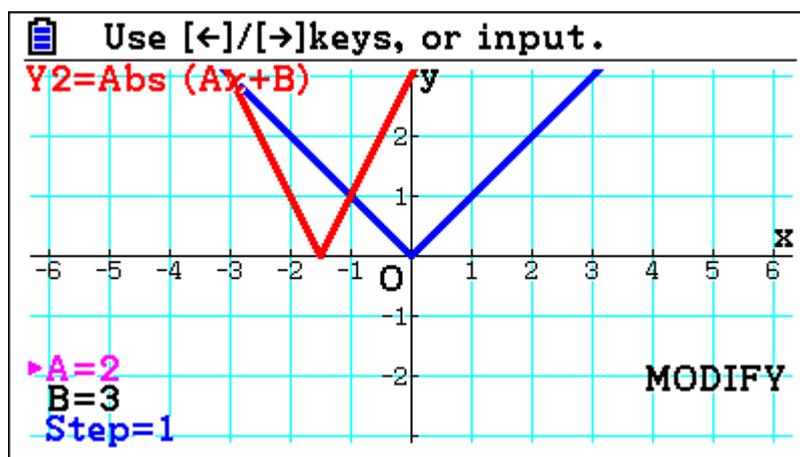


# MEI Casio Tasks for A2 Core

## Task 1: Functions – The Modulus Function

The modulus function,  $abs(x)$ , is found using **OPTN** > **NUMERIC** > **Abs**

1. Add a new Graphs screen: **MENU** **5**
2. Add the graph  $y = |x|$ , **Y1=Abs(x)**: **OPTN** **F5** **F1** **X,θ,T** **EXE**
3. Add the graph  $y = |ax+b|$ , **Y2=Abs(Ax+B)**:  
**OPTN** **F5** **F1** **ALPHA** **X,θ,T** **X,θ,T** **+** **ALPHA** **log** **EXE**
4. Plot the curves using modify: **F5**



### Questions

- What transformation maps the graph of  $y = |x|$  onto the graph of  $y = |ax+b|$ ?
- Where is the vertex on the graph of  $y = |ax+b|$ ?
- Where does the graph of  $y = |ax+b|$  intersect the y-axis?

**Problem** (Try the question with pen and paper first then check it on your calculator)

Sketch the graph of  $y = |3x+2| - 3$  and find the points of intersection with the axes.

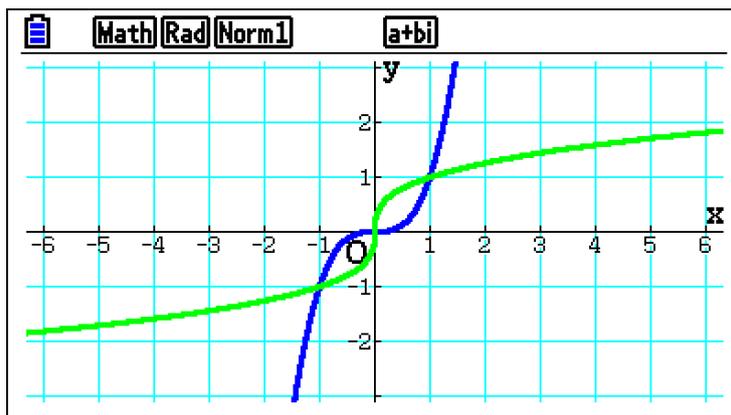
### Further Tasks

- Investigate the graphs of
  - $y = |f(x)|$
  - $y = f(|x|)$for different functions  $f(x)$ , e.g.  $y = \sin(x)$  or  $y = x^3 - x^2$ .
- Investigate the solutions to the inequality  $|x + a| + b > 0$ .

# MEI Casio Tasks for A2 Core

## Task 2: Inverse functions

1. Add a new Graphs screen: **MENU** **5**
2. Enter the graph  $Y1 = x^3$ : **X,θ,T** **^** **3** **EXE**
3. Plot the graph: **F6**
4. Plot the inverse function: Sketch > Inverse **F4** **F4**



### Questions

- What graphical transformation maps the graph of the original function onto its inverse?
- What is the equation of the graph of the inverse function?

Try finding the inverses of some other functions.

