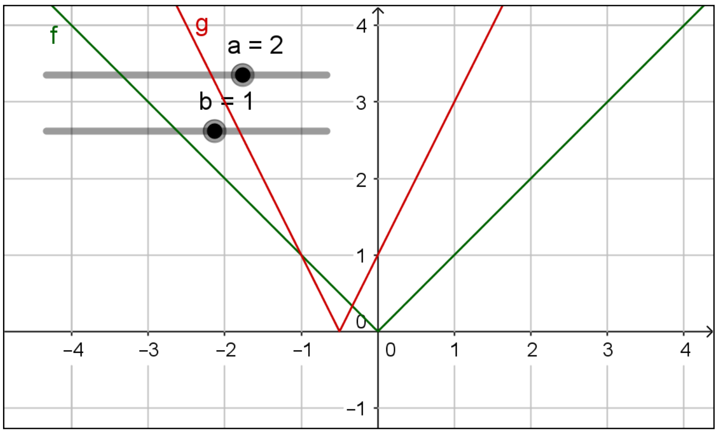
MEI GeoGebra Tasks for A2 Core

Task 1: Functions – The Modulus Function

1. Enter the function $f(x) = |x|$: **f(x)=abs(x)**
2. Use the **Slider** tool to add two sliders named **a** and **b**
3. Enter the function $g(x) = |ax+b|$: **g(x)=abs(a*x+b)**



Questions for discussion

- What combination of transformations maps the graph of $y = |x|$ onto the graph of $y = |ax+b|$?
- Where is the vertex on the graph of $y = |ax+b|$?
- Where does the graph of $y = |ax+b|$ intersect the *y*-axis?

Problem (*Try the question with pen and paper first then check it on your software*)

Sketch the graph of $y = |3x+2| - 3$ and find the points of intersection with the axes.

Further Tasks

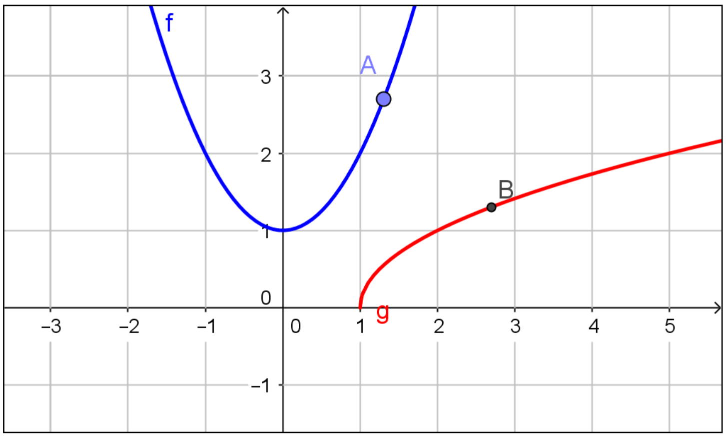
- Investigate the functions of
  - $g(x) = |f(x)|$
  - $g(x) = f(|x|)$

  for different functions $f(x)$, e.g. $f(x) = \sin(x)$ or $f(x) = x^3 - x^2$.
- Investigate the solutions to the inequality $|x + a| + b > 0$.

MEI GeoGebra Tasks for A2 Core

Task 2: Inverse functions

1. Enter the function $\mathrm{f}(x) = x^2 + 1$: **f(x)=x^2**
2. Plot the inverse function: **g(x)=Invert[f]**
3. Use the **Point** tool to add a point **A** on the function $\mathrm{f}(x) = x^2 + 1$
4. Enter the point **B=(y(A),x(A))**



Questions for discussion

- What graphical transformation maps the graph of the original function onto its inverse?
- Why is the inverse function only defined for part of the original function?
- Can you confirm algebraically that the function given for $\mathrm{g}(x)$ is the inverse function?

Try finding the inverses of some other functions.

