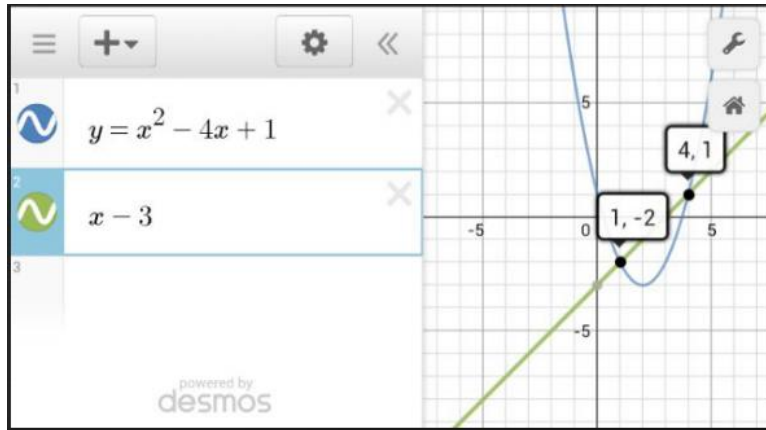


# MEI Desmos Tasks for AS Pure

## Task 1: Coordinate Geometry – Intersection of a line and a curve

1. Add a quadratic curve, e.g.  $y = x^2 - 4x + 1$
2. Add a line, e.g.  $y = x - 3$
3. Select the points of intersection of the line and the curve.



### Questions for discussion

- What is the relationship between the x-coordinates of the points of intersection and the equations of the line and curve?
- Does this work for other curves and lines?

**Problem** (Try the problems with pen and paper first then check it on your software)

Find exact values of the coordinates of the points of intersection of the following:

$$y = x^2 \text{ and } y = 2x + 3 \quad y = x^2 - x \text{ and } y = 2 - x \quad y = x^2 - 2x + 2 \text{ and } y = 2x + 1$$

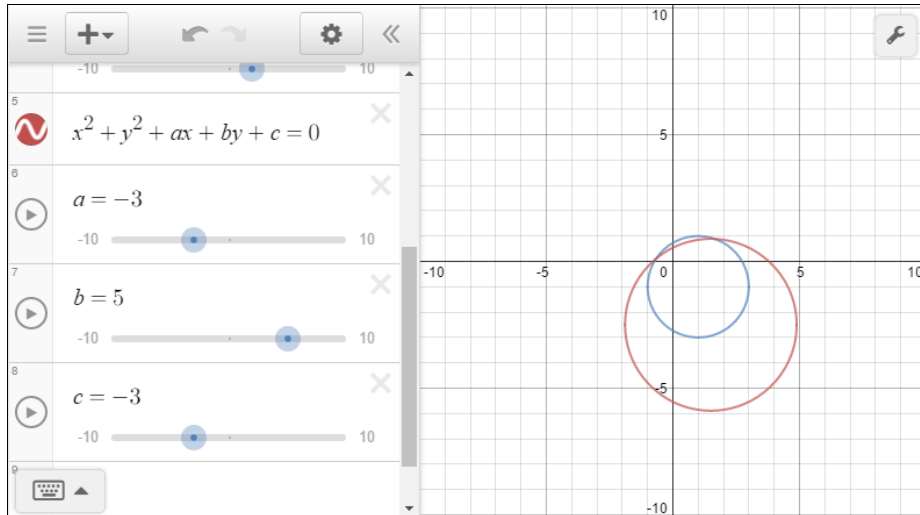
### Further Tasks

- Can you find an example of a line and a curve that would have:
  - Exactly 1 point of intersection?
  - No points of intersection?
- Investigate the number of points of intersection of two curves.
- Investigate the intersection of a line and a circle.

# MEI Desmos Tasks for AS Pure

## Task 2: Coordinate Geometry – Equations of Circles

1. Add the graph:  $(x - p)^2 + (y - q)^2 = r^2$  and add sliders for  $p$ ,  $q$  and  $r$
2. Add the graph:  $x^2 + y^2 + ax + by + c = 0$  and add sliders for  $a$ ,  $b$  and  $c$



### Questions

- For circles of the form  $(x - p)^2 + (y - q)^2 = r^2$  what is the radius and the position of the centre of the circle?
- For circles of the form  $x^2 + y^2 + ax + by + c = 0$  what is the radius and the position of the centre of the circle?

**Problem** (*Try the question with pen and paper first then check it on your calculator*)

Find the radius and the centre of the circle  $x^2 + y^2 - 4x + 2y - 4 = 0$ . Find the exact values of the coordinates of the points of intersection with the  $y$ -axis.