**Problem** (Check your answers by plotting the graphs on your calculator)

Find inverses of the following functions:

$$f(x) = x^2 + 2$$

$$g(x) = \sqrt{x+1}$$

$$h(x) = \frac{1}{x-2}$$

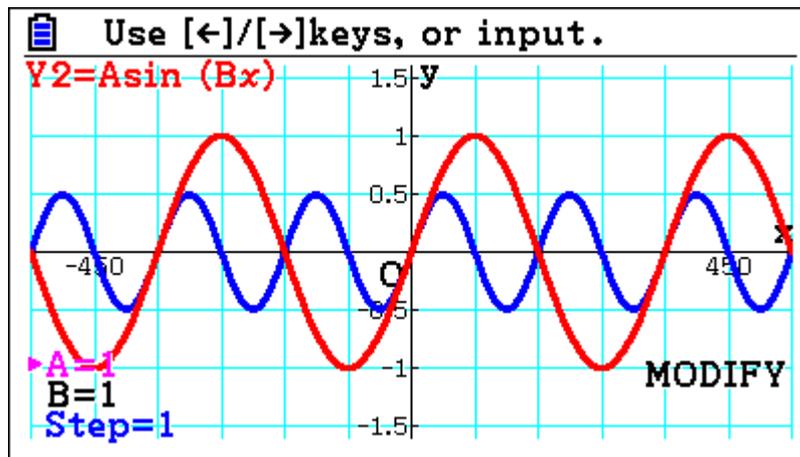
### Further Tasks

- Find the inverse of the function  $f(x) = x^2 + 6x + 7$ . Can you always find the inverse of a quadratic function  $f(x) = ax^2 + bx + c$ ?
- Investigate the graphs of the inverse trigonometric functions (you might find radians more convenient for this).

# MEI Casio Tasks for A2 Core

## Task 3: Trigonometry – Double Angle formulae

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Angle: Degrees** and **V-Window: Trig**.
3. Add the graph  $y = \sin x \cos x$ , **Y1=sin x × cos x** : **sin** **X,θ,T** **×** **cos** **X,θ,T** **EXE**
4. Add the graph  $y = a \sin (bx)$ , **Y2=A sin(B x)**:  
**ALPHA** **X,θ,T** **sin** **(** **ALPHA** **log** **X,θ,T** **)** **EXE**
5. Plot the curves using modify: **F5**



### Questions

- For what values of  $a$  and  $b$  does  $\sin x \cos x = a \sin (bx)$  ?
- Can you find values of  $a$ ,  $b$  and  $c$  so that:
  - $\cos^2 x = a \cos (bx) + c$
  - $\sin^2 x = a \cos (bx) + c$
- How do these relationships link to the double angle formulae for sin and cos?

**Problem** (Try the question with pen and paper first then check it on your calculator)

Solve  $\sin 2\theta - \cos \theta = 0$  in the range  $0^\circ \leq \theta < 360^\circ$ .

### Further Tasks

- Describe the relationship  $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$  graphically.
- Find expressions for  $\sin 3\theta$  and  $\cos 3\theta$  in terms of  $\sin \theta$  and  $\cos \theta$ .

# MEI Casio Tasks for A2 Core

## Task 4: Trigonometry: $R\cos(\theta-\alpha)$

1. Add a new Graphs screen: **MENU** **5**

2. In SET UP set **Angle: Degrees**:

**SHIFT** **MENU** **▼** **▼** **▼** **▼** **▼** **▼** **▼** **▼** **▼** **▼** **F1** **EXIT**

3. Set V-Window to Trig, Ymin: -3, Ymax: 3:

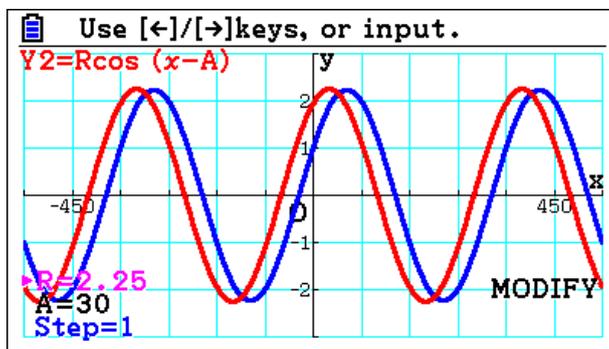
**SHIFT** **F3** **F2** **▼** **▼** **▼** **▼** **(←)** **3** **EXE** **3** **EXIT**

4. Enter the graphs  $y = \cos x + 2\sin x$  and  $y = R\cos(x-A)$ :

**Y1=cos x + 2 sin x, Y2=Rcos(x-A):**

**cos** **(X,θ,T)** **+** **2** **sin** **(X,θ,T)** **EXE** **(ALPHA)** **6** **cos** **(** **(X,θ,T)** **-** **(ALPHA)** **(X,θ,T)** **)** **EXE**

5. Plot the graphs using Modify: **F5**



NB You can vary the values of **A** and **R** by typing directly to them.

