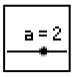
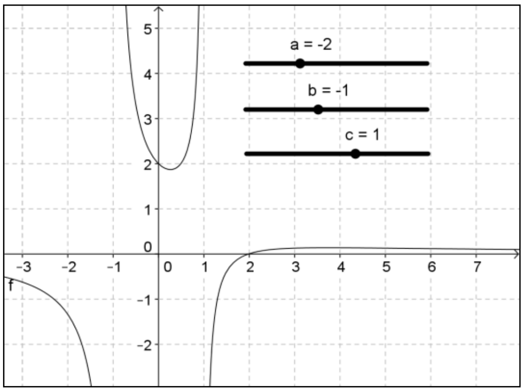
MEI GeoGebra Tasks for Further Pure

Task 1 – Curve sketching: Rational Functions

1. Use  to add 3 sliders **a**, **b** and **c.**

2. In the Input bar enter the function: **f(x)=(x+a)/((x+b)(x+c))**.

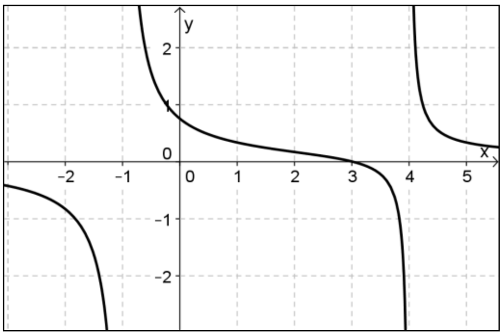


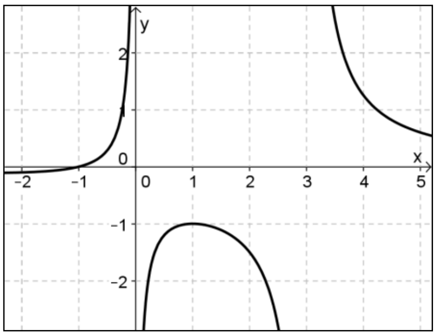
Questions

- What point on the curve does the value of a give you?
- What is the relationship between the shape of the curve and the values of b and c?

Problem

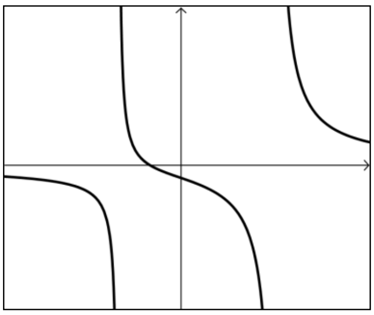
Find the values of *a*, b and c for the following curves:

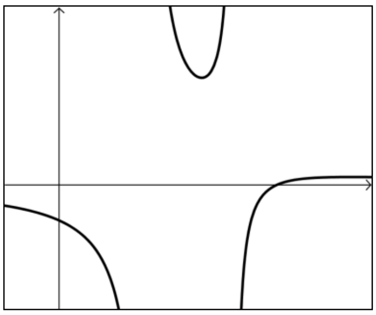


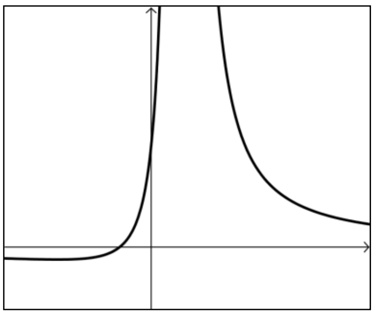


Further Tasks

Can you find conditions on a, b and c so that the curve will have one of these general shapes?







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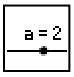
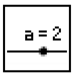
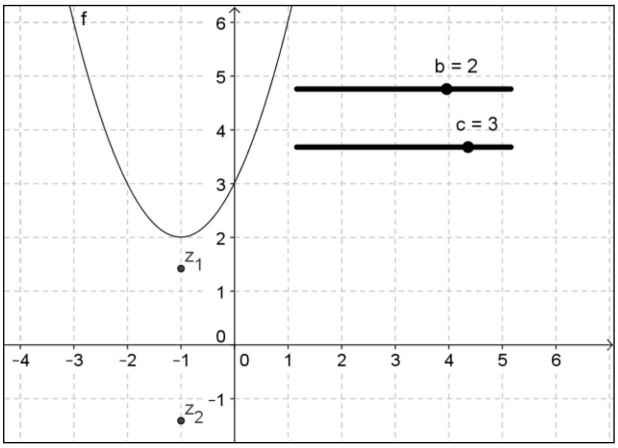
Task 2 – Complex Numbers: Roots of Quadratic Equations

1. Add a slider  and set its name to **b**.

2. Add a slider  and set its name to **c**.

3. In the Input bar enter the function: **f(x)=x^2+b*x+c**

4. In the Input bar enter the function: **ComplexRoot[f]**



Questions

- When are the roots of $f(x) = 0$ real? When are the roots of $f(x) = 0$ complex?

- Can you find values of b and c so that the roots are complex and the real part is 2?
… or 1? … or -1? … or p?

- Can you find a function f with roots $2 \pm 3i$

- Explain how you would find a function f with roots $p \pm qi$ (for some p and q)?