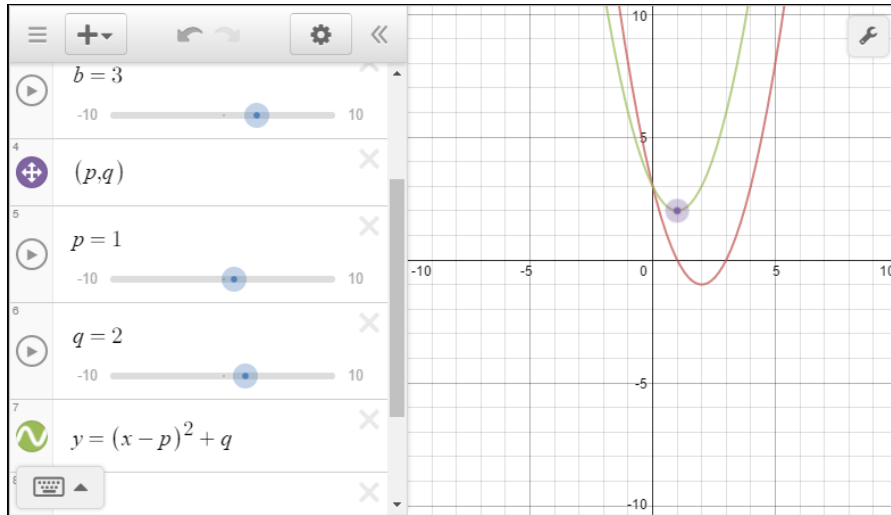
### Further Tasks

- Investigate circles that pass through the origin.
- Investigate circles of the form  $x^2 + y^2 + ax + by + c = 0$  that do not intersect either the  $x$  or  $y$  axes.

# MEI Desmos Tasks for AS Pure

## Task 3: Algebra – Graphs of quadratic functions

1. Add the curve  $y = (x - a)(x - b)$  and add the sliders for  $a$  and  $b$ .
2. Add the point  $(p, q)$  and add the sliders for  $p$  and  $q$ .
3. Add the curve  $y = (x - p)^2 + q$



### Questions for discussion

- Can you find values of  $a$ ,  $b$ ,  $p$  and  $q$  where two graphs are the same?
- What is the relationship between the values of  $a$ ,  $b$ ,  $p$  and  $q$  when the graphs are the same?

**Problem** (Try the problems with pen and paper first then check it on your software)

Solve the equation  $x^2 - 2x - 8 = 0$  by both factorising and completing the square.

### Further Tasks

Change the second curve to  $y = k(x - p)^2 + q$

- Where does this curve cross the  $x$ -axis?
- Can you change the original curve so the two curves are the same?

# MEI Desmos Tasks for AS Pure

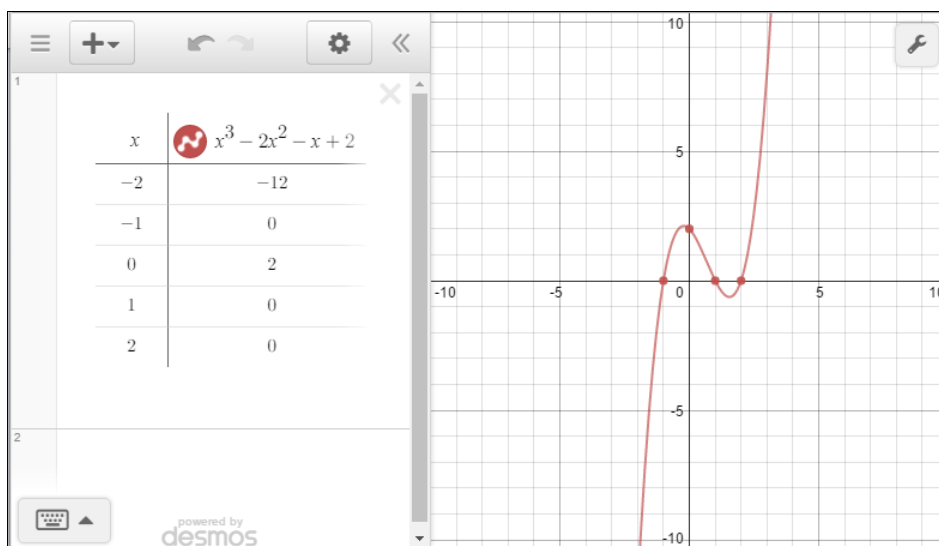
## Task 4: Algebra – The Factor Theorem

1. Add the curve:  $y = x^3 - 2x^2 - x + 2$

2. With the curve selected press Edit



and Convert to table



### Questions for discussion

- How do this table and graph confirm that  $x^3 - 2x^2 - x + 2 = (x + 1)(x - 1)(x - 2)$ ?
- Can you find the factors of the following cubics:  
 $y = x^3 + 4x^2 + x - 6$                        $y = x^3 - 4x^2 - 11x + 30$   
 $y = x^3 - x^2 - 8x + 12$                        $y = x^3 - 7x^2 + 36$