### Questions

- Can you find values of **A** and **R** so that the curves are the same?
- Can you find values for **A** and **R** for any  $a$  and  $b$  where  $a \cos x + b \sin x = R \cos(x - A)$ ?
- Can you explain the relationship using  $R \cos(x - A) = R \cos x \cos A + R \sin x \sin A$ ?

**Problem** (Check your answer by plotting the graphs on your calculator)

Express  $4\cos \theta + 3\sin \theta$  in the form  $R \cos(\theta - \alpha)$  where  $0^\circ < \alpha < 90^\circ$ .

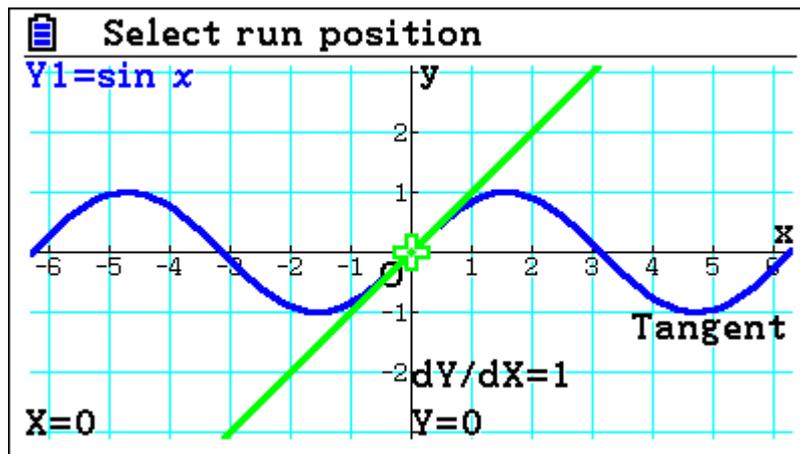
### Further Tasks

- Explore how the form  $R \cos(\theta - \alpha)$  can be used to find the maximum value of  $a \cos \theta + b \sin \theta$  and the angle at which it occurs.
- Investigate the height of a rectangle as it is rotated through an angle  $\theta$  about one of its corners.

# MEI Casio Tasks for A2 Core

## Task 5: Differentiation – Trigonometric functions

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Derivative: On** and **Angle:Radians**:  
**SHIFT** **MENU** **▼** **▼** **▼** **▼** **▼** **▼** **F1** **▼** **▼** **▼** **▼** **F2** **EXIT**
3. Draw the graph  $y = \sin x$ , **Y1=sin(x)**: **sin** **X,θ,T** **EXE** **F6**
4. Add the tangent at the point (*Sketch > Tangent*): **F4** **F2**



Use **◀**/**▶** to move the position of the point on the curve.

### Questions

- How does the gradient of the tangent vary as  $x$  varies:
  - What are its maximum and minimum values?
  - When is the gradient of the tangent 0?
  - For what values of  $x$  do these occur?
- Can you suggest a function for the derivative?
- Can you suggest a function for the derivative of  $y = \cos(x)$ ?

**Problem** (*Check your answer by plotting the graph and the tangent on your calculator*)

Find the equation of the tangent to the curve  $y = \sin x$  at the point  $x = \frac{\pi}{3}$ .

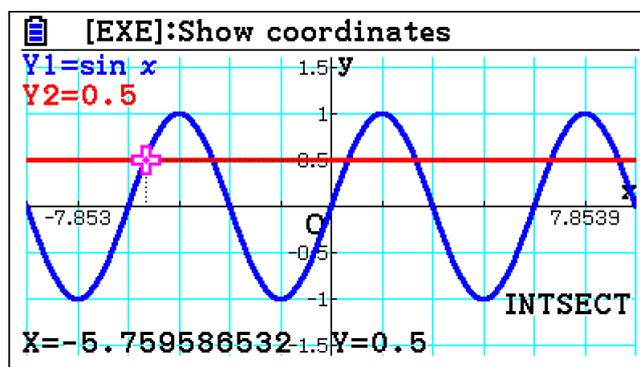
### Further Tasks

- Investigate the derivatives of  $y = \sin ax$  and  $y = b \sin x$ .
- Explain why this wouldn't work as neatly if the angle was measured in degrees.

