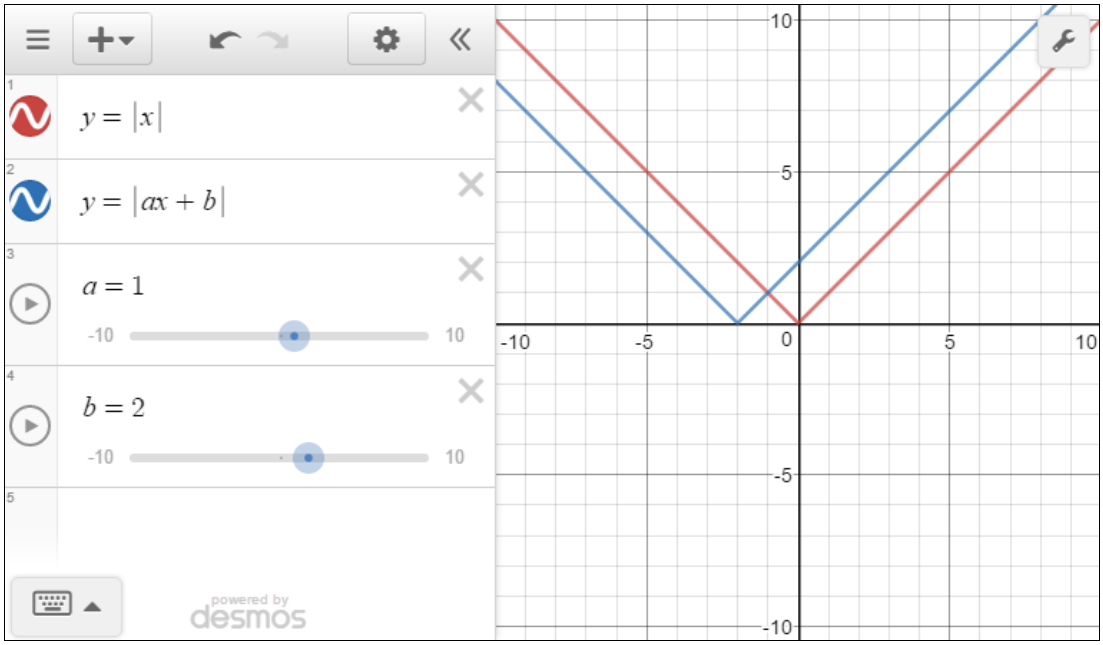
MEI Desmos Tasks for A2 Core

Task 1: Functions – The Modulus Function

1. Plot the graph: $y = |x|$

2. Plot the graph: $y = |ax+b|$ and add sliders **a** and **b**



Questions for discussion

- What combination of transformations maps the graph of $y = |x|$ onto the graph of $y = |ax+b|$?
- Where is the vertex on the graph of $y = |ax+b|$?
- Where does the graph of $y = |ax+b|$ intersect the y-axis?

Problem (*Try the question with pen and paper first then check it on your software*)

Sketch the graph of $y = |3x+2| - 3$ and find the points of intersection with the axes.

Further Tasks

- Investigate the functions of
  - $y = |f(x)|$
  - $y = f(|x|)$

  for different functions $f(x)$, e.g. $f(x) = \sin(x)$ or $f(x) = x^3 - x^2$.
- Investigate the solutions to the inequality $|x + a| + b > 0$.

