MEI AS Maths Programming Tasks

Section 1: Warm up!

1. Print "Hello World". Print "My name is" followed by your name.

2. Define *apple* to be 6, *pear* to be 9, *banana* to be 300, *pineapple* to be 7. Add them up and assign this value to *fruit*. Print the value of *fruit*.

3. Print all the natural numbers up to and including 100 using a 'for' loop.

4. Print all the natural numbers up to and including 100 which are divisible by 3.

5. Sum the natural numbers up to and including 100 which aren't divisible by 3. Print the result.

6. Print all the numbers which are multiples of 3 or 5 up to and including 100.

7. Print all the numbers which are multiples of 3 and 5 up to and including 100.

8. Write a function, which takes the input n, and outputs $3n+2$. Test your function by using it to print the value it outputs when 5 is input.

9. For all the numbers up to and including 100 which are divisible by 7, multiply by 3 and add 2. Print these numbers.

10. Use a while loop to print all the numbers from 0 to 500.

11. Use a while loop to find the biggest perfect square less than 100,000. Print it and print the value that it is the square of.

12. You need to halve 987654321 at least how many times to get a value less than 1?

Section 2: Some problems from A-level Mathematics

1. Using 'for' loops calculate the following by direct summing:

   $$\sum_{r=1}^{5}\frac{1}{4^r},\ \sum_{r=1}^{10}\frac{1}{4^r},\sum_{r=1}^{20}\frac{1}{4^r},\sum_{r=1}^{50}\frac{1}{4^r}$$

2. Write a function that takes, for an arithmetic sequence, the first term, the common difference and a positive integer n as input and returns the n^{th} term.

   Use your function to calculate the following summation by direct summing with a 'for' loop:

   $$7 + 23 + 39 + 55 + \ldots + 160$$

3. Write a program that prints the coordinates of those points (a, b) with $1 < a < 10$ and $1 < b < 10$, a and b integers, which are inside the circle centred at (4,5) with radius 3.

4. Assuming $a \neq c$, write a program that takes inputs of a, b, c, d and t and prints to the screen the y-coordinate of the point on the straight line joining (a,b) and (c, d) with x-coordinate of t.

5. You can invest £1000 for one year in any of the following banks:

   Bank 1 pays you 100% interest once, at the end of the year.

   Bank 2 pays you 50% interest after six months, and then a further 50% interest after the second sixth months.

   Bank 3 pays you 33.3333…% interest after 4 months, then 33.3333…% after another 4 months and the final 33.3333…% after the last four months.

   and so on…

   Write a function which takes input of a positive integer n and returns the amount in your account at Bank n after one year should you invest your £1000 pounds there. What is the smallest value of n such that bank n returns more than £2700 after one year?

6. $\sum_{r=1}^{\infty} \frac{1}{3^r} = 1.5$. How many terms are needed so that the total differs from 1.5 by less than a) 0.1, b) 0.01, c) 0.001

7. Without using any direct function to calculate a fifth root, use a 'while' function to find the root of the equation $x^5 - 100 = 0$ correct to four decimal places.

8. This question is related to definition of differentiation ('first principles').

   Here you'll find an approximation to the derivative of $f(x) = x^2$ at $x = 3$.

   Starting with $h = 1$, calculate $\frac{(3+h)^2 - 3^2}{h}$. Then continually halve h and recalculate. Stop doing this when the distance between two successive results is less than 0.00001 and print the value of the latest calculation to the screen.

9. Write a function that takes a, b, n (a positive integer) and f (a function defined between a and b and at a and b) as inputs and calculates the estimate to $\int_a^b f(x)dx$ using the trapezium rule with n trapezia.

Section 3: Extra challenges

1. Consider the sequence of Fibonacci numbers. The first two terms are 1, and each successive term is the sum of the two previous terms. What is the sum of the first 100 Fibonacci numbers?

2. How many terms of the harmonic series $\sum_{r=1}^{\infty} \frac{1}{r}$ are needed for the total to exceed:

   a. 10
   b. 100
   c. 1000

3. Here is a description of the Collatz conjecture:

   Consider a sequence defined iteratively as follows.

   x_0 is a positive integer and then:

   - If x_n is even then $x_{n+1} = \frac{x_n}{2}$
   - If x_n is odd then $x_{n+1} = 3x_n + 1$

   The Collatz conjecture states that this sequence eventually reaches 1 no matter which positive integer is chosen as the starting value.

   Write a program which prints the sequence generated by an initial value $x_0 = m$ where m is an integer. Find the starting value between 1 and 200 whose sequence takes the longest to reach 1.