Programming challenges (suitable for GCSE students and above)

Challenge 1

The smallest prime factor of 1001 is 7.

It is the smallest prime number that divides exactly into 1001.

Write a program to find the smallest prime factor of a positive integer (whole number).

Use your program to find the smallest prime factor of:

a) 72
b) 377
c) 1147
d) 2015
e) 2011

Challenge 2

The largest prime factor of 1001 is 13.

It is the largest prime number that divides exactly into 1001.

Write a program to find the largest prime factor of a positive integer (whole number).

Use your program to find the largest prime factor of:

a) 72
b) 377
c) 1147
d) 2015
e) 2011

Challenge 3

Write a program to find whether a positive integer has any square factors (other than 1).

Use your program to investigate which of these numbers have any square factors (other than 1):

a) 72
b) 2011
c) 14700

Challenge 4

Prime pairs are integers (whole numbers) that are prime and differ by 2 (i.e. one is two more than the other).

3 and 5 are a prime pair. We'd write the pair as (3,5)

Write a program to find all of the prime pairs less than a maximum integer, m.

List all the prime pairs less than 200.

Look at the prime pairs you have. Ignore the pair (3,5) and look at the other pairs.

What do you notice about all of the other pairs? (Hint: look at the number between the values in each pair)

Why do you think this is?

Challenge 5

The Highest Common Factor (HCF) of two numbers is the highest integer (whole number) that divides exactly into both numbers

Write a program to find the HCF of any two numbers.

Use you program to find the HCF of the following pairs of numbers

a) 36 and 54
b) 275 and 915
c) 1324 and 4502
d) 9763 and 14201

Challenge 6

The Lowest Common Multiple (LCM) of two numbers is the lowest integer (whole number) that both numbers divide exactly into. For example, the lowest common multiple of 16 and 24 is 48.

Write a program to find the LCM of any two numbers.

Use you program to find the LCM of the following pairs of numbers

a) 36 and 54
b) 275 and 915
c) 1324 and 4502
d) 9763 and 14201

Challenge 7

The “Look and Say” sequence is a famous mathematical novelty.

It looks like this:

1
11
21
1211
111221
312211
13112221

Each term in the sequence is found by reading out the one before it e.g. if a term is 1211, the next term is found by reading out what you see – in this case, you can see one one, followed by one two, followed by two ones. The next number is therefore 111221.

John Conway proved that the ratio of a term divided by the previous term approaches a limit.

Write a program to find a good approximation to that limit.

Write a program that creates a look and say sequence for any input

Challenge 8

A *balanced* number is one with $2n$ digits such that the sum of the first n digits is equal to the sum of the last n digits. Numbers may not start with 0. Digits may be repeated

3241 is a balanced number: $3 + 2 = 4 + 1$.

3324 is a balanced number

Write a program to find all of the four digit balanced numbers that use only the digits 0,1,2,3 and 4.

Adapt your program to find all of the four digit balanced numbers that use the digits 0 to k where k is a single digit number that the user enters.

Challenge 9

The multiples of 3 and 5 that are below 10 are 3, 5, 6 and 9. These numbers add up to 23.

Find the sum of all the multiples of 3 or 5 below 1000.

Write a program to find the sum of all the multiples of 3 or 5 below 1000.

Adapt your program to find the sum of all the multiples of 3 or 5 below k where k is a positive integer that the user enters.

Challenge 10

The Fibonacci sequence starts 1, 1. Subsequent terms are found by adding the two previous terms together.

The first 8 terms of the Fibonacci sequence are 1, 1, 2, 3, 5, 8, 13, 21

Write a program to find the sum of all the even terms of the Fibonacci sequence that are below 10,000.

Adapt your program to find the sum of all the odd terms of the Fibonacci sequence that are below 10,000.

A level challenges

Diophantine Equations

1. (i) Write a program to find all the positive integer solutions to $ax+by=c$

   (ii) Use your program to find the positive integer solutions to
   (a) $3x+5y=60$
   (b) $2x+4y=77$
   (c) $12x+18y=100$
   (d) $12x+18y=120$

   (iii) Show that if c is not a multiple of the highest common factor of a and b then $ax+by=c$ has no solutions that are positive integers.

2. (i) Write a program to count how many Pythagorean triples there are with hypotenuse < 60.

   (ii) Edit your program so that it only outputs primitive Pythagorean triples with hypotenuse < 60.

   (iii) Which value/values occur as the hypotenuse in more than one triple in (i)? Use your answers to part (ii) to explain why.

3. (i) Write a program to show that Fermat's Last Theorem is true for $n \le 7$, $x, y \le 100$.

   (ii) Can you find any examples of values of x and y such that $x^3 + y^3$ is one more or one less than a perfect cube and both x and y are greater than 1?

4. (i) Write a program that will give all the positive integer solutions to $x^2 - ny^2 = 1$ where x and y are both less than or equal to some maximum value, m.

   (ii) Use your program to find all the positive integer solutions to $x^2 - 2y^2 = 1$ and $x^2 - 7y^2 = 1$ for $1 \le x \le 200$ and $1 \le y \le 200$.

   (iii) Use your answers to part (ii) to give a rational approximation to $\sqrt{14}$, as accurately as possible. How accurate is this approximation?

Further Investigation

5. The Indian Mathematician Ramanujan famously stated the number 1729 is a very interesting number as it is the smallest number expressible as the sum of two positive cubes in two different ways. Write a program to find a smaller number that is expressible as the sum of two integer cubes in two different ways.

6 (i) Will $m^2 - n^2$, $2mn$, $m^2 + n^2$ always generate a primitive Pythagorean triple?

(ii) Can all primitive Pythagorean triples be written as $m^2 - n^2$, $2mn$, $m^2 + n^2$?

7 Investigate $x^2 - ny^2 = -1$. For which values of n will the equation have solutions?