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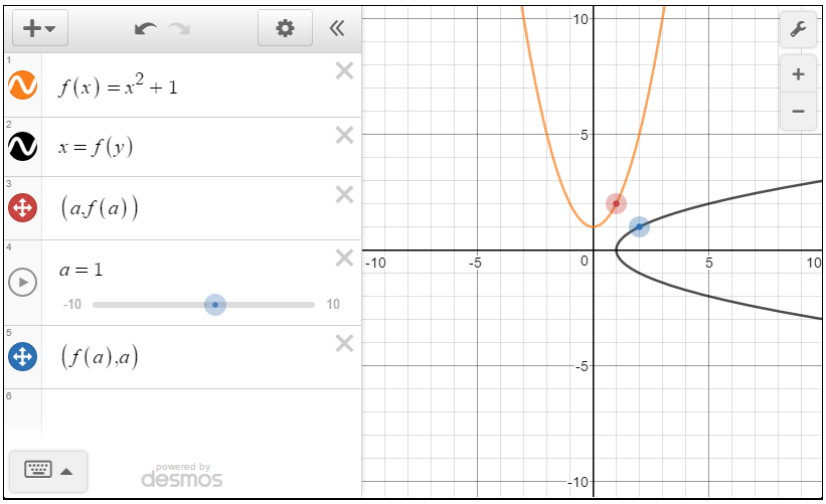
Task 2: Inverse functions

1. Plot function: $\mathrm{f}(x) = x^2 + 1$
2. Plot the inverse function: $x = \mathrm{f}(y)$
3. Plot the point $(a, \mathrm{f}(a))$ and add a slider for **a**
4. Plot the point $(\mathrm{f}(a), a)$



Questions for discussion

- What graphical transformation maps the graph of the original function onto its inverse?
- What is the equation of the graph of the inverse function?
- Why is the inverse function only defined for part of the original function?

Try finding the inverses of some other functions.

Problem (*Try the questions with pen and paper first then check them on your software*)

Find inverses of the following functions:

$\mathrm{f}(x) = (x+3)^2$ $\qquad$ $g(x) = x^3$ $\qquad$ $\mathrm{h}(x) = \dfrac{1}{x-2}$

Further Tasks

- Find the inverse of the function $\mathrm{f}(x) = x^2 + 6x + 7$. Can you always find the inverse of a quadratic function $\mathrm{f}(x) = ax^2 + bx + c$?
- Investigate the graphs of the inverse trigonometric functions (you might find radians more convenient for this).

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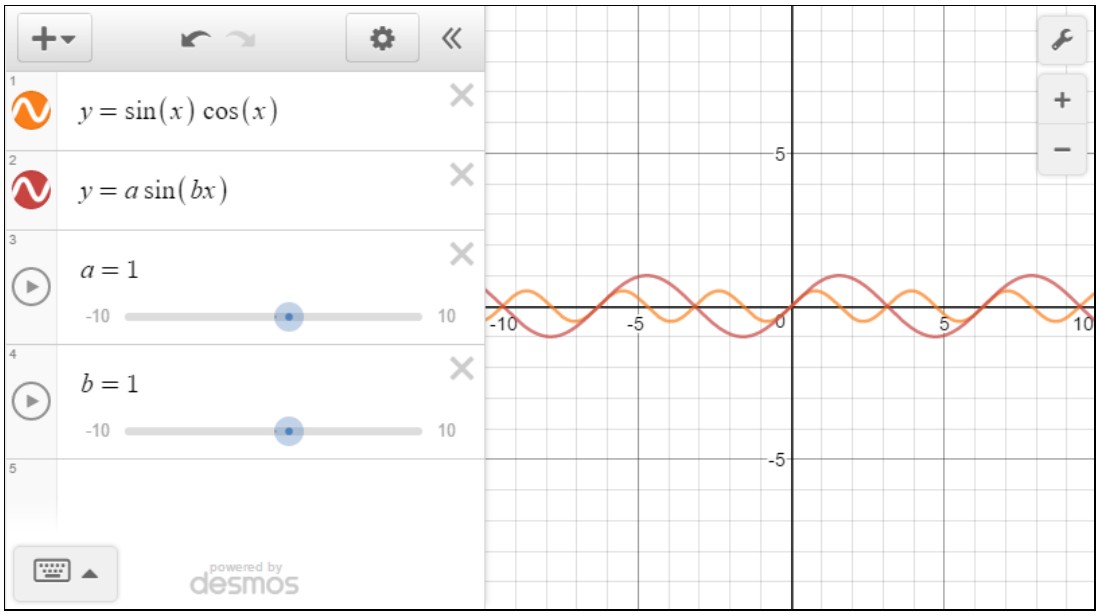
Task 3: Trigonometry – Double Angle formulae

1. Plot the graph: $y = \sin(x)\cos(x)$

2. Plot the graph $y = a\sin(bx)$ and add the sliders **a** and **b**



Questions for discussion

- For what values of a and b does $\sin x \cos x = a \sin bx$?
- Can you find values of a, b and c so that:
  - $\cos^2 x = a \cos(bx) + c$
  - $\sin^2 x = a \cos(bx) + c$
- How do these relationships link to the double angle formulae for sin and cos?

