Autograph for AS Core Mathematics: Coordinate Geometry/Algebra

Straight lines

1. Generate a dynamic $y = mx + c$

   Questions:
   a. How does m affect the line?
   b. How does c affect the line?
   c. For a given x-value what is the y-value (or vice-versa)?
   d. Can the line ever be horizontal/vertical?
   e. What does m tell you about any two points on the line?
   f. Give me an example of a line
      i. that passes through (2,5).
      ii. that passes through the origin.
      iii. that passes through the x-axis at $x = 3$.

2. Generate a line from a point and a gradient using $y - y_1 = m(x - x_1)$:
   Add a point to the screen**.**
   Select the point and right-click (or use the object menu) and select gradient line.
   Enter the gradient as **m**.
   With the line selected display the text.

   Questions
   a. Autograph will display the line in the form $y = mx + c$. Can you show why this is this equivalent to $y - y_1 = m(x - x_1)$ where (x_1, y_1) are the coordinates of the point?
   b. What does the equation tell you about other points on the line? Can you find some other points on the line?
   c. Can the line ever be horizontal/vertical?
   d. Give me an example of a line
      i. that passes through (2,5).
      ii. that passes through the origin.
      iii. that passes through the x-axis at $x = 3$.

3. Generate a dynamic $ax + by = c$:
   Add a point on the x-axis and a point on the y-axis.
   Create a line between the 2 points.
   With the line selected display the text.

   Questions:
   a. Can you rewrite the equation given so that all the coefficients are whole numbers?
   b. Is there only one version of the equation for each pair of points?
   c. How are the intersections with the axes related to the equation?
   d. Is there more than one equation that gives the same line?
   e. Can the line ever be horizontal/vertical?
   f. Find an example of a line in this form:

i. that passes through (2,5).
ii. that passes through the origin.
iii. that passes through the x-axis at $x = 3$.
iv. with negative gradient.

4. Generate a straight line and construct parallel and perpendiculars through a point.

   Questions
   a. Can you explain the equations?
   b. Why is the gradient of the perpendicular line $-1/m$?
   c. How does this relate to vertical and horizontal lines?
   d. What is the relationship between perpendicular lines in the form $ax + by = c$?

Quadratics

5. Generate a dynamic version of $y = ax^2 + bx + c$

   Questions
   a. What is the effect of varying c?
   b. What is the effect of varying a?
   c. What is the effect of varying b?
   d. Give me an example of a quadratic
      i. that passes through the origin.
      ii. that passes through (2,5)
      iii. that cuts both the positive and negative x-axis.
      iv. that passes through (1,1) and (2,5).
      v. that doesn't cut the x-axis.

6. Generate a dynamic version of $y = ax^2 + bx + c$

   Questions
   a. With $a = 1$ describe the conditions on b and c needed so that:
      i. The curve crosses the x-axis at two different points.
      ii. The curve touches the x-axis at a single point.
      iii. The curve doesn't cross or touch the x-axis?
   b. Vary a. Describe the conditions the conditions on a, b and c needed so that:
      i. The curve crosses the x-axis at two different points.
      ii. The curve touches the x-axis at a single point.
      iii. The curve doesn't cross or touch the x-axis?

7. Plot a parabola based on its vertex and find its equation and roots:
   Create a general point by entering the coordinates **(p, q)**.
   Enter the equation **$y=(x-p)^2+q$**.
   Questions
   a. What is the equation of the curve in the form $y = x^2 + bx + c$?
   b. Where does the curve cut the x-axis?

c. Where does the curve cut the *y*-axis?
d. Can you find a parabola in this form that cuts the *x*-axis at:
    i. (0,0) and (2,0)?
    ii. (2,0) and (8,0)?
e. Also see: http://www.furthermaths.org.uk/files/gcseextension/NA1.pdf

8. Plot $y = ax^2 + bx + c$ and $y = mx + k$. Solve f(x)=g(x) to find their points of intersection.

Questions

a. For a given parabola can you find examples of lines that cut the curve at
    i. 2 distinct points?
    ii. 1 point?
    iii. 0 points?
b. Can you show why these are the case algebraically?
c. For a given quadratic can you find examples of lines that cuts the curve at 2 distinct points that are both to the right of the vertex of the parabola?
d. Add a point to the line. Can you find a parabola where the line is a tangent at the point?

Circles

9. Create a dynamic circle: $(x - a)^2 + (y - b)^2 = r^2$

Questions

a. Give an example of values of **a**, **b**, and **c** such that the circle:
    i. is completely in the top-right quadrant
    ii. cuts the positive x-axis twice and the positive y-axis twice
    iii. cuts the positive and negative x-axis and the positive and negative y-axis.
    iv. cuts the positive x-axis twice and touches the y-axis.
    v. goes through the origin.
b. If the scales are ignored how many different cases are there?
N.B. A *distinct case* is defined by the quadrant/axis that contains the centre and the number of distinct intersections with the positive and negative x and y axes
c. Give values of a, b and r that are an examples of each case

10. Create a dynamic circle: $x^2 + y^2 = r^2$

Questions

a. Can you find a value of **r** so that the circle that passes through:
    i. exactly 4 points with integer coordinates.
    ii. exactly 8 points with integer coordinates.
    iii. exactly 12 points with integer coordinates.
    iv. exactly 16 points with integer coordinates.
    v. exactly 20 points with integer coordinates.