**Problem** (Try the question with pen and paper first then check it on your software)

Show that  $(x - 2)$  is a factor of  $f(x) = x^3 + 4x^2 - 3x - 18$ . Hence find all the factors of  $f(x)$ .

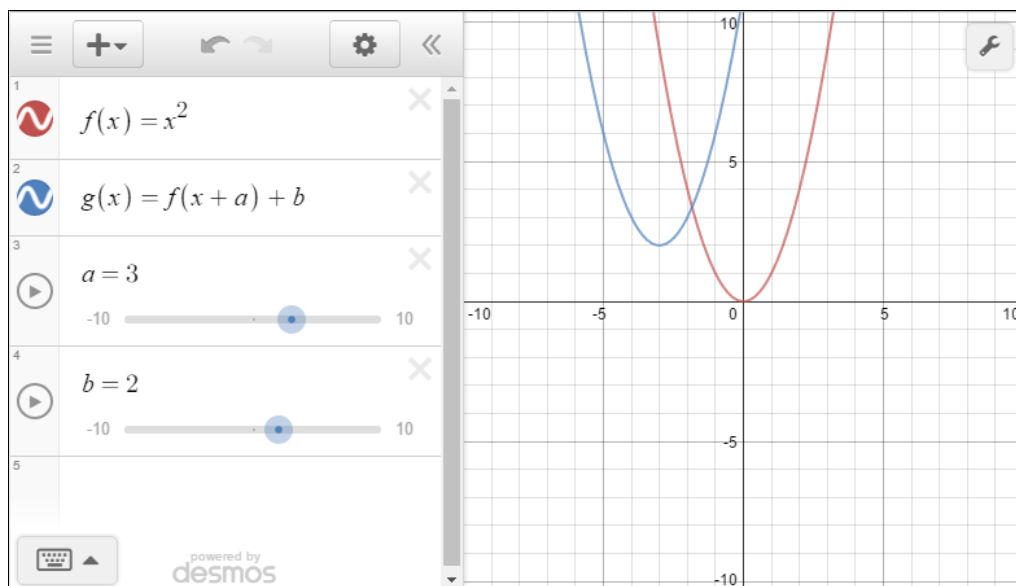
### Further Tasks

- Find examples of cubics that only have one real root.
- Investigate using the factor theorem for polynomials of other degrees, e.g. quadratics or quartics.

# MEI Desmos Tasks for AS Pure

## Task 5: Functions – Transformations

1. Add the function:  $f(x) = x^2$
2. Add the function:  $g(x) = f(x+a) + b$  and add the sliders for  $a$  and  $b$ .



### Questions for discussion

- What transformation maps  $f(x)$  onto  $g(x)$ ?
- Does this work if other functions are entered for  $f(x)$ ?

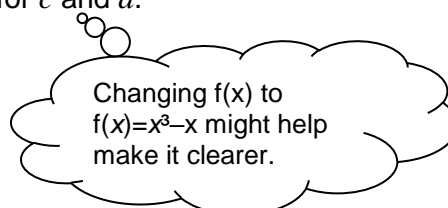
**Problem** (Try the problem with pen and paper first then check it on your software)

Show that  $(x+2)^3 + 3 = x^3 + 6x^2 + 12x + 11$ .

Hence sketch the graph of  $y = x^3 + 6x^2 + 12x + 11$ .

### Further Tasks

- Show that  $f(x) = x^4 - 8x^3 + 24x^2 - 32x + 13$  can be written in the form  $(x+a)^4 + b$  and hence find the coordinates of the minimum point on the graph of  $y = f(x)$ .
- Add the graph:  $h(x) = c f(dx)$  and add sliders for  $c$  and  $d$ .  
What transformation maps  $f(x)$  onto  $h(x)$ ?