Problem (*Try the question with pen and paper first then check it on your software)*

Solve $\sin 2\theta - \cos\theta = 0$ in the range $0 \le \theta < 2\pi$.

Further Tasks

- Describe the relationship $\tan 2\theta \equiv \dfrac{2\tan\theta}{1-\tan^2\theta}$ graphically.
- Find expressions for $\sin 3\theta$ and $\cos 3\theta$ in terms of $\sin\theta$ and $\cos\theta$.

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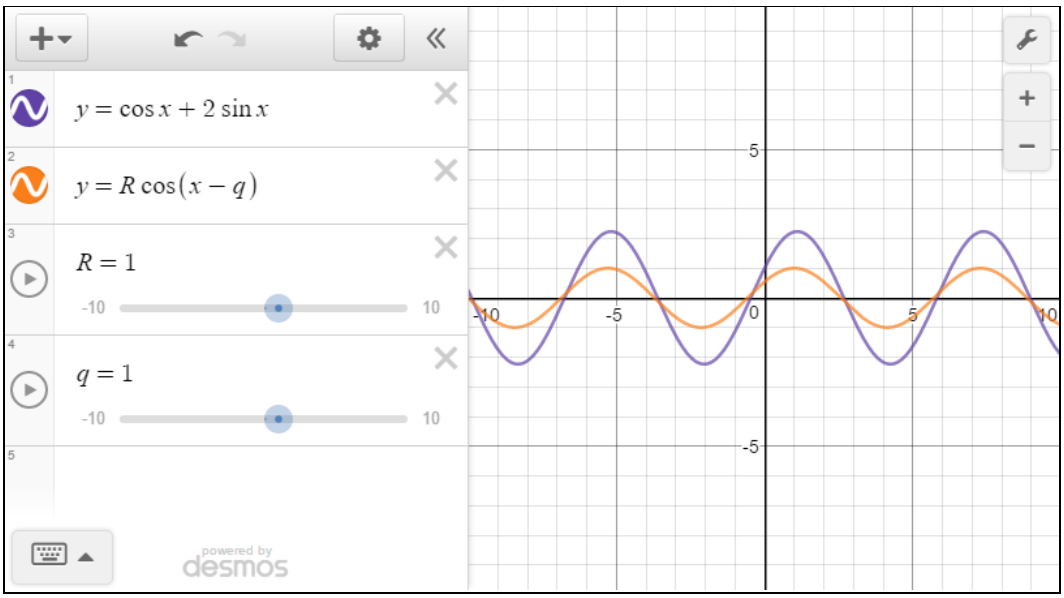
Task 4: Trigonometry: *R*cos(*θ*–*α*)

NB **q** is being used as a substitute for **α** when plotting graphs in the software.

1. Plot the graph $y = \cos x + 2\sin x$

2. Plot the graph $y = R\cos(x - q)$ and add sliders **R** and **q**



Questions for discussion

- Can you find values of **q** and **R** so that the curves are the same?
- Can you find values for **q** and **R** for any *a* and *b* where $a\cos x + b\sin x = R\cos(x - q)$?
- Can you explain the relationship using $R\cos(x - \alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha$?

Problem (*Try the question with pen and paper first then check it on your software)*

Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $0 < \alpha < \frac{\pi}{2}$.

Further Tasks

- Explore how the form $R\cos(\theta - \alpha)$ can be used to find the maximum value of $a\cos\theta + b\sin\theta$ and the angle at which it occurs.
- Investigate the height of a rectangle as it is rotated through an angle *θ* about one of its corners.