# MEI Casio Tasks for A2 Core

## Task 6: Solutions of Trigonometric Equations (Radians)

1. Select Graphs mode: **MENU** **5**
2. Check the angle type is set to radians: **SHIFT** > **SET UP** and scroll down to Angle.
3. Enter the graph  $Y1=\sin x$  : **sin** **X,θ,T** **EXE**
4. Enter the graph  $Y2=0.5$  : **0** **.** **5** **EXE**
5. Set the View-Window to TRIG: **SHIFT** **F3** **F2** **EXIT**
6. Draw the graphs: **F6**
7. Use G-Solve to find the points of intersection: **SHIFT** **F5** **F5**



You can use the cursor (**◀**/**▶**) to move between the points of intersection. Try finding the points of intersection for other values of  $Y2$  (e.g.  $Y2 = 0.75$  or  $Y2 = -0.3$ ).

### Questions

- What symmetries are there in the positions of the points of intersection?
- How can you use these symmetries to find the other solutions based on the value of  $\sin^{-1}x$  given by your calculator? (This is known as the "principal value".)

**Problem** (Try the question just using the  $\sin^{-1}$  function first then check it using the graph)

Solve the equation:  $\sin x = 0.2$  ( $-2\pi \leq x \leq 4\pi$ )

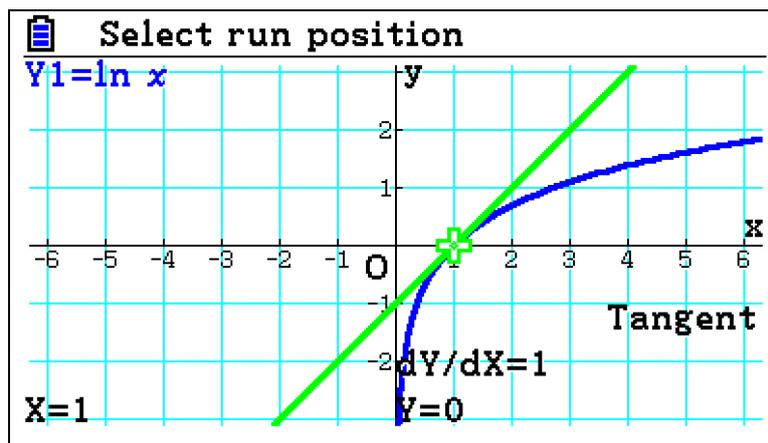
### Further Tasks

- Investigate the symmetries of the solutions to  $\cos x = k$  and  $\tan x = k$ .
- Investigate the symmetries of the solutions to  $\sin 2x = k$ .

# MEI Casio Tasks for A2 Core

## Task 7: Gradients of tangents to the natural logarithm $y = \ln x$

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Derivative: On:** **SHIFT** **MENU** **▼** **▼** **▼** **▼** **▼** **▼** **F1** **EXIT**
3. Draw the graphs  $y = e^x$  and  $y = \ln x$ . **Y1=ln x**:  
**In** **X,θ,T** **EXE** **F6**
4. Add the tangent at the point (*Sketch > Tangent*): **F4** **F2**



Use **◀**/**▶** to move the position of the point on the curve.

### Questions

- What is the relationship between the point and the gradient of the tangent on  $y = \ln x$ ?
- How does this relationship change for the graphs of  $y = \ln 2x$ ,  $y = \ln 3x$  ... ?

**Problem** (*Check your answer by plotting the graph and the tangent on your calculator*)

Find the equation of the tangent to the curve  $y = \ln x$  at the point  $x = 2$ .

