ICT for AS Core Mathematics

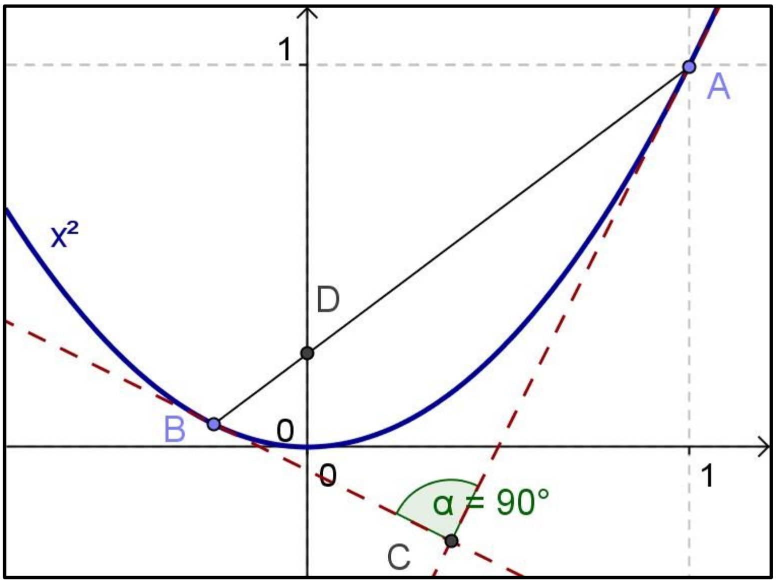
Differentiation/Integration

1. Plot a function and find the gradient of the chord between two points:
    a. What happens to the gradient of the chord as the chord reduces in length?
    b. How is this related to the gradient of the tangent?
    c. Could you predict what the gradient of the chord would be if it was infinitely short?

2. Explore the gradient at a point on a curve (student-centred):
    a. Plot a curve: e.g. $\mathbf{f(x) = x^2}$
    b. Measure the gradient of the tangent to the curve at a point:
        i. Add a point on the curve.
        ii. Find the tangent to the curve at the point.
        iii. Measure the slope of the tangent.
    c. In a spreadsheet, record the value of the gradient for different x-values.
    d. Find a formula, in terms of x, that fits the gradient values.
    e. Try changing the initial equation of the curve to a different function, e.g. $\mathbf{f(x) = x^2}$, $\mathbf{f(x) = x^3}$, $\mathbf{f(x) = x^4}$, $\mathbf{f(x) = 5x^2}$, $\mathbf{f(x) = x^2 + 3x}$

3. Gradient functions and stationary points (student-centred):
    a. Plot a function and its gradient function.
        i. Plot a curve, e.g. $\mathbf{f(x) = x^3 – 3x^2 – 2x}$ (a cubic function works well for this task).
        ii. Plot the gradient function for **f(x)**.
    b. Add a point to the original curve and display the tangent to the curve at this point.
    c. Describe the gradient of the tangent in terms of the gradient function.
    d. When is the tangent horizontal?
    e. When is the gradient of the tangent at a maximum/minimum?

4. Tangents and Normals:
   a. Draw the curve **f(x) = x²**.
   b. Add two points on the curve **A** and **B**.
   c. Display the tangents to the curve at each point,
   d. Find the point of intersection of the two tangents.
   e. Measure the angle between the two tangents.
   f. Add the line segment **AB** and find where this intersects the y-axis.
   g. Find positions of the two tangent so that they are perpendicular.
      i. Record the point of intersection of **AB** with the y-axis.
      ii. Record the coordinates of the point of intersection of the tangents.
   h. Repeat f. for different positions of the tangents that are perpendicular. What is the relationship between the points of intersection?
   i. Can you use this to construct a perpendicular tangent to a given tangent?



5. Stationary points:
   a. Find a cubic with two distinct stationary points and plot it, e.g. **f(x) = x³ + 3x² – 5x – 3**.
   b. Find the turning stationary points for $f(x)$.
   c. Find the inflection point for $f(x)$.
   d. Repeat this for different cubics and observe a relationship between the points.
      i. Prove this relationship using differentiation.

6. Introduction to integration (finding areas under lines):
   a. Add sliders for **m** and **c**.
   b. Draw a line **y = mx + c** (starting with +ve m and c is easiest).
   c. Find the area under the line up to an x-value (**x_1**):
      i. Split the area into a rectangle ($x_1 \times c$) and the triangle above it.
      ii. Sum the area of the rectangle and the triangle.
   d. Can you generalise this? What is the area under $y = mx + c$ up to $x = x_1$?
   e. What is the relationship between this and differentiation?
   f. Does it work if either m or c are negative?
   g. Can you extend this to the area between $x = x_1$ and $x = x_2$?

7. Introduction to integration (finding areas under curves):
    a. Plot a function and its integral.
        i. Plot a curve, e.g. $\mathrm{f}(x) = x^2$.
        ii. Add a point **A** at the origin and a point **B** on the x-axis.
        iii. Set **a** as the x-coordinate of **A** and **b** as the x-coordinate of **B**.
        iv. Find the integral of the curve between the points **a** and **b**.
        v. Plot a point **C** which shows the area for a given value of **b**: **(b,Area)**.
        vi. Switch **trace** to *on* for point **C** and vary the point **B**.
    b. What function would pass through these points?
    c. Try some different functions for f. How is the Area function related to the original function?

8. Integration (reverse of differentiation):
    a. Add sliders for **a** and **b**.
    b. Plot **g(x) = a*x + b**.
    c. Find a function $\mathrm{f}(x)$ that would have $\mathrm{g}(x)$ as its gradient function and verify that this is the case.
    d. Repeat c. for different values of **a** and **b**.
    e. Add a third slider for **c**.
    f. Change the function $\mathrm{g}(x)$ to **g(x) = ax^2 + bx + c**.
    g. Find a function $\mathrm{f}(x)$ that would have $\mathrm{g}(x)$ as its gradient function.

9. Finding the general cases of cubics:
    a. Add sliders for **a**, **b**, **c** and **d**.
    b. Plot **f(x) = ax^3 + bx^2 + cx + d**.
    c. Find values of **a**, **b**, **c** and **d** such that the curve has…
        i. …two stationary points.
        ii. …one stationary point.
        iii. …no stationary points.
    d. What are the conditions on a cubic that determine whether it has 0, 1 or 2 stationary points.

10. Trapezium rule:
    a. Plot a function that you can't integrate.
    e.g. $\mathrm{f}(x) = \sin x$, $\mathrm{f}(x) = x\cos x$, $\mathrm{f}(x) = 2^x$, $\mathrm{f}(x) = \sqrt{1-x^2}$
    b. In a spreadsheet, estimate the area under the curve between 0 and 1 by the trapezium rule with a strip size of 0.2:
        i. In cells **A1:B6** enter 0, 0.2, 0.4, 0.6, 0.8, 1.
        ii. In cells **B1 – B6** calculate $\mathrm{f}($ **A1 – A6** $)$**.**
        iii. In cells **C1** and **C6** enter 1, and in cells **C2:C5** enter 2.
        iv. In cell **D1**, enter **B1*C1** and repeat this for **D2 – D6**.
        v. In cell **D7,** find the sum from **D1 – D6**.
        vi. In cell **D8**, enter 0.2 (the strip size).
        vii. In cell **D9** enter .5***D8*****D7**.
    c. Compare the value of the area to integral on the graphs page.
    What is the percentage error in the trapezium rule estimate?
    d. How could you find an improved trapezium rule estimate?