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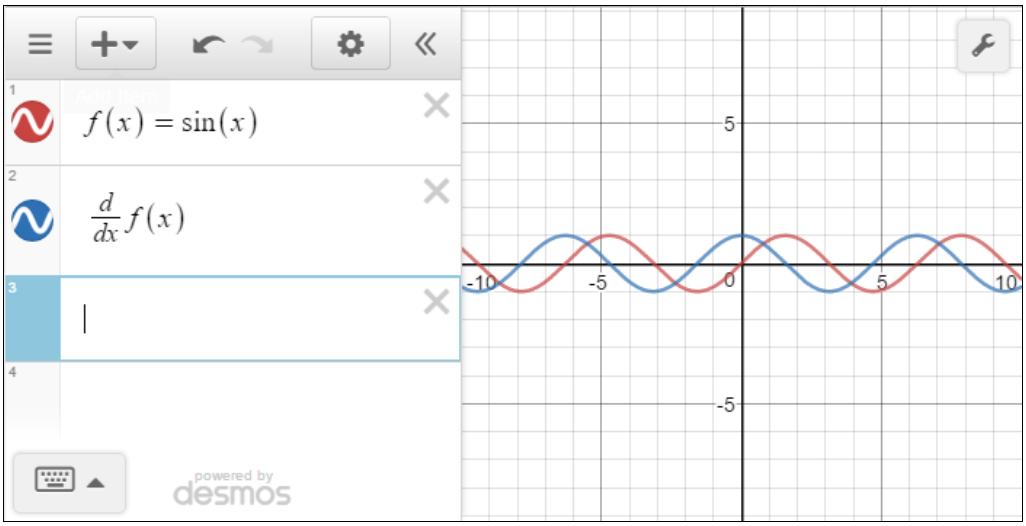
Task 5: Differentiation – Trigonometric functions

NB It is essential that the Graph Settings are set as Radians (not Degrees)

1. Plot the function $f(x) = \sin x$
2. Plot the derivative: $\frac{d}{dx}f(x)$ ∘ ○ ○

You can type d/dx or use the d/dx key in: funcs > misc > d/dx



Questions for discussion

- How does the derivative vary as x varies:
  - What are its maximum and minimum values?
  - For what values of x do these max/min occur?
  - When is the derivative 0?
- Can you suggest a function for the derivative?
- Can you suggest a function for the derivative of $f(x) = \cos(x)$?

Problem (*Check your answer by plotting both the graph and the tangent on your software)*

Find the equation of the tangent to the curve $y = \sin x$ at the point $x = \frac{\pi}{3}$.

Further Tasks

- Investigate the derivatives of $f(x) = \sin ax$ and $f(x) = b \sin x$.
- Explain why this wouldn't work as neatly if the angle was measured in degrees.