# MEI Desmos Tasks for AS Pure

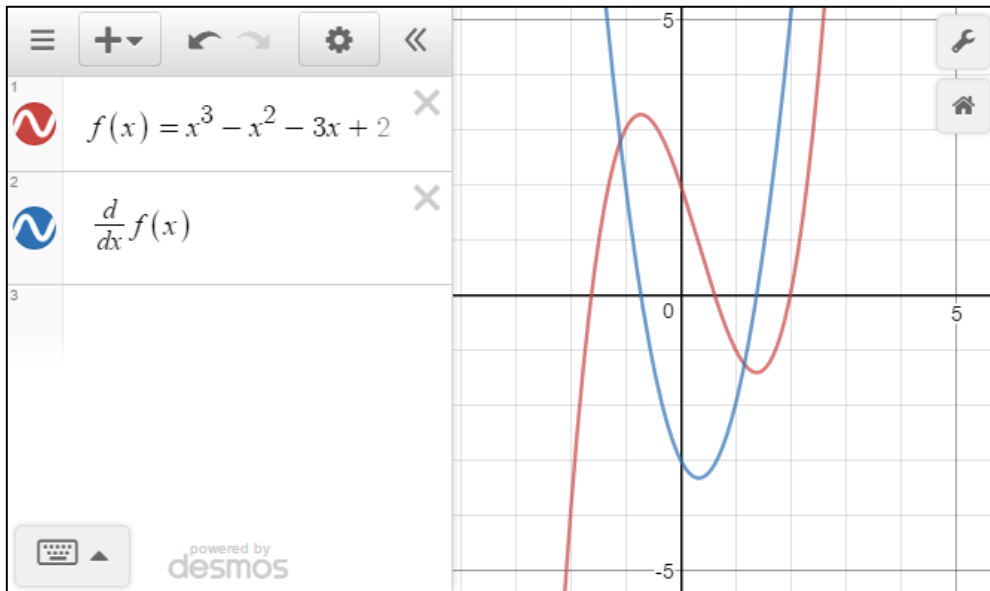
## Task 6: Differentiation – Exploring the gradient on a curve

1. Enter a cubic function: e.g.  $f(x) = x^3 - x^2 - 3x + 2$

2. Enter:  $\frac{d}{dx} f(x)$

• • •

You can type d/dx or use the d/dx key in: funcs > misc > d/dx



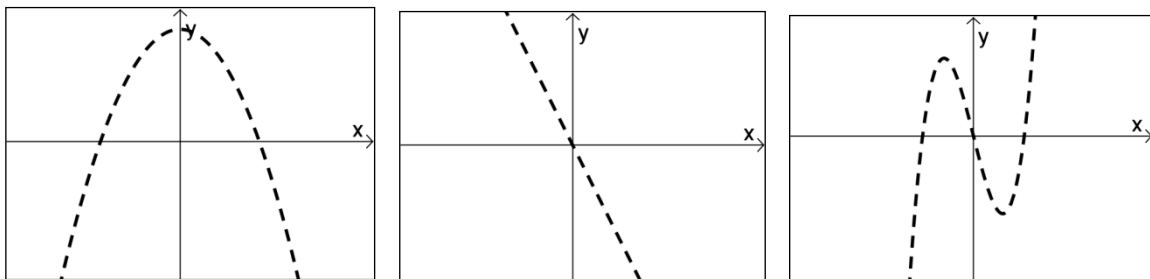
### Question for discussion

- How is the gradient of the tangent (as the point moves) related to the shape of the gradient graph?

Verify your comments by trying some other functions for  $f(x)$ .

### Problem

Change your original function so the gradient function has one of the following graphs:



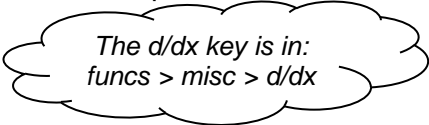
### Extension Task

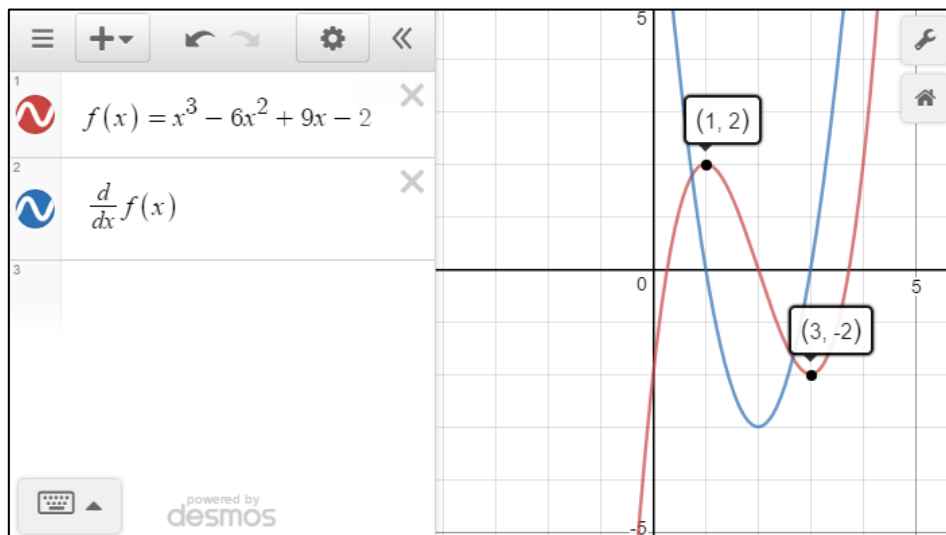
Find the point on the function  $f(x) = x^3 - 6x^2 + 9x - 1$  where the tangent has its maximum downwards slope. Investigate the point with maximum downward slope for other cubic functions.

