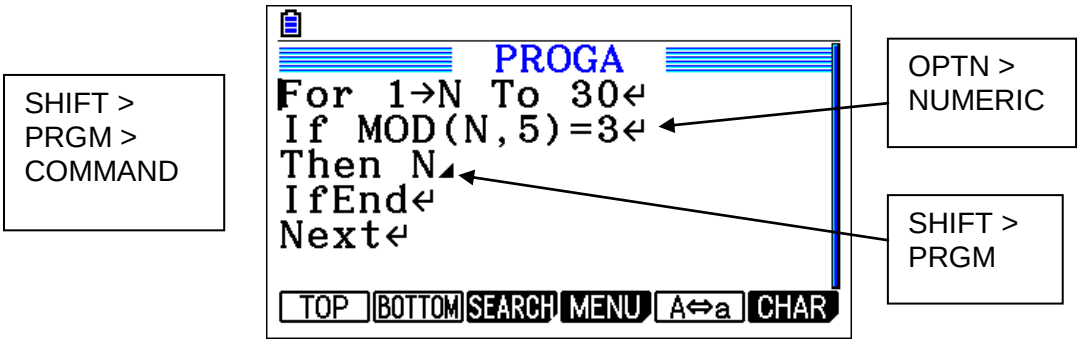
MEI Casio Tasks

Introduction to Programming for Number Theory

Task A

Write a program to find all the positive integers n $(n \leq 30)$ such that $n \equiv 3 \pmod 5$.

Program



Task B

Write a program to find whether an inputted number is prime.

Program

?→N
1→A
Int (√N)→M
For 2→C To M
If MOD(N,C)=0
Then 0→A
IfEnd
Next
A

Problem

Write a program to solve the congruence $7x \equiv 3 \pmod{19}$.
Edit this to solve the congruence $4x \equiv 2 \pmod{18}$.

Investigate the solutions to $ax \equiv b \pmod m$ for different values of *a*, *b* and *m* and explain how the solution differs when *m* is prime and *m* is not prime.

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Investigation

Investigate the simultaneous congruences $\begin{matrix} ax+by \equiv c \pmod p \\ dx+ey \equiv f \pmod p \end{matrix}$ with $0 \le x < p, 0 \le y < p$.

Suggested programs:

Find the positive integer solutions to $3x+5y \equiv 7 \pmod{17}$ with $0 \le x < 17, 0 \le y < 17$.

Find the positive integer solutions, *x* and *y*, to $\begin{matrix} 3x+5y \equiv 7 \pmod{17} \\ 2x+7y \equiv 1 \pmod{17} \end{matrix}$.

Find the number of solutions, *x* and *y*, to the simultaneous congruences $\begin{matrix} 7x+y \equiv 6 \pmod{17} \\ x+5y \equiv 13 \pmod{17}. \end{matrix}$

Investigate the number of solutions, *x* and *y*, to the simultaneous congruences

$$\begin{matrix} kx+5y \equiv 7 \pmod{17} \\ 2x+7y \equiv 1 \pmod{17} \end{matrix}$$

for different integer values of *k* where $0 \le k < 17$.

Examination question: MEI Further Pure with Technology – June 2014

3 This question concerns Pythagorean triples: positive integers *a*, *b* and *c* such that $a^2+b^2=c^2$. The integer *n* is defined by $c=b+n$.

(i) Create a program that will find all such triples for a given value of *n*, where both *a* and *b* are less than or equal to a maximum value, *m*. You should write out your program in full.

For the case *n* = 1, find all the triples with $1 \le a \le 100$ and $1 \le b \le 100$.

For the case *n* = 3, find all the triples with $1 \le a \le 200$ and $1 \le b \le 200$. **[9]**

(ii) For the case *n* = 1, prove that there is a triple for every odd value of *a* where *a* > 1. **[4]**

(iii) For the case *n* = *p*, where *p* is prime, show that *a* must be a multiple of *p*. **[3]**

(iv) For the case *n* = *b*, determine whether there are any triples. **[4]**

(v) Edit your program from part **(i)** so that it will only find values of *a* and *b* where *b* is not a multiple of *n*. Indicate clearly all the changes to your program.

Use the edited program to find all such triples for the case *n* = 2 with $1 \le a \le 100$ and $1 \le b \le 100$. **[4]**