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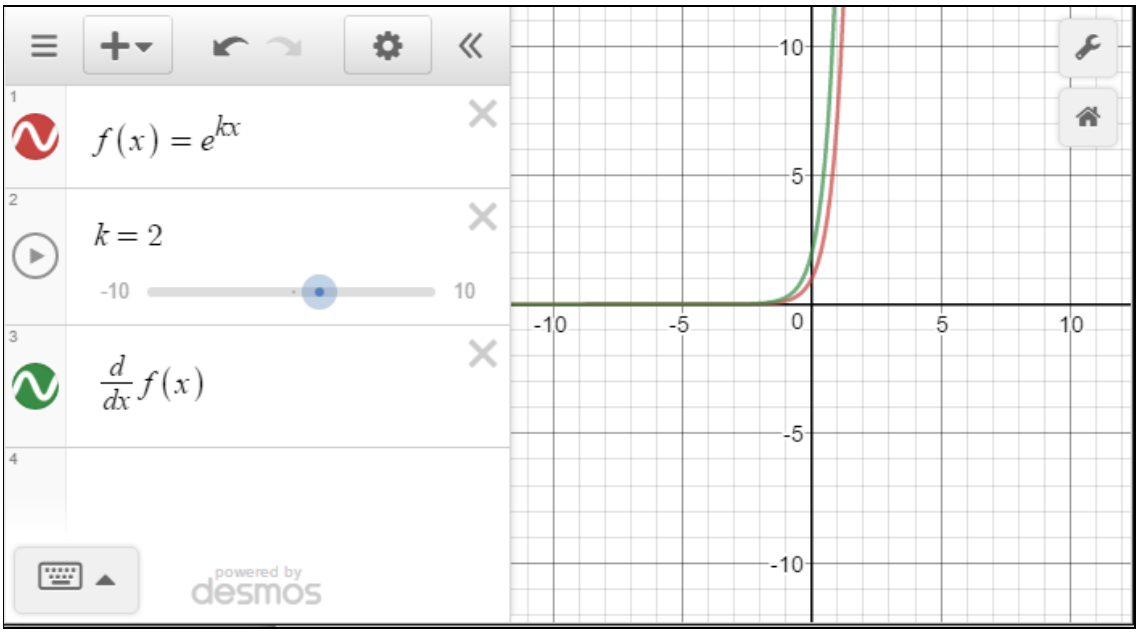
Task 6: Derivate of exponential functions $y=e^{kx}$

1. Plot the function $f(x) = e^{kx}$ and add a slider for ***k***
2. Plot the derivative: $\frac{d}{dx}f(x)$ ∘ ○ ○

You can type d/dx or use the d/dx key in: funcs > misc > d/dx



Question for discussion

- How is the derivative related to the function?

Problem (*Check your answer by plotting the graph and the tangent on your software)*

Find the equation of the tangent to the curve $y = e^{2x}$ at the point $x = 1$.

Further Tasks

- Find the tangent to $y = e^x$ that passes through the origin.
- Find the gradient of the tangent to $y = 3^x$ when $x = 0$.

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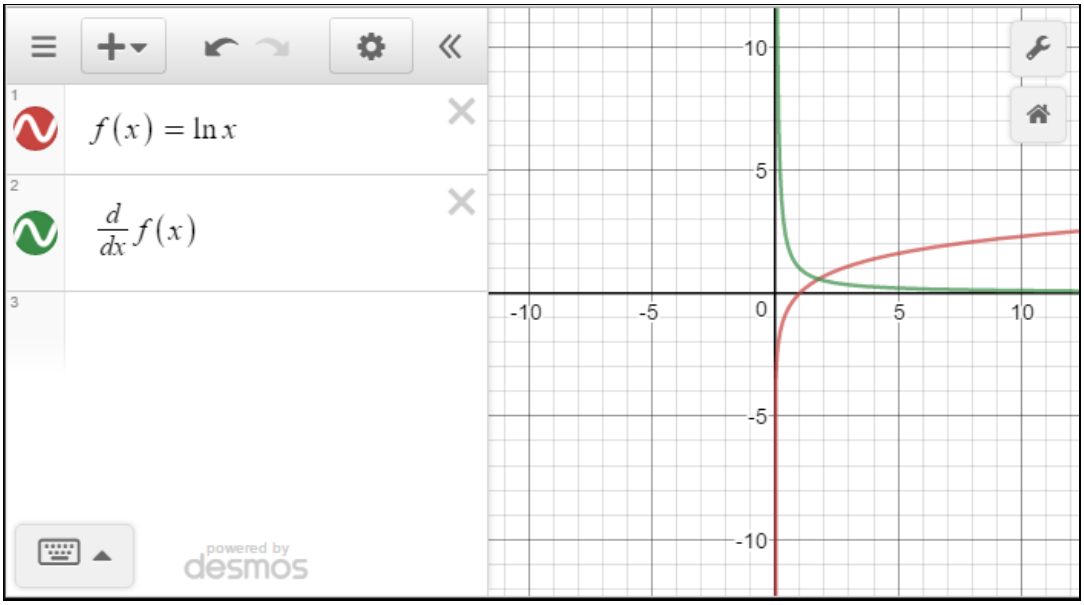
Task 7: Derivative of the natural logarithm $y=\ln x$

1. Plot the function $f(x) = \ln x$ and add a slider for ***k***
2. Plot the derivative: $\frac{d}{dx} f(x)$

You can type d/dx or use the d/dx key in: funcs > misc > d/dx



Questions for discussion

- What is equation of the derivative?
- How does this derivative change for the graphs of $f(x) = \ln 2x$, $f(x) = \ln 3x$ … ?