# MEI Desmos Tasks for AS Pure

## Task 7: Differentiation – Stationary points

1. Add the cubic function:  $f(x) = x^3 - 6x^2 + 9x - 1$
2. Select the maximum and minimum points to show their values

3. Enter:  $\frac{d}{dx} f(x)$ . ° ○ ○ 



### Question for discussion

- How can you use the graph of the gradient function to explain why the function has a local maximum and a local minimum at the points?

Verify your comments by trying some other functions for  $f(x)$ .

**Problem** (Try this on paper first then check the answer on your software)

For the following curves plot the graphs and their derivatives. Use the derivative graph to find where the curve has a maximum or minimum:

$$y = x^2 + 4x + 1$$

$$y = 4 - 6x - x^2$$

$$y = x^3 - 3x$$

$$y = x^3 - 3x^2 + 3x$$

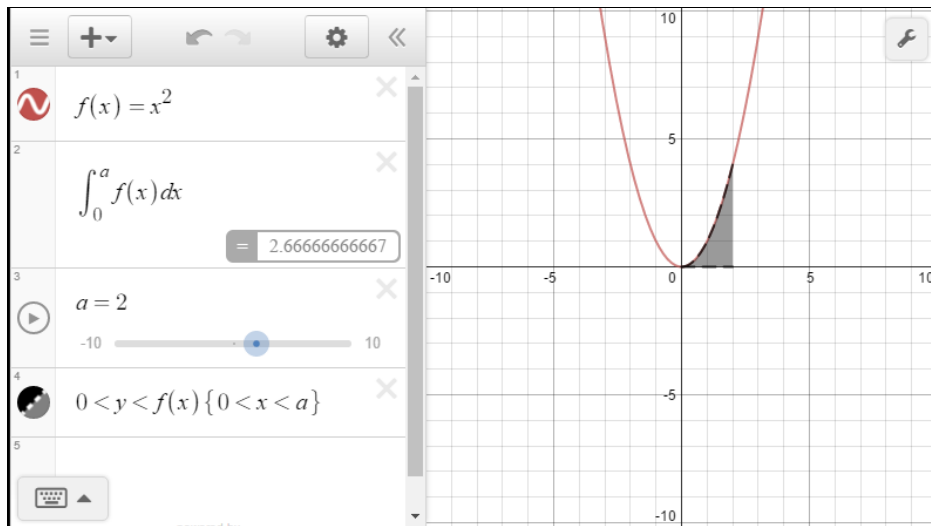
### Extension Task

- Use your software to find the gradient function of  $f(x) = x^3 - 6x^2 + 12x - 5$ . Explain why the function has a stationary point that is neither a maximum nor a minimum (a stationary point of inflection).
- Find some other functions that have stationary points of inflection.

# MEI Desmos Tasks for AS Pure

## Task 8: Integration – Area under a curve

1. Add the function:  $f(x) = x^2$
2. Add  $\int_0^a f(x)dx$ : For the integral type: int and add the slider for  $a$
3. Shade the area under the curve:  $0 < y < f(x) \{0 < x < a\}$



### Questions for discussion

- What is the relationship between the area and the value of  $a$ ?
- What is the relationship if  $f(x)$  is changed to a different power of  $x$ ?

**Problem** (Try the problem with pen and paper first then check it on your software)

Find the area under  $f(x) = x^5$  between  $x = 0$  and  $x = 3$ .