Problem *(Check your answers by plotting the graphs on your software)*

Find inverses of the following functions:

$\mathrm{f}(x) = (x+3)^2$ $\qquad g(x) = x^3$ $\qquad \mathrm{h}(x) = \dfrac{1}{x-2}$

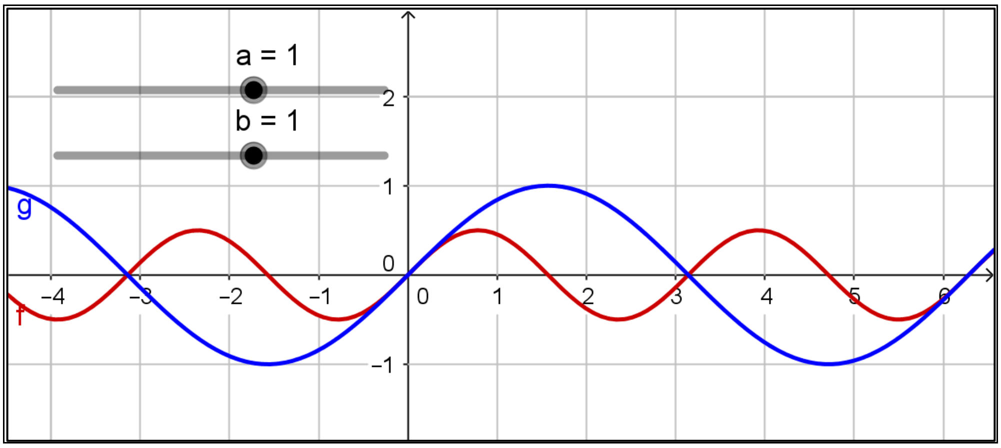
Further Tasks

- Find the inverse of the function $\mathrm{f}(x) = x^2 + 6x + 7$. Can you always find the inverse of a quadratic function $\mathrm{f}(x) = ax^2 + bx + c$?
- Investigate the graphs of the inverse trigonometric functions (you might find radians more convenient for this).

MEI GeoGebra Tasks for A2 Core

Task 3: Trigonometry – Double Angle formulae

1. Enter the function $f(x) = \sin x \cos x$: **f(x)=sin(x)*cos(x)**
2. Use the **Slider** tool to add two sliders named **a** and **b**
3. Enter the function $g(x) = a \sin bx$: **g(x)=a*sin(b*x)**



Questions for discussion

- For what values of a and b does $\sin x \cos x = a \sin bx$?
- Find values of a, b and c so that:
  - $\cos^2 x = a \cos(bx) + c$
  - $\sin^2 x = a \cos(bx) + c$
- How do these relationships link to the double angle formulae for sin and cos?

Problem (*Try the question with pen and paper first then check it on your software)*

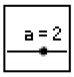
Solve $\sin 2\theta - \cos\theta = 0$ in the range $0 \le \theta < 2\pi$.

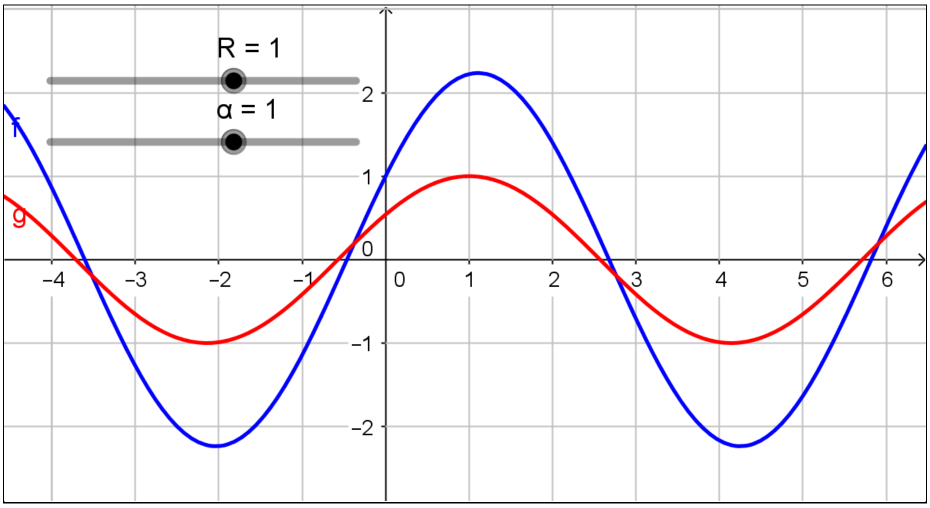
Further Tasks

- Describe the relationship $\tan 2\theta \equiv \dfrac{2\tan\theta}{1-\tan^2\theta}$ graphically.
- Find expressions for $\sin 3\theta$ and $\cos 3\theta$ in terms of $\sin\theta$ and $\cos\theta$.