Problem (*Check your answer by plotting both the graph and the tangent on your software)*

Find the equation of the tangent to the curve $y = \ln x$ at the point $x = 2$.

Further Tasks

- Find the tangent to $y = \ln x$ that passes through the origin.
- Explain the relationship between the derivatives of $y = e^x$ and $y = \ln x$.
  Hint: consider the point (a,b) on $y = e^x$ and the reflected point (b,a) on $y = \ln x$

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Task 8: Converting parametric equations to cartesian equations

1. Plot the parametric curve $x = t^2$, $y = t^3$ by entering: (t^2, t^3)

2. Change the domain to $-10 \leq t \leq 10$

3. Plot the curve $y = x^{\frac{3}{2}}$

4. Plot the curve $y = x^{\frac{2}{3}}$

Questions for discussion

- Which of the cartesian equations gives the same graph as the parametric equation and can you explain why algebraically?
- Can you find a cartesian equation for other parametric curves:

  e.g. $x = 2t + 1,\ y = \frac{1}{t}$ $x = \cos t, y = \sin t$

Problem (*Try the question with pen and paper first then check it on your software)*

Find a cartesian equation of the curve $x = e^t, y = t^2 + 1.$

Further Tasks

- Compare finding the points of intersection with the x and y axes for the same curve written in parametric and cartesian form.
- Explore converting from cartesian to parametric form.

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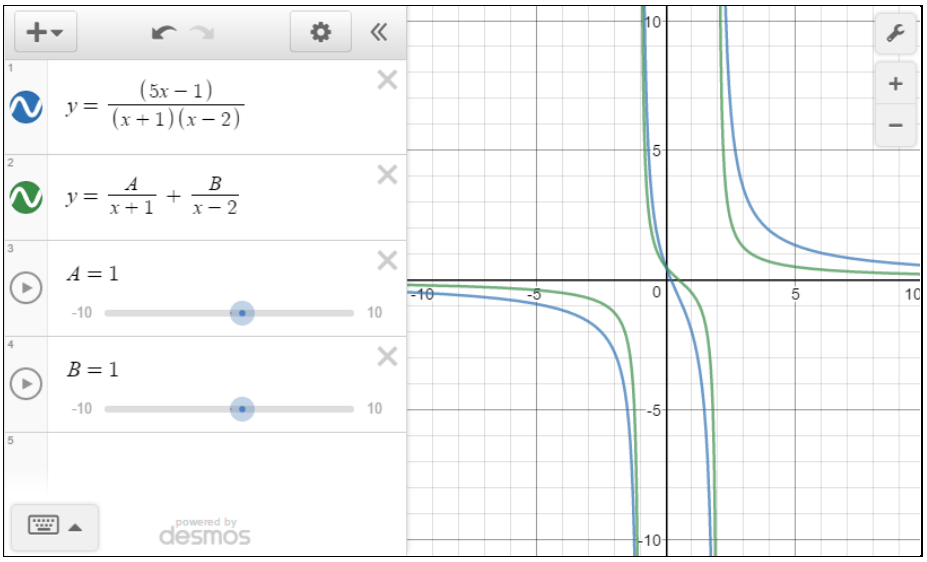
Task 9: Partial Fractions

1. Enter the function $y = \dfrac{5x-1}{(x+1)(x-2)}$
2. Enter the function $y = \dfrac{A}{x+1} + \dfrac{B}{x-2}$ and add sliders for **A** and **B**



Find values of **A** and **B** so that the graphs of the functions are the same.