### Further Tasks

- Investigate the area under  $f(x) = x^n$  between  $x = a$  and  $x = b$ .
- Investigate the areas under functions that are the sums of powers of  $x$ :  
e.g.  $f(x) = x^3 + 3x^2 + 4x + 1$



# MEI Desmos Tasks for AS Pure

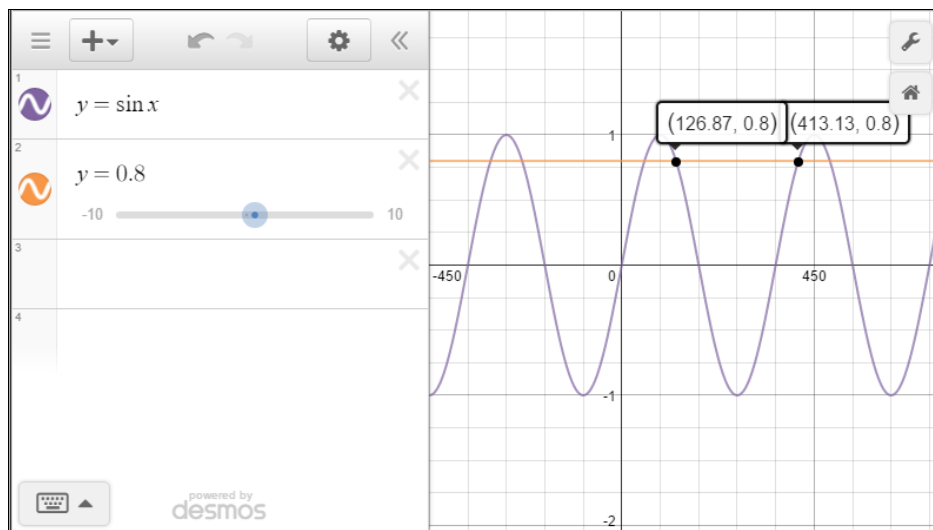
## Task 9: Trigonometry – Trigonometric equations

1. Press the graph setting wrench:



Set the range  $-400 < x < 800$  Step: 90,  $-2 < y < 2$ , Angles: Degrees

2. Add the graph:  $y = \sin x$
3. Add the graph:  $y = 0.8$
4. Click on the points of intersection to see their values



### Questions for discussion

- What symmetries are there in the positions of the points of intersection?
- How can you use these symmetries to find the other solutions based on the value of  $\sin^{-1}x$  given by your calculator? (This is known as the “principal value”.)

**Problem** (Try the question just using the  $\sin^{-1}$  function on your calculator first then check it using the software)

Solve the equation:  $\sin x = 0.2$  ( $-360^\circ \leq x \leq 720^\circ$ )

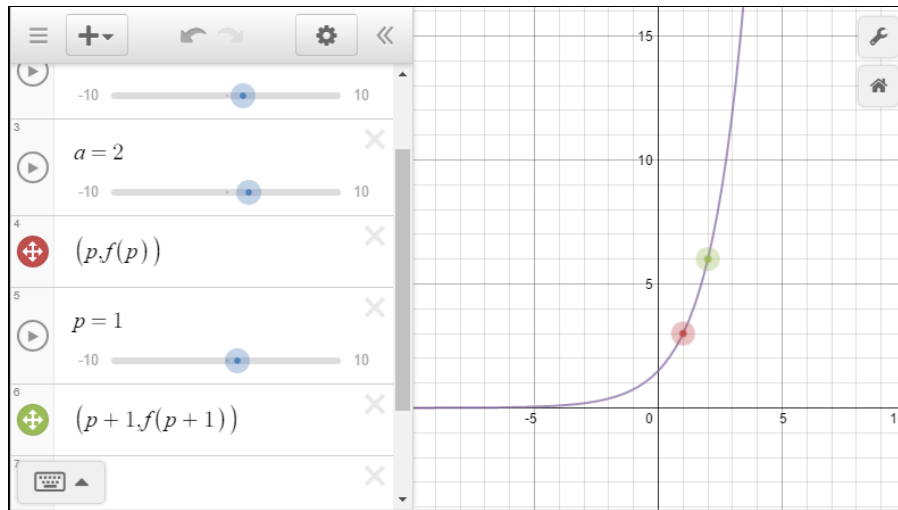
### Further Tasks

- Investigate the symmetries of the solutions to  $\cos x = k$  and  $\tan x = k$ .
- Investigate the symmetries of the solutions to  $\sin 2x = k$ .

# MEI Desmos Tasks for AS Pure

## Task 10: Exponentials and logarithms – Graph of $y = ka^x$

1. Add the function:  $f(x) = ka^x$  and add the sliders for  $a$  and  $k$
2. Add the point:  $(p, f(p))$  and add the slider for  $p$
3. Add the point:  $(p+1, f(p+1))$



### Questions for discussion

- How does varying  $k$  affect the curve?
- How does varying  $a$  affect the curve? Why is it sensible to restrict  $a$  to positive values?
- What is the relationship between the  $y$ -coordinates of two points?