MEI GeoGebra Tasks for A2 Core

Task 4: Trigonometry: *R*cos(*θ*–*α*)

1. Enter the function $f(x) = \cos x + 2\sin x$: **f(x)=cos(x)+2sin(x)**
2. Use the **Slider** tool  to add two sliders named **R** and **α**
3. Enter the function $g(x) = R\cos(x - \alpha)$: **g(x)=R*cos(x–α)**



Questions for discussion

- Can you find values of **α** and **R** so that the curves are the same?
- Can you find values for **α** and **R** for any *a* and *b* where $a\cos x + b\sin x = R\cos(x - \alpha)$?
- Can you explain the relationship using $R\cos(x - \alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha$?

Problem *(Check your answer by plotting the graphs on your software)*

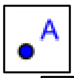
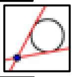
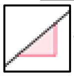
Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $0 < \alpha < \frac{\pi}{2}$.

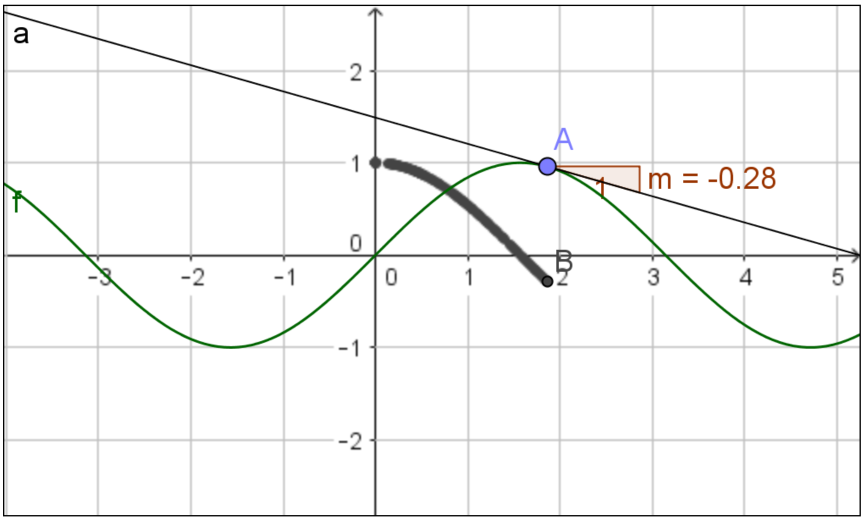
Further Tasks

- Explore how the form $R\cos(\theta - \alpha)$ can be used to find the maximum value of $a\cos\theta + b\sin\theta$ and the angle at which it occurs.
- Investigate the height of a rectangle as it is rotated through an angle *θ* about one of its corners.

MEI GeoGebra Tasks for A2 Core

Task 5: Differentiation – Trigonometric functions

1. Enter the function $\mathrm{f}(x) = \sin x$: **f(x)=sin(x)**
2. Use the **Point** tool  to add a point **A** on the function $\mathrm{f}(x) = \sin x$
3. Use the **Tangent** tool  to add a tangent to $\mathrm{f}(x) = \sin x$ at **A**
4. Use the **Slope** tool  to measure the gradient of the tangent, **m**
5. Enter the point **B=(x(A),m)**
6. Add a trace to the point **B**



Questions for discussion

- How does the gradient of the tangent vary as x varies:
  - What are its maximum and minimum values?
  - When is the gradient of the tangent 0?
  - For what values of x do these (max, min or 0) occur?
- Can you suggest a function for the derivative?
- Can you suggest a function for the derivative of $y = \cos(x)$?

Problem (*Check your answer by plotting the graph and the tangent on your software)*

Find the equation of the tangent to the curve $y = \sin x$ at the point $x = \frac{\pi}{3}$.

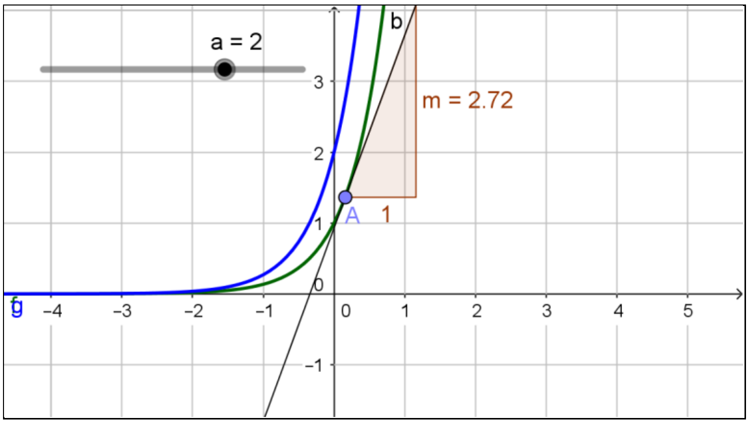
Further Tasks

- Investigate the derivatives of $y = \sin ax$ and $y = b \sin x$.
- Explain why this wouldn't work as neatly if the angle was measured in degrees.