Questions for discussion

- How could you find the values using $\dfrac{5x-1}{(x+1)(x-2)} = \dfrac{A}{x+1} + \dfrac{B}{x-2}$?
- Does this method work for $\dfrac{2x+7}{(x+2)(x+3)} = \dfrac{A}{x+2} + \dfrac{B}{x+3}$?

Problem (*Try the question with pen and paper first then check it on your software)*

Find values of *A* and *B* so the following can be expressed as partial fractions:

$$\frac{7x-14}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$$

Further Tasks

- Find *A*, *B* and *C* such that $\dfrac{5x^2+3x+7}{(x+2)(x^2+3)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+3}$
- Find *A*, *B* and *C* such that $\dfrac{7x^2+29x+28}{(x-1)(x+3)^2} = \dfrac{A}{x-1} + \dfrac{B}{x+3} + \dfrac{C}{(x+3)^2}$

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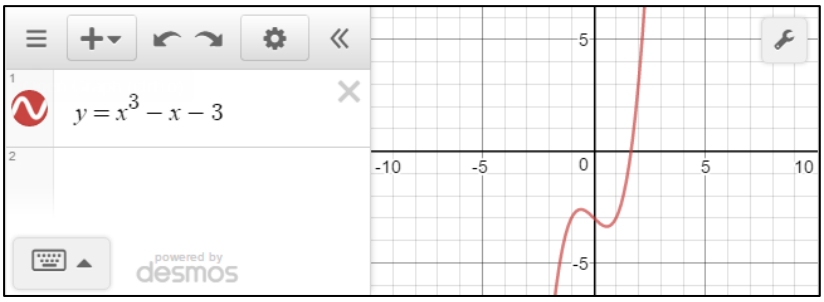
Task 10 – Numerical Methods: Change of sign

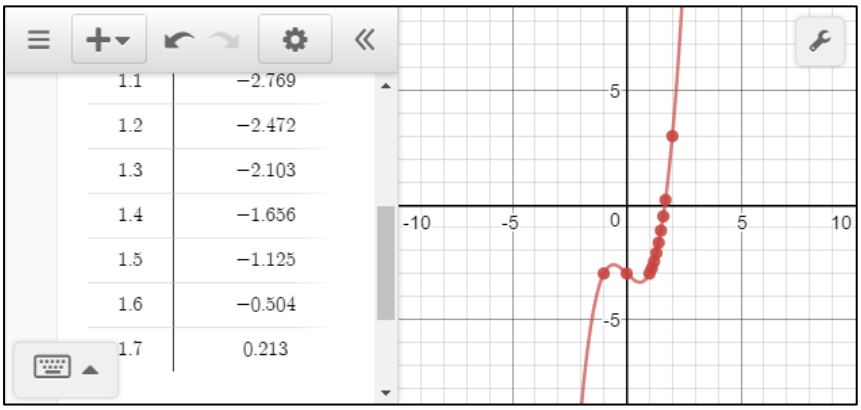
1. Plot the graph of $y = x^3 - x - 3$



In this example you can see that the root lies between $x = 1$ and $x = 2$.

2. With the curve selected press Edit  and Convert to table 

3. In the x-column enter the values 1.1, 1.2, 1.3, … until you observe a change of sign in y.



In this example there is a change of sign between $x = 1.6$ and $x = 1.7$.

4. You can now investigate further by entering x-values from 1.6 to 1.7 in steps of 0.01.

5. You can check your answer by zooming-in and selecting the point of intersection with the x-axis.

Try using your software to find the roots of other equations using the change of sign method.

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Teacher guidance

Task 1: The Modulus Function

Students should consider how this relates to the graph of $y = ax+b$

Problem solution: $x = -\frac{5}{3}, \frac{1}{3}\ y = -1$

Students might need some help structuring the investigation into $|x + a|+b > 0$. One strategy is to fix either a or b and investigate changing the other parameter.