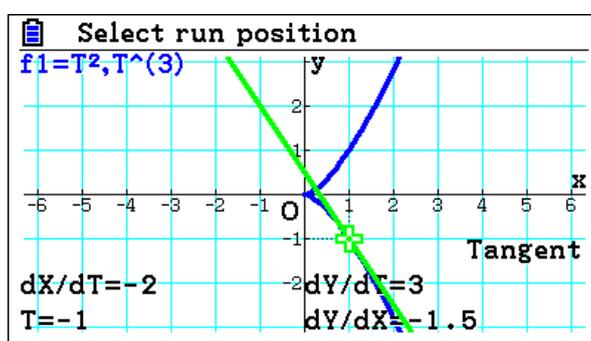
### Further Tasks

- Find the tangent to  $y = \ln x$  that passes through the origin.
- Explain the relationship between the derivatives of  $y = e^x$  and  $y = \ln x$ .  
*Hint: consider the point  $(a, b)$  on  $y = e^x$  and the reflected point  $(b, a)$  on  $y = \ln x$*

# MEI Casio Tasks for A2 Core

## Task 8: Tangents to parametric curves

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Derivative: On** and **Angle: Radians**:  
**SHIFT** **MENU** **▼** **▼** **▼** **▼** **▼** **▼** **F1** **▼** **▼** **▼** **▼** **F2** **EXIT**
3. Set the graph Type to Parametric: **F3** **F3**
4. Use V-Window to set the range of T to **Tmin: -4, Tmax: 4, pitch: 0.05**  
**SHIFT** **F3** **▼** **▼** **▼** **▼** **▼** **▼** **▼** **(←)** **4** **EXE** **4** **EXE** **0** **.** **0** **5** **EXE** **EXIT**
5. Draw the graph  $x = t^2, y = t^3$ . **X1t=T^2, Y1t=T^3**: **X,θ,T** **x^2** **EXE** **X,θ,T** **^** **3** **EXE** **F6**
6. Add the tangent at the point (*Sketch > Tangent*): **F4** **F2**



Use **◀**/**▶** to move the position of the point on the curve.

### Questions

- What is the relationship between  $\frac{dy}{dx}$ ,  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$ ?
- Does this relationship for other parametric curves?  
 e.g.  $x = 2t + 1, y = \frac{1}{t}$  or  $x = \cos t, y = \sin t$ .

**Problem** (*Check your answer by plotting the graph and the tangent on your calculator*)

Find the coordinates of the points on the curve  $x = 2t + \cos t, y = \sin t, -2\pi < t \leq 2\pi$  for which the tangent to the curve is parallel to the x-axis.

### Further Tasks

- Explore how you can find the equation of the tangent to a parametric curve at a point.
- Describe how to find the tangent to a parametric curve that passes through a specific point that is not on the curve.

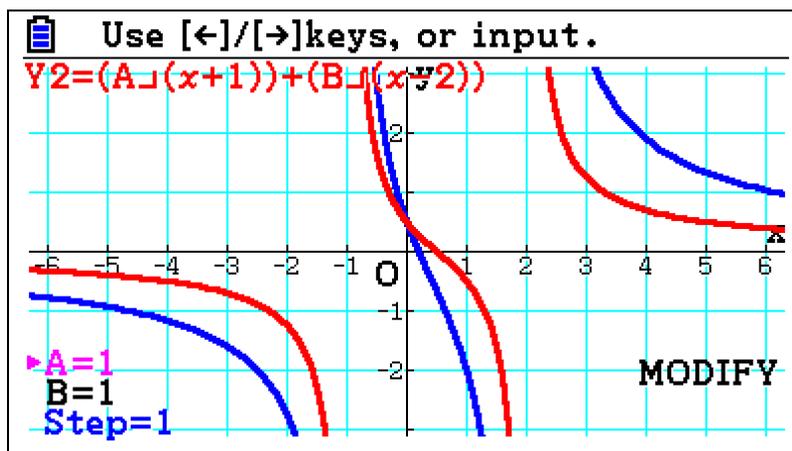
# MEI Casio Tasks for A2 Core

## Task 9: Partial Fractions

1. Add a new Graphs screen.

2. Enter the graphs  $Y1 = \frac{5x-1}{(x+1)(x-2)}$  and  $Y2 = \frac{A}{x+1} + \frac{B}{x-2}$ .

3. Plot the graphs using Modify: **F5**



Use  $\leftarrow$ / $\rightarrow$  to vary the values of **A** and **B** so that the two curves are the same.

### Question

- How could you find the values of  $A$  and  $B$  using  $\frac{5x-1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$ ?
- Does this method work for  $\frac{2x+7}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$ ?