**Problem** (Try the problem with pen and paper first then check it on your software)

The graph of  $y = ka^x$  passes through the points  $(1,10)$  and  $(3,160)$ .  
Find the values of  $a$  and  $k$ .

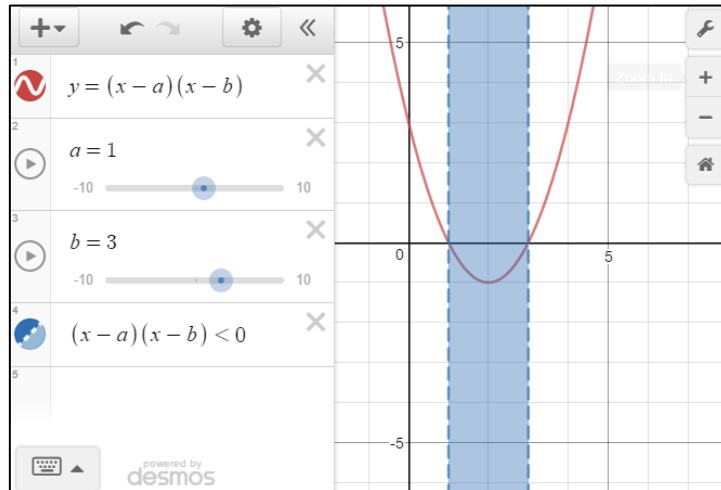
### Further Tasks

- Investigate solving  $ka^x = b$  graphically and by using logs.
- Use the graph of  $y = a^x$  to explain why  $a^{m+n} = a^m a^n$ .

# MEI Desmos Tasks for AS Pure

## Task 11: Quadratic Inequalities

1. Add the curve  $y = (x - a)(x - b)$  and add sliders for  $a$  and  $b$ .
2. Add the inequality  $(x - a)(x - b) < 0$ .



### Questions for discussion

- If the product of two numbers is negative what does this tell you about the numbers?
- Will you always be able to find  $x$ -values for which a quadratic is negative?
- What would the solution to  $(x - a)(x - b) > 0$  look like?

**Problem** *(Try the problem with pen and paper first then check it on your software)*

Sketch the graph of  $y = 2x^2 - x - 6$  and hence solve the inequality  $2x^2 - x - 6 \geq 0$ .

### Further Tasks

- Find the range of values for  $k$  such that  $x^2 - 4x + 3 = kx$  has two distinct roots.
- Investigate  $y > mx + c$  and  $y > ax^2 + bx + c$  graphically.

# MEI Desmos Tasks for AS Pure

## Teacher guidance

### Task 1: Coordinate Geometry – Intersection of a line and a curve

This task can be used to introduce the intersection of a line and a curve.

Students should consider the equation formed by subtracting the linear function from the quadratic and observing its roots. Some students might find it helpful to plot this.

Problem solutions:

$$\begin{aligned} y = x^2 \text{ and } y = 2x + 3 & \quad (-1, 1) \text{ and } (3, 9) \\ y = x^2 - x \text{ and } y = 2 - x & \quad (-2, 4) \text{ and } (1, 1) \\ y = x^2 - 2x + 2 \text{ and } y = 2x + 1 & \quad (-\sqrt{3}+2, -2\sqrt{3}+5) \text{ and } (\sqrt{3}+2, 2\sqrt{3}+5) \end{aligned}$$

### Task 2: Coordinate Geometry – Equations of circles

Students who have not done much investigative work before might need some support structuring their approach: suggest that they change one value at a time and then record what is happening for each. They might need help with a hint that  $c$  should be negative in the second equation initially so they can see a circle.

Students should link the form  $x^2 + y^2 + ax + by + c = 0$  to the completed square form of a quadratic.

Problem solution: Centre  $(2, -1)$ , radius 3.

### Task 3: Algebra – Graphs of quadratic functions

Students should attempt to solve some quadratic equations by completing the square and then making  $x$  the subject. Students might find it helpful to observe the line of symmetry of the curve or the relationship between the completed square solution and solving with the standard formula.

$$\begin{aligned} \text{Problem solution: } x^2 - 2x - 8 = 0 \\ (x - 4)(x + 2) = 0 & \quad x = 4 \text{ or } x = -2 \\ (x - 1)^2 - 9 = 0 & \quad x = 1 \pm 3 & \quad x = 4 \text{ or } x = -2 \end{aligned}$$

### Task 4: The Factor Theorem

This task is intended to reinforce the link between the numerical values of roots, algebraic factors and points of intersection with the  $x$ -axis. In discussions students should be encouraged to explain how *both* the table and the graph indicate what the factors are.