MEI GeoGebra Tasks for A2 Core

Task 6: Derivate of exponential functions $y=e^{kx}$

1. Use the Slider tool to add a slider named **k**
2. Enter the function $f(x) = e^{kx}$: **f(x)=exp(k*x)**
3. Use the **Point** tool to add a point **A** on the function $f(x) = e^{kx}$
4. Use the **Tangent** tool to add a tangent to $f(x) = e^{kx}$ at **A**
5. Use the **Slope** tool to measure the gradient of the tangent, **m**
6. Enter the gradient function: **g(x)=f '(x)**



Question for discussion

- How can the *y*-coordinate of A and the slope of the tangent be used to explain the gradient function for $f(x) = e^{kx}$?

Problem (*Check your answer by plotting the graph and the tangent on your software)*

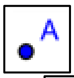
Find the equation of the tangent to the curve $y = e^{2x}$ at the point $x = 1$.

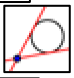
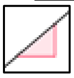
Further Tasks

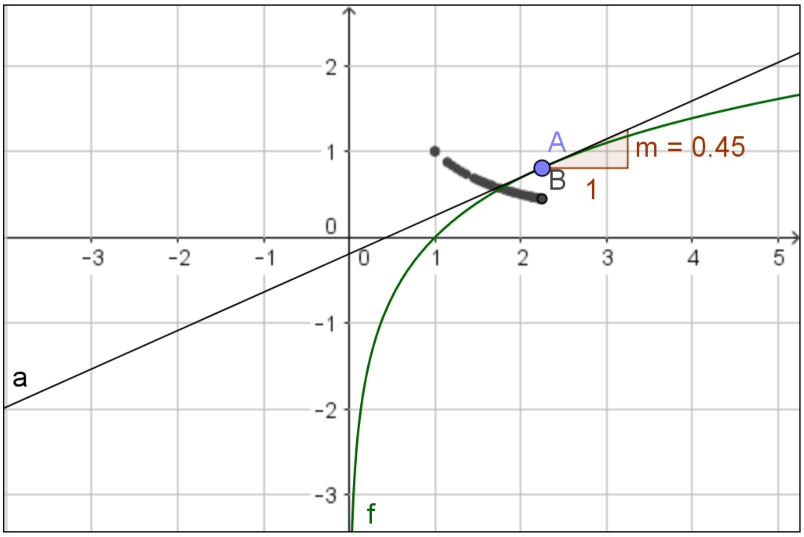
- Find the tangent to $y = e^{x}$ that passes through the origin.
- Find the gradient of the tangent to $y = 3^{x}$ when $x = 0$.

MEI GeoGebra Tasks for A2 Core

Task 7: Gradients of tangents to the natural logarithm $y=\ln x$

1. Enter the function $f(x) = \ln x$: **f(x)=ln(x)**
2. Use the **Point** tool 

to add a point **A** on the function $f(x) = \ln x$
3. Use the **Tangent** tool  to add a tangent to $f(x) = \ln x$ at **A**
4. Use the **Slope** tool  to measure the gradient of the tangent, **m**
5. Enter the point **B=(x(A),m)**
6. Add a trace to the point **B**



Questions for discussion

- What is the relationship between the point and the gradient of the tangent on $y = \ln x$?
- How does this relationship change for the graphs of $y = \ln 2x$, $y = \ln 3x$ … ?

Problem (*Check your answer by plotting the graph and the tangent on your software)*

Find the equation of the tangent to the curve $y = \ln x$ at the point $x = 2$.