**Problem** (Check your answers by plotting the graphs on your calculator)

Find values of  $A$  and  $B$  so the following can be expressed as partial fractions:

$$\frac{7x-14}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$$

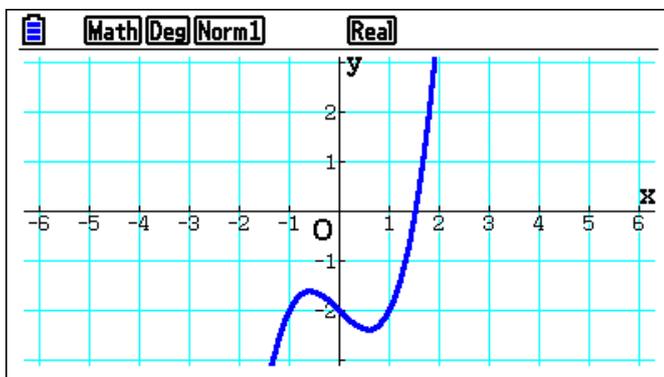
### Further Tasks

- Find  $A$ ,  $B$  and  $C$  such that  $\frac{5x^2+3x+7}{(x+2)(x^2+3)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+3}$
- Find  $A$ ,  $B$  and  $C$  such that  $\frac{7x^2+29x+28}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

# MEI Casio Tasks for A2 Core

## Task 10 – Numerical Methods: Change of sign

1. Add a new Graphs screen: **MENU** **5**
2. Plot the graph, e.g.  $y = x^3 - x - 2$ , **Y1** =  $x^3 - x - 2$ :  
**X,θ,T** **^** **3** **▶** **-** **X,θ,T** **-** **2** **EXE** **F6**



In this example you can see that the root lies between  $x = 1$  and  $x = 2$ .

3. Now go into the Table mode: **MENU** **7**
4. Using SET (F5), set the Table Start to 1, End to 2 and Step to 0.1:  
**1** **EXE** **2** **EXE** **0** **.** **1** **EXE** **EXIT**
5. Generate the Table: **F6**

The image shows the calculator's table mode. At the top, the mode is 'Math Deg Norm1 d/c Real'. The table has two columns: 'X' and 'Y1'. The values are as follows:

X	Y1
1.3	-1.103
1.4	-0.656
1.5	-0.125
1.6	0.496

At the bottom of the screen, the value '1.6' is displayed, and the menu options are: **FORMULA** **DELETE** **ROW** **EDIT** **GPH-CON** **GPH-PLT**.

In this example there is a change of sign between  $x = 1.5$  and  $x = 1.6$ .

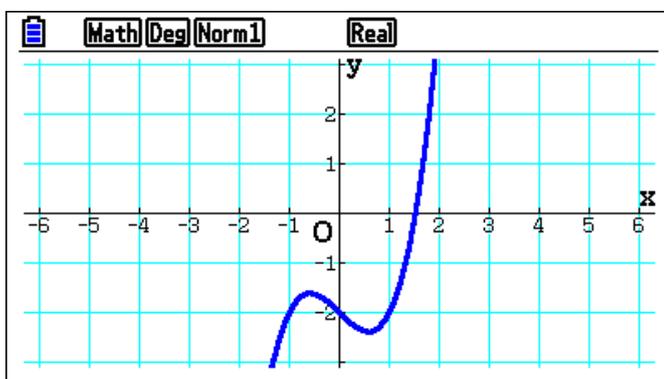
6. You can now investigate further by changing the set-up of the table so that it goes from 1.5 to 1.6 in steps of 0.01.  
(Alternatively you can do this by overtyping x-values in the table or inserting new rows).
7. You can check your answer in Graphs mode using G-Solv > Root: **F5** **F1**

Try using your calculator to find the roots of other equations using the change of sign method.

# MEI Casio Tasks for A2 Core

## Task 11 – Numerical Methods: Fixed Point Iteration

1. Add a new Graphs screen: **MENU** **5**
2. Plot the graph, e.g.  $y = x^3 - x - 2$ , **Y1** =  $x^3 - x - 2$ :  
**X,θ,T** **^** **3** **▶** **-** **X,θ,T** **-** **2** **EXE** **F6**



In this example you can see that the root lies between  $x = 1$  and  $x = 2$ .