Problem *(Try the problem with pen and paper first then check it on your software)*

The function $f(z) = z^3 - 10z^2 + 34z - 40$ has a root $z = 3 + i$. Find the other two roots.

Further Tasks

- Explain why the roots of a cubic with real coefficients will always form an isosceles triangle in the Argand diagram.

- Find a cubic $f(z)$ where the roots of $f(z)$ form an equilateral triangle in the Argand diagram.

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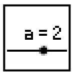
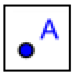
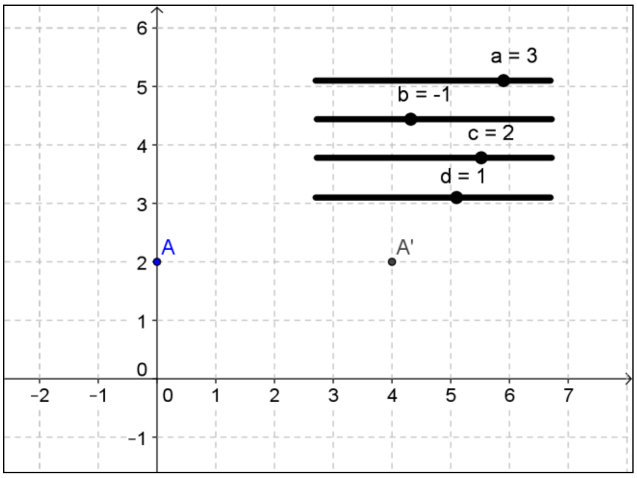
Task 3 – Matrices: Transformation matrices

1. Use  to add 4 sliders **a**, **b**, **c** and **d.**
2. Create the matrix $\mathrm{M}=\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ by entering: **M={{a,c},{b,d}}** in the input bar.
3. Use **New Point** (2nd menu)  to add a point, **A**.
4. Create the image of A under the transformation M by entering **A'=M*A** in the input bar.



Answer the following questions for different matrices, M:

- What is the relationship between A' and A when A is on the x-axis?
- What is the relationship between A' and A when A is on the y-axis?
- How can you use these relationships to find the position of A' for any point A?

Problem *(Try the problem with pen and paper first then check it on your software)*

The image of the quadrilateral OABC under the transformation $\mathrm{M}=\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$ is a square.

O=(0,0), A=(2,4) and B=(4,0). Find the coordinates of C.

Further Tasks

- Under the matrix $\mathrm{M}=\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$ the image of the point A=(–1,3) is A'=(2,4).

  Find other matrices M and points A such that A'=(2,4).

- A=(2,1) is an invariant point under the matrix $\mathrm{M}=\begin{pmatrix} 3 & -4 \\ 2 & -3 \end{pmatrix}$ because the image of the point A=(2,1) is A'=(2,1). Find the invariant points of other matrices M.

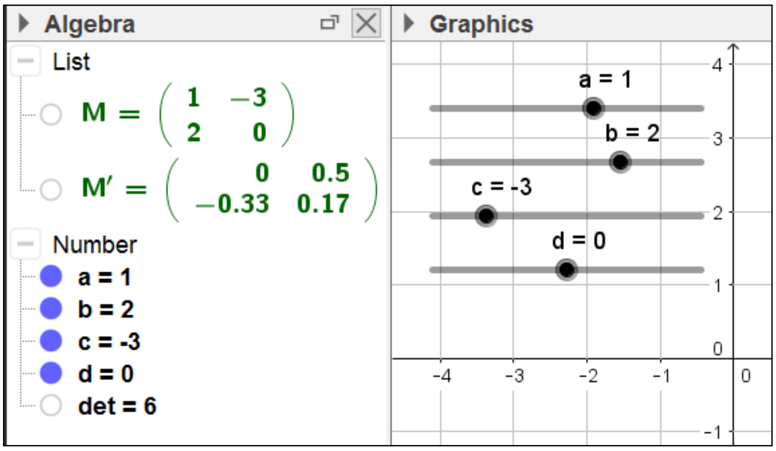
Task 4 – Matrices: Determinants and inverse matrices

1. Use [slider tool icon] to add 4 sliders **a**, **b**, **c** and **d.**

2. Create the matrix $M = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ by entering: **M={{a,c},{b,d}}** in the input bar.

3. Find the determinant of M by entering: **det=Determinant[M]** in the input bar.

4. Find the inverse of M by entering: **M'=Invert[M]** in the input bar.



Questions for discussion

- What is the relationship between the matrix, the determinant and the inverse?
- What is the answer when a matrix is multiplied by its inverse?
- Are there any matrices that don't have an inverse?

Problem *(Try the question with pen and paper first then check it on your software)*

For the matrices $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$ find A^{-1}, B^{-1} and $(AB)^{-1}$.

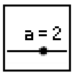
Further Tasks

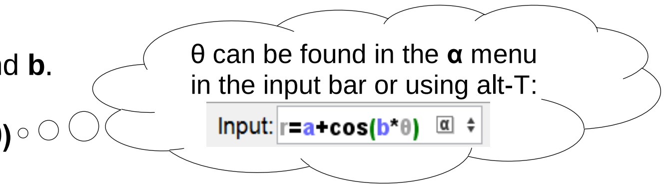
- Investigate the determinants and inverse of matrices for standard transformations: reflection, rotation and stretches.