It might be useful for some students to practise expanding products of three factors before attempting this task.

Students will also need to be shown, or to develop, strategies for dividing by a factor, such as equating coefficients, long division or division by the box method.

## MEI Desmos Tasks for AS Pure

Questions:

$$\begin{array}{ll} y = x^3 + 4x^2 + x - 6 : & y = (x-1)(x+2)(x+3) \\ y = x^3 - 4x^2 - 11x + 30 & y = (x-5)(x-2)(x+3) \\ y = x^3 - x^2 - 8x + 12 & y = (x-2)^2(x+3) \\ y = x^3 - 7x^2 + 36 & y = (x-6)(x-3)(x+2) \end{array}$$

The third question can be used to demonstrate an example of a cubic with a repeated root.

Problem solution:

$$x^3 + 4x^2 - 3x - 18 = (x-2)(x+3)^2$$

### Task 5: Functions – Transformations

If students have met trigonometric functions then these work well for this task.

For the problem students should expand the function using either a binomial expansion or by multiplying out the brackets.

The graph of the function is the graph of  $f(x) = x^3$  translated by  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ .

Be careful with second further task (horizontal stretches) – they can look like vertical stretches for many functions but this is an excellent discussion point.

Students should also take care with the scales on the axes here as these can cause confusion.  $f(x) = \sin x$  or  $f(x) = x^3 - x$  are good functions to use for this.

### Task 6: Differentiation – Exploring the gradient on a curve

The aim of this task is for students to investigate (or verify if they have already met it) the shape of derivative functions. They should be encouraged to discuss why the derivatives have the shape they do in terms of the gradient of the tangent to the curve at different points. It can be used as an introduction to the topic or to consolidate what they have already learnt.

Problem solution (possible solutions):

$$f(x) = -x^3 + 2x \qquad f(x) = -x^2 \qquad f(x) = x^4 - 2x^2$$

### Task 7: Differentiation – Introduction to Stationary Points

This task highlights the link between the derivative and determining the nature of stationary points on curves. Students should be encouraged to consider how the derivative crosses the x-axis (+ve to -ve or -ve to +ve) to determine the nature of the stationary points.

Problem solutions:

$$\begin{array}{ll} y = x^2 + 4x + 1 & \text{min: } (-2, -3) \\ y = 4 - 6x - x^2 & \text{max: } (-3, 13) \\ y = x^3 - 3x & \text{min: } (1, -2) \quad \text{max: } (-1, 2) \\ y = x^3 - 3x^2 + 3x & \text{no maxima or minima} \end{array}$$

The final example can be used to discuss stationary points that are points of inflection and this can lead into the extension tasks.

# MEI Desmos Tasks for AS Pure

## Task 8: Integration – Area under a curve

The aim of this task is for students to investigate (or verify if they have already met it) the rule for integrating/finding the area under polynomials. It can be used as an introduction to the topic or to consolidate what they've already learnt.

Problem solution: The area is 20.25.

The first of the further tasks is an opportunity for students to investigate:

$$\int_a^b f(x)dx = \int_0^b f(x)dx - \int_0^a f(x)dx.$$

## Task 9: Solutions of Trigonometric Equations (Degrees)

This task encourages students to think about the symmetries of the trigonometric graphs and use these in finding solutions to equations.

Students should be familiar with scaling one of the axes independently by either dragging the axis (as in instruction 2) or by setting it via the Graphics properties.

Problem solution:

$$x = -348.46^\circ, -191.54^\circ, 11.54^\circ, 168.46^\circ, 371.54^\circ, 528.46^\circ.$$

## Task 10: Exponentials and logarithms – Graph of $y = ka^x$

This task allows students to investigate exponential functions of the form  $y = ka^x$ . The points A and B encourage students to focus on the growth factor as a multiple when increasing the value of  $x$  by 1.

Problem solution:  $y = 2.5 \times 4^x$ .

## Task 11: Quadratic Inequalities

This task focusses on solving quadratic inequalities by sketching graphs. Students should be encouraged to relate the roots of the quadratic with the possible values of the factors to determine whether the product is positive or negative. Substituting in some values can help confirm the solution is valid.

The software shades the region that satisfies the inequality as a vertical strip. It is useful to discuss with students when the convention of shading the region satisfied is helpful, as opposed to shading the region outside the inequality. You should also highlight that the strip is equivalent to marking a set of values on the number line.

It is important to highlight where the solution can be written as a single inequality and where it should be written as two separate inequalities.

Problem solution:  $x \leq -\frac{3}{2}$  or  $x \geq 2$