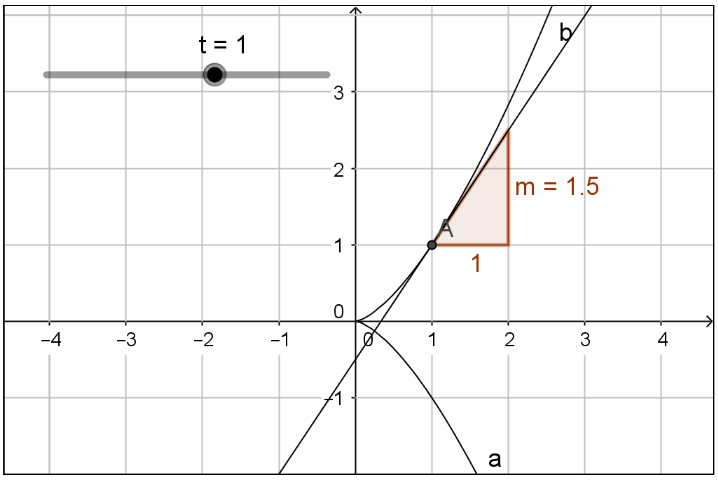
Further Tasks

- Find the tangent to $y = \ln x$ that passes through the origin.
- Explain the relationship between the derivatives of $y = e^x$ and $y = \ln x$.
  Hint: consider the point (a,b) on $y = e^x$ and the reflected point (b,a) on $y = \ln x$

MEI GeoGebra Tasks for A2 Core

Task 8: Tangents to parametric curves

1. Enter the parametric curve $x = t^2$, $y = t^3$: **X1t=T²**, **Y1t=T³**:
2. Use the **Slider** tool to add a slider and name it **t**
3. Enter the point **A=(t^2,t^3)**
4. Use the **Tangent** tool to add a tangent to the curve at **A**
5. Use the **Slope** tool to measure the gradient of the tangent



Questions for discussion

- What is the relationship between $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$?
- Does this relationship for other parametric curves?
  e.g. $x = 2t + 1,\ y = \frac{1}{t}$ or $x = \cos t, y = \sin t$.

Problem *(Check your answer by plotting the graph and the tangent on your software)*

Find the coordinates of the points on the curve $x = 2t + \cos t, y = \sin t, -2\pi < t \le 2\pi$ for which the tangent to the curve is parallel to the *x*-axis.

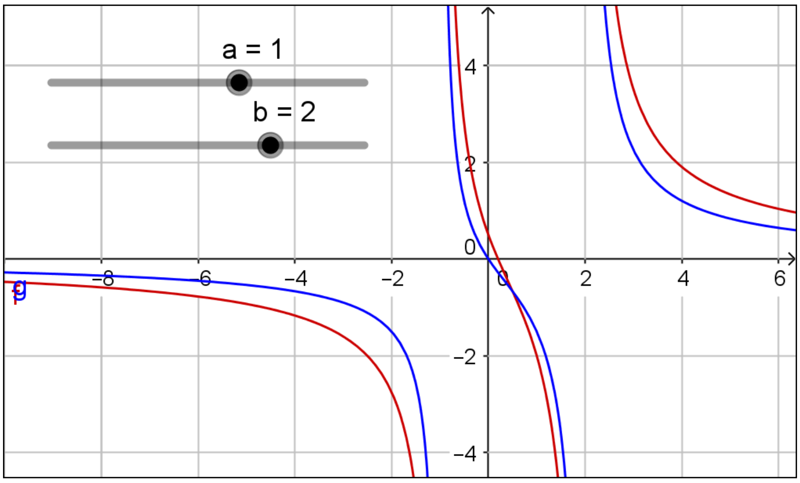
Further Tasks

- Explore how you can find the equation of the tangent to a parametric curve at a point.
- Describe how to find the tangent to a parametric curve that passes through a specific point that is not on the curve.

MEI GeoGebra Tasks for A2 Core

Task 9: Partial Fractions

1. Enter the function $f(x)=\frac{5x-1}{(x+1)(x-2)}$: **f(x)=(5x-1)/((x+1)(x-2))**
2. Use the **Slider** tool [a=2] to add two sliders named **a** and **b**
3. Enter the function $g(x)=\frac{A}{x+1}+\frac{B}{x-2}$: **g(x)=a/(x+1)+b/(x-2)**



Find values of **a** and **b** so that the graphs of the functions are the same.

Question for discussion

- How could you find the values using $\frac{5x-1}{(x+1)(x-2)}=\frac{A}{x+1}+\frac{B}{x-2}$?
- Does this method work for $\frac{2x+7}{(x+2)(x+3)}=\frac{A}{x+2}+\frac{B}{x+3}$?