3. Now go into the Recursion mode: **MENU** **8**
4. Add the recurrence relation  $x_{n+1} = \sqrt[3]{x_n + 2}$ , this should appear as  $a_{n+1} = \sqrt[3]{a_n + 2}$ :  
**SHIFT** **^** **3** **▶** **F2** **+** **2** **EXE**
5. Using SET (F5), set the Table Start to 1, End to 5 and  $a_0$  to 1:
6. Generate the Table: **F6**

The image shows the Recursion mode screen on a Casio calculator. The recurrence relation  $a_{n+1} = \sqrt[3]{a_n + 2}$  is displayed. Below it, a table is shown with columns for 'n+1' and 'a\_{n+1}'. The values for n=2, 3, 4, and 5 are listed, with the value for n=5, 1.5213, highlighted. At the bottom, the value 1.521345368 is shown, along with function keys like FORMULA, DELETE, WEB-GPH, GPH-CON, and GPH-PLT.

n+1	a <sub>n+1</sub>
2	1.5098
3	1.5197
4	1.5211
5	1.5213

In this example after 5 iterations the approximation is accurate to 3 decimal places.

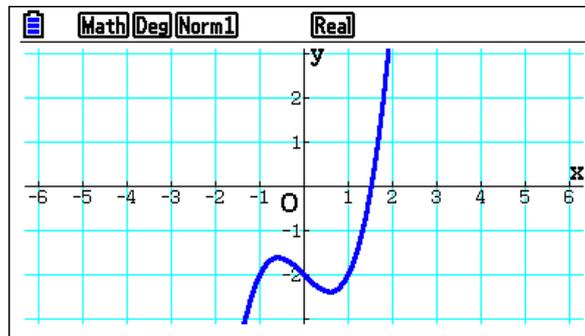
7. You can check your answer in Graphs mode using G-Solv > Root: **F5** **F1**

Try using your calculator to find the roots of other equations using a fixed-point iteration.

# MEI Casio Tasks for A2 Core

## Task 12 – Numerical solutions of equations: Newton-Raphson

1. Add a new Graphs screen: **MENU** **5**
2. Plot the graph, e.g  $y = x^3 - x - 2$ , **Y1** =  $x^3 - x - 2$ :  
**X,θ,T** **^** **3** **▶** **-** **X,θ,T** **-** **2** **EXE** **F6**



In this example you can see that the root lies between  $x = 1$  and  $x = 2$ .

3. Now go into the Recursion mode: **MENU** **8**
4. Add the recurrence relation  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$   $a_{n+1} = a_n - \frac{a_n^3 - a_n - 2}{3a_n^2 - 1}$ :  
**F4** **F2** **-** **(** **F2** **^** **3** **▶** **-** **F2** **-** **2** **)** **÷** **(** **3** **F2** **x<sup>2</sup>** **-** **1** **)** **EXE**
5. Using SET (F5), set the Table Start to 1, End to 5 and  $a_0$  to 2:  
**F5** **1** **EXE** **5** **EXE** **2** **EXE** **EXIT**
6. Generate the Table: **F6**

n	a <sub>n</sub>
2	1.5303
3	1.5214
4	1.5213
5	1.5213

1.521379707

In this example after 5 iterations the approximation is accurate to more than 3 decimal places.

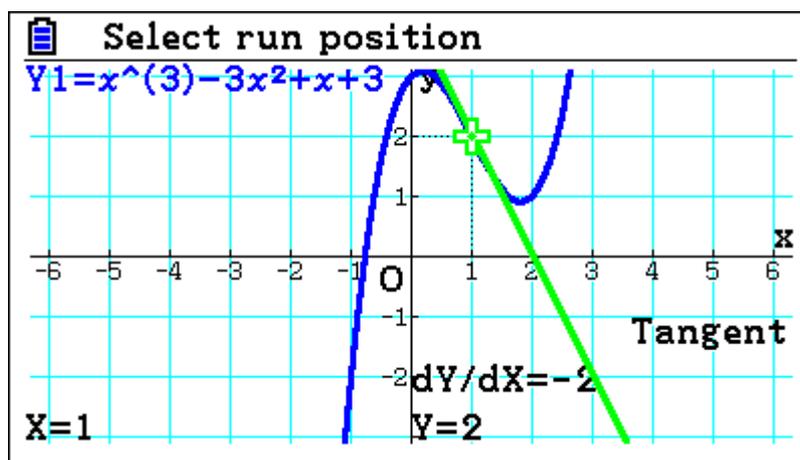
7. You can check your answer in Graphs mode using G-Solv > Root: **F5** **F1**

Try using your calculator to find the roots of other equations using the Newton-Raphson method.

