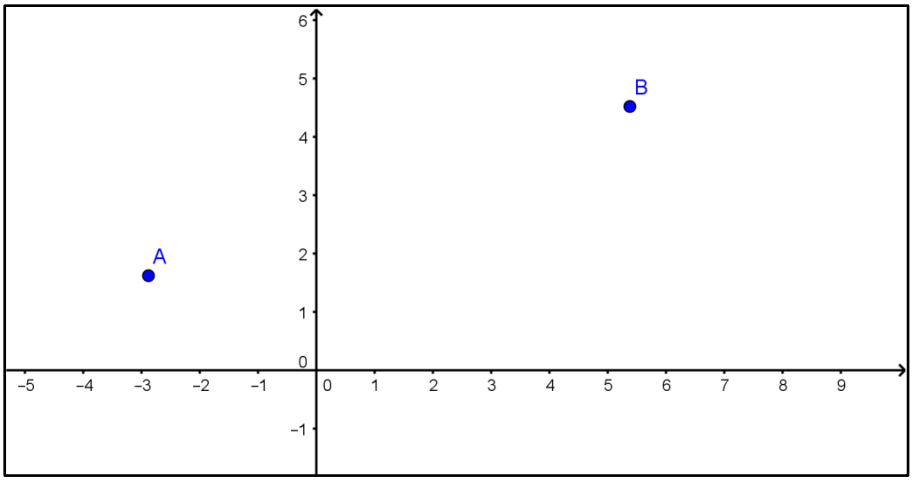
GeoGebra:	www.geogebra.org/
GeoGebra Tube:	www.geogebra.org/materials/
MEI Use of IT support page:	mei.org.uk/geogebra
MEI GeoGebra Institute page:	tube.geogebra.org/institute-mei
Simon Clay GeoGebra materials:	www.geogebra.org/simonclay

GeoGebra files for a number of the problems included in this session can be found in the GeoGebra Book 'MEI Conference 2017: GeoGebra & problem solving' via this link: https://ggbm.at/eTNVdACK

As you arrive: Finding the midpoint

Spend a few minutes thinking of the different mathematical ways for finding the midpoint of two points A and B. You may want to list them below.

Using GeoGebra see if you can reproduce some or all of your approaches.



Problem-solving

Overarching theme in new A level mathematics

OT2 Mathematical problem-solving

Ref	Knowledge/Skill
OT2.1	**[Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved]**
OT2.2	**[Construct extended arguments to solve problems presented in an unstructured form, including problems in context]**
OT2.3	**[Interpret and communicate solutions in the context of the original problem]**
OT2.4	Understand that many mathematical problems cannot be solved analytically, but numerical methods permit solution to a required level of accuracy
OT2.5	**[Evaluate, including by making reasoned estimates, the accuracy or limitations of solutions]**, including those obtained using numerical methods
OT2.6	**[Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle]**
OT2.7	**[Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems, including in mechanics]**

Note that **bold text** within **[square brackets]** is AS content. Extract taken from *'Mathematics AS and A level content'* (2014), DfE https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/516949/GCE_AS_and_A_level_subject_content_for_mathematics_with_appendices.pdf

Attributes of problems

A	Tasks have little or no scaffolding: there is little guidance given to the candidate beyond a start point and a finish point. Questions do not explicitly state the mathematical process(es) required for the solution.
B	Tasks provide for multiple representations, such as the use of a sketch or a diagram as well as calculations.
C	The information is not given in mathematical form or in mathematical language; or there is a need for the results to be interpreted or methods evaluated, for example, in a real-world context.
D	Tasks have a variety of techniques that could be used.
E	The solution requires understanding of the processes involved rather than just application of the techniques.
F	The task requires two or more mathematical processes or may require different parts of mathematics to be brought together to reach a solution.

Taken from 'A Level Mathematics Working Group Report on Mathematical Problem Solving, Modelling and the Use of Large Data Sets in Statistics in AS/A Level Mathematics and Further Mathematics' (2015), Ofqual https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/481857/a-level-mathematics-working-group-report.pdf

Change one aspect….

https://ggbm.at/eTNVdACK

Change one aspect of the equation $(x-4)^2+(y-5)^2=25$
so that the point $(1,1)$ lies inside the circle.