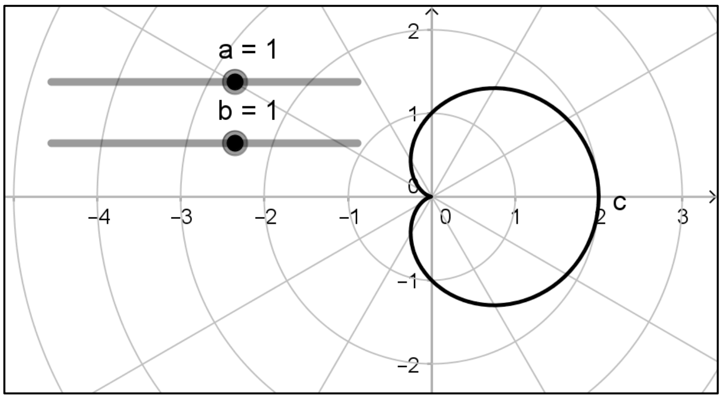
  You might find it useful to draw a shape (**poly1**) with the polygon tool [polygon tool icon] and then use the command: **ApplyMatrix[M,poly1]**

- For other 2×2 matrices, A and B, investigate the relationship between A^{-1}, B^{-1} and $(AB)^{-1}$.

Task 5 – Polar curves

1. Use  to add sliders: **a** and **b**.
2. Enter the function **r=a +cos(b*θ)**





To display a polar grid:

- *Computer software: right-click in the graphics area and select **Graphics** (the last option), on the grid option select **Grid type: Polar**.*
- *Tablet/web version: the polar grid can be found in the graphics style bar.*

Questions for discussion

- What is the maximum/minimum distance from the pole and for what values of θ does this occur?
- How is this polar curve related to the Cartesian curve $y = a + \cos bx$?

Problem *(Try the questions with pen and paper first then check it on your software)*

Plot the following curves:

$r = 2 + \cos\theta$ $\qquad r = 2$ $\qquad r = 1 + \sin 2\theta$ $\qquad r = 3 + \cos\dfrac{\theta}{2}$ $\qquad r = 3 - 2\sin\theta$

Further Tasks

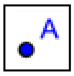
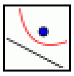
- For what values of θ does r take its maximum and minimum values? How can these be deduced from the polar equation?

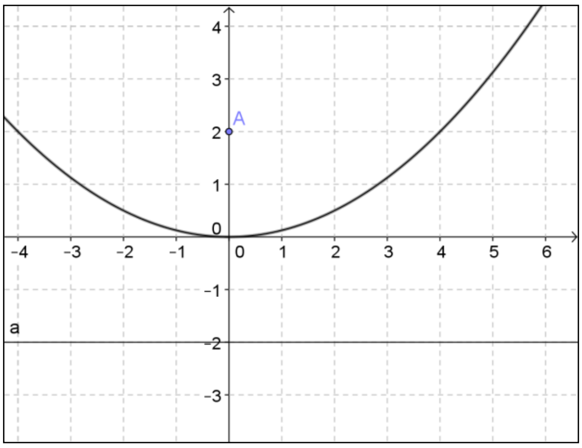
- For which parts of the graph does r take negative values? What are the conditions such that $r = a + b\cos\theta$ and $r = a + b\sin\theta$ doesn't take negative values?

- The default setting is to plot values of θ from 0 to 2π (this can be changed by selecting the function). Are there any curves for which this results in the same graph being traced over again? Are there any graphs for which the graph is incomplete using this range?

MEI GeoGebra Tasks for Further Pure

Parabolas, Hyperbolas and Ellipses

1. Use **New Point** (2nd menu)  to add a point on the y-axis, **A**.
2. In the Input bar enter: **y=-y(A)**
3. Use **Parabola** (7th menu)  to create a parabola with focus **A** and directrix **y=-y(A)**.



Question

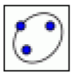
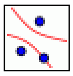
What is the relationship between the equation of the curve and position of **A**?

Problem *(Try the problem with pen and paper first then check it on your software)*

Show that the line joining the points A:(4,4) and B:(–1, ¼) passes through the focus of $x^2 = 4y$.

Show that the tangents to $x^2 = 4y$ at the points A:(4,4) and B:(–1, ¼) meet at right-angles on the directrix to the parabola.

Further Tasks

On the 7th menu there are tools for creating an Ellipse:  and a Hyperbola: 

These are based the position of the two foci (**A** and **B**) and a point on the curve **C**.

Can you find positions for **A**, **B** and **C** for the following curves?

$$\frac{x^2}{9}+\frac{y^2}{5}=1 \qquad \frac{x^2}{25}+\frac{y^2}{34}=1 \qquad y^2-\frac{x^2}{8}=1$$

$$\frac{x^2}{8}-\frac{y^2}{8}=1 \qquad y=\frac{1}{x} \qquad y=\frac{k}{x}$$