Task 2: Inverse functions

The aim of this task is to reinforce the link between the reflection in the line $y = x$ and rearranging $y = \mathrm{f}(x)$ to express x in terms of y. The software can plot this using $x = \mathrm{f}(y)$

Problem solutions:

$\mathrm{f}^{-1}(x) = \sqrt{x} - 3$ $\qquad$ $g^{-1}(x) = \sqrt[3]{x}$ $\qquad$ $\mathrm{h}^{-1}(x) = \frac{1}{x} + 2$

It is important to emphasise that the domain of the original function needs to be restricted so that it is one-to-one for the inverse to be a function.

Task 3: Trigonometry – Double Angle formulae

Students might need some help structuring the investigation into $\sin x \cos x = a \sin (bx)$. One strategy is to fix b and investigate changing a first to find a curve with the correct amplitude.

Use of the compound angle formulae for $\sin(a+b)$ and $\cos(a+b)$ might be useful for some students to verify their results.

Problem solution: $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Task 4: Trigonometry: *R*cos(*θ*–*α*)

Students are expected to be able to relate their findings to the expansion of $R\cos(x - A) = R\cos x \cos A + R\sin x \sin A$.

Problem solution:
$4\cos\theta + 3\sin\theta = 5\cos(\theta - 0.644)$.

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Task 5: Differentiation – Trigonometric functions

By considering key points the students should be able to observe that this has the same shape as cos(x).

Problem solution:

$y = \frac{x}{2} + \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ or $y = 0.5x + 0.342$

Task 6: Derivatives of exponential functions $y=e^{kx}$

This task can be done on its own or with task 7. The aim of this task is for students to be able to find the gradients and equations of tangents to exponential functions.

Students should observed that the derivative is the same as the *y*-coordinate for $y = e^x$ before exploring other curves of the form $y = e^{kx}$.

Problem solution:

$y = 14.778x - 7.389$

The second of the further tasks requires students to rewrite $y = 3^x$ as $y = e^{(\ln 3)x}$.

Task 7: Derivative of the natural logarithm $y=\ln x$

This task can be done on its own or with task 6. The aim of this task is for students to be able to find the gradients and equations of tangents to the natural logarithm function.

For the second discussion point students might be surprised that the result doesn't change but they should be encouraged to think of this in terms of laws of logs.

Problem solution:

$y = 0.5x - 0.307$

Task 8: Converting parametric equations to cartesian equations

In the discussion questions students could also consider why the cartesian version does not plot the full curve given by the parametric version.

Solutions to discussion questions:

$x = 2t + 1,\ y = \frac{1}{t}$: $\quad y = \frac{2}{x-1}$

$x = \cos t, y = \sin t$: $\quad x^2 + y^2 = 1$

Trig-based parametric equations will often require identities to convert to cartesian form.

Problem solution:

$y = (\ln x)^2 + 1$

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Task 9: Partial Fractions

This task can be used as an introduction to partial fractions or as a consolidation exercise. Students should be encouraged to express their methods algebraically.

Solutions to partial fractions:

$$\frac{5x-1}{(x+1)(x-2)} = \frac{2}{x+1} + \frac{3}{x-2} \qquad \frac{2x+7}{(x+2)(x+3)} = \frac{3}{x+2} - \frac{1}{x+3}$$

$$\frac{7x-14}{(x+3)(x-4)} = \frac{5}{x+3} + \frac{2}{x-4} \qquad \frac{5x^2+3x+7}{(x+2)(x^2+3)} = \frac{3}{x+2} + \frac{2x-1}{x^2+3}$$

$$\frac{7x^2+29x+28}{(x-1)(x+3)^2} = \frac{4}{x-1} + \frac{3}{x+3} - \frac{1}{(x+3)^2}$$

Task 10: Numerical Methods – Change of sign

This task is a set of instructions for how to implement the change of sign method on the software. Students are encouraged to work through these instructions and then try solving some equations of their own.

It is useful to have some additional equations for students to be finding the roots of once they have completed this sheet.