Problem *(Check your answers by plotting the graphs on your software)*

Find values of *A* and *B* so the following can be expressed as partial fractions:

$$\frac{7x-14}{(x+3)(x-4)}=\frac{A}{x+3}+\frac{B}{x-4}$$

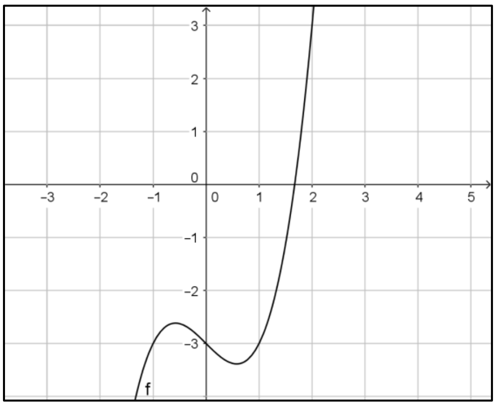
Further Tasks

- Find *A*, *B* and *C* such that $\frac{5x^2+3x+7}{(x+2)(x^2+3)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+3}$
- Find *A*, *B* and *C* such that $\frac{7x^2+29x+28}{(x-1)(x+3)^2}=\frac{A}{x-1}+\frac{B}{x+3}+\frac{C}{(x+3)^2}$

MEI GeoGebra Tasks for A2 Core

Task 10 – Numerical Methods: Change of sign

1. Enter the function $f(x) = x^3 - x - 3$: **f(x)=x^3-x-3**



In this example you can see that the root lies between $x = 1$ and $x = 2$.

2. Enable the spreadsheet: **View > Spreadsheet**

3. In cells A1 and A2 enter the values 1, 1.1 and then fill down to 2.

4. In cell B1 enter **f(A1)** and then fill down.

   In this example there is a change of sign between x = 1.6 and x = 1.7 .

5. You can now investigate further by changing the column A so that it goes from 1.6 to 1.7 in steps of 0.01.

	A	B
1	1	-3
2	1.1	-2.77
3	1.2	-2.47
4	1.3	-2.1
5	1.4	-1.66
6	1.5	-1.12
7	1.6	-0.5
8	1.7	0.21
9	1.8	1.03
10	1.9	1.96
11	2	3

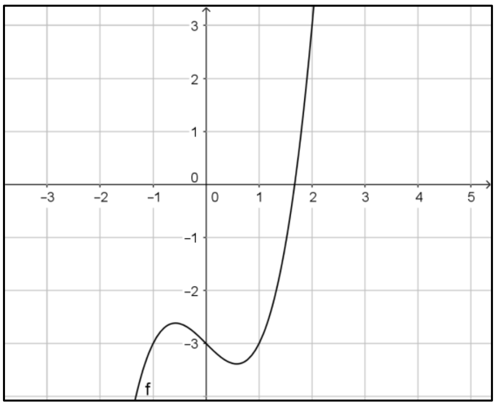
6. You can check your answer in the Graphics view by finding the point of intersection of the function and the *x*-axis.

NB The number of decimal places can be set with **Options > Rounding**.

Try using your software to find the roots of other equations using the change of sign method.

Task 11 – Numerical Methods: Fixed Point Iteration

1. Enter the function $f(x) = x^3 - x - 3$: **f(x)=x^3-x-3**



In this example you can see that the root lies between $x = 1$ and $x = 2$.

2. Enable the spreadsheet: **View > Spreadsheet**

3. To add the recurrence relation $x_{n+1} = \sqrt[3]{x_n + 2}$, enter the function $g(x) = \sqrt[3]{x+2}$: **g(x)=cbrt(x+3)**

4. In Cell A1 enter 0 and in Cell B1 enter 1.

5. In Cell A2 enter **A1+1** and in cell B2 enter **g(B1)** and fill down.

	A	B
1	0	1
2	1	1.5874
3	2	1.6616
4	3	1.6705
5	4	1.6716
6	5	1.6717
7	6	1.6717

6. You can check your answer in the Graphics view by finding the point of intersection of the function and the *x*-axis.

NB The number of decimal places can be set with **Options > Rounding**.

Try using your software to find the roots of other equations using a fixed-point iteration.

MEI GeoGebra Tasks for A2 Core

Teacher guidance

Task 1: The Modulus Function

Students should consider how this relates to the graph of $y = ax+b$

Problem solution: $x = -\frac{5}{3}, \frac{1}{3}$ $y = -1$

Students might need some help structuring the investigation into $|x + a|+b > 0$. One strategy is to fix either a or b and investigate changing the other parameter.

