ICT for A2 Core Mathematics: e, natural logarithms and exponentials

Autograph version

e

1. Definition of e
   a. Plot **y = a^x**.
   b. Select the curve and click Gradient Function
   c. Plot y**=k*a^x.**
   d. Use the constant controller to set the value of **a**.
   e. Use the constant controller to vary **k** to verify that the gradient function is **y=k*a^x** for some value of **k**.
   f. Find the value of **k** for different values of **a**.
   g. Is there a value of **a** where the **y=a^x** is its own gradient function (i.e. **k**=1)?

2. Modelling growth functions in terms of e
   a. Plot **y = a^x**.
   b. Add a point on the curve. You may find it useful to enable the Status Box to display its coordinates, alternatively you can select the point and diplay a dynamic textbox.
   c. Select the point and a tangent to the curve at the point. The gradient of the tangent can be deduced from its equation.
   d. Plot **y = *e*^(b*x)**.
   e. Use the constant controller to vary **y = *e*^(b*x)** so that it sits on top of **y = a^x)**.
   f. What is the approximate relationship between the y-value of the point, the gradient of the slope and the value of **b**?
   g. Move the point and verify that the relationship still holds.
   h. Vary the value of **a** and verify that the relationship still holds.

3. Use a spreadsheet to investigate the following ways of generating the number e: (NB you may have to change the Documents settings: Calculation mode to Approximate for these).
   a. It is possible to conjecture that an infinite polynomial that would differentiate to itself is: $\mathrm{f}(x)=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+...$ Using this as the definition of e^x and substituting $x=1$ into it gives $\mathrm{e}^x=\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+...$
   b. $\mathrm{e}^x=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Natural log

4. Inverse of e^x (see prepared file)
   The file shows the graphs of **y=*e*^x**, **y=ln(x)** and **y=x**.
   a. The gradient on **y= *e*^x** at the point **(a, *e*^{a})** is ***e*^{a}**, this is displayed as a gradient triangle (i.e. one with base 1).
   b. The image of **(a, *e*^{a})** when reflected in **y=x** is **(*e*^{a}, a)**. The gradient at this point is **1/*e*^{a}**, as displayed by the image of the gradient triangle under the reflection.
   c. The gradient on y=ln(x) is 1/(x-value at the point), i.e. the derivative of ln(x) is 1/x.

5. Area under 1/x
   a. Plot **y=1/x**. You may find it easier to resize the axes so the x-axis goes from –2 to 12 and the y-axis from–0.5 to 1.5.
   b. Find Area under the curve from 1 to a for different values of a. Use the trapezium rule with a relatively large number of divisions.
   c. What is the relationship between the integral and **a**?

Functions

6. Transformations of functions
   a. f(*x*+*a*)+*b*
      i. Define a function **f(x)**.
      ii. Plot **y=f(x)**.
      iii. Plot **y=f(x+a)+b**
      iv. Use the constant controller to vary the values of **a** and **b**.
      v. Describe the relationship between the graphs of **y=f(x)** and **y= f(x+*a*)+*b***
      vi. Verify that this relationship works for different functions for **f(x)**.
   b. *c*f(*x*) and f(*dx*)
      i. Define a function **f(x)**.
      ii. Plot **y=f(x)**.
      iii. Plot **y=cf(x)**
      iv. Use the constant controller to vary the values of **a** and **b**.
      v. Describe the relationship between the graphs of **y=f(x)** and **y=cf(x)**
      vi. Plot **y= f(dx)**
      vii. Describe the relationship between the graphs of **y=f(*x*)** and **y=f(dx)**
      viii. Verify that these relationships work for different functions for **f(x)**.
   c. f(–*x*) and –f(*x*)
      i. Define a function **f(x)**.
      ii. Plot **y=f(x)**.
      iii. Plot **y=f(–x)**
      iv. Describe the relationship between the graphs of **y=f(x)** and **y=f(–x)**
      v. Plot **y==–f(x)**
      vi. Describe the relationship between the graphs of **y=f(x)** and **y=f(–x)**
      vii. Verify that these relationships work for different functions for **f(x)**

7. Composite functions
    a. Define functions for **f(x)** and **g(x)**.
    b. Plot **y=f(g(x))**
    c. What single function would define **f(g(x))**.
    d. Try different functions for **f(x)** and **g(x)** and generalise what you find.

8. Inverse functions
    a. Define a function **f(x)**.
    b. Plot **y=f(x)**.
    c. Reflect the graph in y=x
    d. What function would give the reflected curve?
    e. Is the reflected curve a function?
        i. Is there a function which fits all the points on the curve?
        ii. If there isn't a function that fits all the points how would you need to limit the domain of the original function so the inverse is a function? NB to limit the domain of the original curve right-click > select Edit Equation and change the startup Options.

9. Odd and even
    a. Define a function **f(x)**.
    b. Plot **y=f(x)**.
    c. Select the curve and add a point with x-coordinate **a**. Select the point and display a dynamic textbox.
    d. Select the curve and add a point with x-coordinate –**a**. Select the point and display a dynamic textbox.
    e. Use the constant controller and investigate **f(–a)** for different functions:
        i. Can you find functions where **f(–a)= f(a)** for all **a**? What properties/symmetries do this curves display?
        ii. Can you find functions where **f(–a)= –f(a)** for all **a**? What properties/symmetries do this curves display?

10. Periodic
    a. Define a function **f(x)**.
    b. Plot **y=f(x)**.
    c. Plot **y=f(x+a)**.
    d. Investigate functions for **f(x)** and (non-zero values of) **a** such that the functions are the same.
        i. Try **sin(x), cos(x), tan(x)**
        ii. Try **sin(2x), sin (3x) …**
        Hint: you may find it easier to set the graphing angle to degrees.