Change one aspect….

https://ggbm.at/eTNVdACK

Change one aspect of the quadratic $y = x^2 + 6x - 8$
so that the graph touches the x-axis

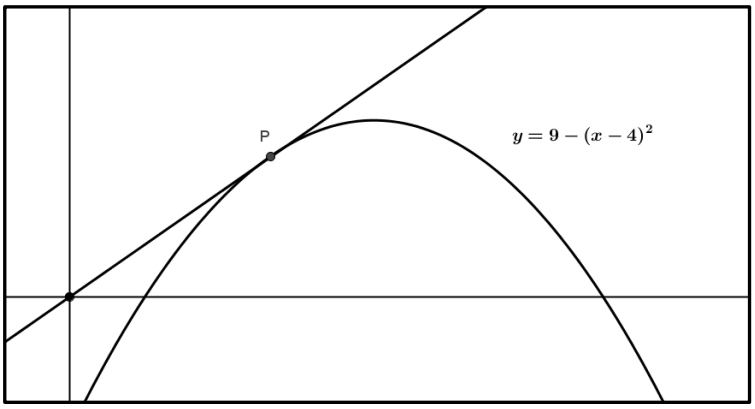
Other topics:

Surprising midpoints?

https://ggbm.at/eTNVdACK

Given a (movable) point A on a graph (e.g. $y = x^2$), and a (fixed) point M, find the locus of points, B, such that M is the midpoint of AB.

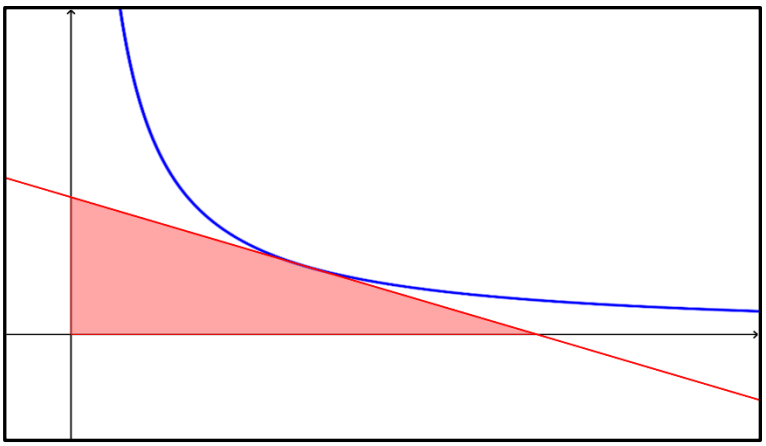
Where's P?



The tangent to $y = 9 - (x - 4)^2$ at P passes through the origin. Find P.

A curve and a triangle

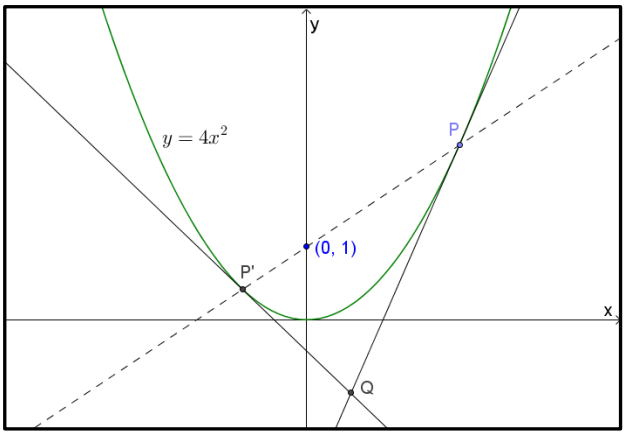
https://ggbm.at/eTNVdACK



Find the area enclosed by the tangent to $y = \frac{1}{x}$ and the axes.

A tangent and a normal

https://ggbm.at/eTNVdACK



Choose a point P on the curve and draw the line through P and (0,1). This crosses the curve at P'. Find the y-coordinate of the point Q where the tangents at P and P' meet. Repeat for another point P. What do you notice?

Exploring polynomials: Some questions from the accredited A levels

(i) OCR Spec B (MEI) A level Mathematics Paper 1 Q13

In this question you must show detailed reasoning.

Determine the values of k for which part of the graph of $y = x^2 - kx + 2k$ appears below the x-axis. **[4]**

(ii) OCR (Spec A) AS level Mathematics Paper 1 Q4

The curve $y = 2x^3 + 3x^2 - kx + 4$ has a stationary point where $x = 2$.

(i) Determine the value of the constant k. **[5]**

(ii) Determine whether this stationary point is a maximum or a minimum point. **[2]**

(iii) Edexcel AS level Mathematics Paper 1 Q13

(a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

Some questions from the accredited A levels

AQA AS level Mathematics Paper 1 Q7

Determine whether the line with equation $2x + 3y + 4 = 0$ is parallel to the line through the points with coordinates (9, 4) and (3, 8).

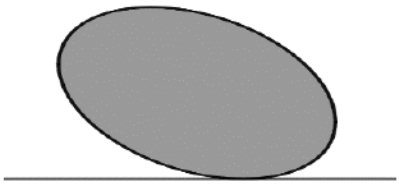
[4 marks]

AQA A level Mathematics Paper 1 Q12

A sculpture formed from a prism is fixed on a horizontal platform, as shown in the diagram.

The shape of the cross-section of the sculpture can be modelled by the equation $x^2 + 2xy + 2y^2 = 10$, where x and y are measured in metres.

The x and y axes are horizontal and vertical respectively.



Find the maximum vertical height above the platform of the sculpture.

[8 marks]

Notes and reflections