Task 2: Inverse functions

The aim of this task is to reinforce the link between the reflection in the line $y = x$ and rearranging $y = \mathrm{f}(x)$ to express x in terms of y.

Problem solutions:

$\mathrm{f}^{-1}(x) = \sqrt{x} - 3$ $\qquad g^{-1}(x) = \sqrt[3]{x}$ $\qquad \mathrm{h}^{-1}(x) = \frac{1}{x} + 2$

It is important to emphasise that the domain of the original function needs to be restricted so that it is one-to-one for the inverse to be a function.

The **invert[f]** command will only work on functions with a single instance of x so students should compare the completed square form to the expanded form of a quadratic.

Task 3: Trigonometry – Double Angle formulae

Students might need some help structuring the investigation into $\sin x \cos x = a \sin (bx)$. One strategy is to fix b and investigate changing a first to find a curve with the correct amplitude.

Use of the compound angle formulae for $\sin(a+b)$ and $\cos(a+b)$ might be useful for some students to verify their results.

Problem solution: $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Task 4: Trigonometry: *R*cos(*θ*–*α*)

Students are expected to be able to relate their findings to the expansion of $R\cos(x - A) = R\cos x \cos A + R\sin x \sin A$.

Problem solution:
$4\cos\theta + 3\sin\theta = 5\cos(\theta - 0.644)$.

Task 5: Differentiation – Trigonometric functions

By considering key points the students should be able to observe that this has the same shape as cos(x).

Problem solution:

$y=\frac{x}{2}+\frac{\sqrt{3}}{2}-\frac{\pi}{6}$ or $y=0.5x+0.342$

Task 6: Derivates of exponential functions $y=e^{kx}$

This task can be done on its own or with task 7. The aim of this task is for students to be able to find the gradients and equations of tangents to exponential functions.

Students should observed that the derivative is the same as the *y*-coordinate for $y = e^x$ before exploring other curves of the form $y = e^{kx}$.

Problem solution:

$y=14.778x-7.389$

The second of the further tasks requires students to rewrite $y=3^x$ as $y=e^{(\ln 3)x}$.

Task 7: Gradients of tangents to the exponential function $y=\ln x$

This task can be done on its own or with task 6. The aim of this task is for students to be able to find the gradients and equations of tangents to the natural logarithm function.

For the second discussion point students might be surprised that the result doesn't change but they should be encouraged to think of this in terms of laws of logs.

Problem solution:

$y=0.5x-0.307$

Task 8: Tangents to parametric curves

Students might find it useful to make a table of values for $\frac{dy}{dx}$, $\frac{dx}{dt}$ and $\frac{dy}{dt}$. Students should be encouraged to explore further parametric equations and to consider their relationship in terms of $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$.

Problem solution:

$t=\frac{\pi}{2},\frac{3\pi}{2}$: points are $(2,1),(2,-1)$.

Task 9: Partial Fractions

This task can be used as an introduction to partial fractions or as a consolidation exercise. Students should be encouraged to express their methods algebraically.

Solutions to partial fractions:

$$\frac{5x-1}{(x+1)(x-2)}=\frac{2}{x+1}+\frac{3}{x-2}$$

$$\frac{2x+7}{(x+2)(x+3)}=\frac{3}{x+2}-\frac{1}{x+3}$$

$$\frac{7x-14}{(x+3)(x-4)}=\frac{5}{x+3}+\frac{2}{x-4}$$

$$\frac{5x^2+3x+7}{(x+2)(x^2+3)}=\frac{3}{x+2}+\frac{2x-1}{x^2+3}$$

$$\frac{7x^2+29x+28}{(x-1)(x+3)^2}=\frac{4}{x-1}+\frac{3}{x+3}-\frac{1}{(x+3)^2}$$

Task 10: Numerical Methods – Change of sign

This task is a set of instructions for how to implement the change of sign method on the software. Students are encouraged to work through these instructions and then try solving some equations of their own.

It is useful to have some additional equations for students to be finding the roots of once they have completed this sheet.

Task 11: Numerical Methods – Fixed Point Iteration

This task is a set of instructions for how to implement the change of sign method on the software. Students are encouraged to work through these instructions and then try solving some equations of their own.

It is useful to have some additional equations for students to be finding the roots of once they have completed this sheet.