# MEI Casio Tasks for A2 Core

## Task 13: Points of Inflection

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Derivative: On:** **SHIFT** **MENU** **▼** **▼** **▼** **▼** **▼** **▼** **F1** **EXIT**
3. Draw the graph  $y = x^3 - 3x^2 + x + 3$ , **Y1 =  $x^3 - 3x^2 + x + 3$ :**  
**X,θ,T** **^** **3** **▶** **-** **3** **X,θ,T** **x<sup>2</sup>** **+** **X,θ,T** **+** **3** **EXE** **F6**
4. Add the tangent at the point (*Sketch > Tangent*): **F4** **F2**
5. Move the point at which the tangent is drawn to (1, 2).



### Questions for discussion

- What is special about the point (1, 2), i.e. what are its unique features?
- Can you find the equivalent point for some other cubics?

**Problem** (*Check your answer by plotting the graph and the tangent on your calculator*)

Find the point of inflection of curve with equation  $y = x^3 + 6x^2 + 7x + 2$ .

### Further Tasks

- Find some example of cubic functions that have no stationary points. Will these have points of inflection?
- Will a quartic function always, sometimes or never have a point of inflection?

# MEI Casio Tasks for A2 Core

## Task 14: Sum of an Arithmetic Progression

- Go into Table mode: **MENU** **7**
- Add  $y = 2x+1$  and display the table. **Y1=2x+1** : **2** **X,θ,T** **+** **1** **EXE** **F6**
- Go into Run-Matrix mode: **MENU** **1**
- Find  $\sum_{x=1}^5 2x+1$ : **Math** **▶** **Σ(**  
**F4** **F6** **F2** **2** **X,θ,T** **+** **1** **▼** **◀** **X,θ,T** **▶** **1** **▲** **5** **EXE**

X	Y1
1	3
2	5
3	7
4	9

$\sum_{x=1}^5 (2x+1)$		35
-----------------------	--	----

Now find the sum of the terms of some other arithmetic progressions.

You can also try entering the sequence in Recursion mode **MENU** **8**

### Questions

- Find the sum of the first 10 terms of the AP that begins: 4, 7, 10, 13, ...
- Why will the terms of  $bn+c$  (for  $n = 1,2,3,\dots$ ) be an arithmetic progression (AP)?
- How can you express the link between the terms of an AP and its sum?

**Problem** (Try the question with pen and paper first then check it on your calculator)

What is the first term of an arithmetic progression if the 3<sup>rd</sup> term is 11 and the sum of the first 10 terms is 185?

### Further Tasks

- Investigate arithmetic progressions with the same sum, e.g. how many APs can you find that have a sum of 100?
- Express the  $n^{\text{th}}$  triangle number as the sum of an AP. Investigate whether the  $n^{\text{th}}$  hexagonal number can be expressed as the sum of an AP.

# MEI Casio Tasks for A2 Core

## Task 15: Sum of an Geometric Progression

- Go into Table mode: **MENU** **7**
- Add  $y = 3 \times 2^x$  and display the table. **Y1=** : **2** **X,θ,T** **+** **1** **EXE** **F6**
- Go into Run-Matrix mode: **MENU** **1**
- Find  $\sum_{x=1}^5 3 \times 2^x$  : **Math** **▶** **Σ(**  
**F4** **F6** **F2** **3** **×** **2** **^** **X,θ,T** **▼** **◀** **X,θ,T** **▶** **1** **▲** **5** **EXE**

X	Y1
1	6
2	12
3	24
4	48

Math Rad Norm1 d/c Real	
$\sum_{x=1}^5 (3 \times 2^x)$	186

Now find the sum of the terms of some other geometric progressions.

You can also try entering the sequence in Recursion mode **MENU** **8**

### Questions

- Find the sum of the first 5 terms of the GP that begins: 5, 4, 3.2, ...
- Why will the terms of  $b \times c^n$  (for  $n = 1, 2, 3, \dots$ ) be a geometric progression (GP)?
- How can you express the link between the terms of a GP and its sum?

**Problem** (Try the question with pen and paper first then check it on your calculator)

What is the common ratio of a geometric progression if the 2<sup>nd</sup> term is 30 and the sum of the first 6 terms is 3640?

### Further Tasks

- Investigate the sums of geometric series where the common ratio is negative and/or less than 1.
- Investigate how savings or loans can be expressed as the sum of a geometric progression.

# MEI Casio Tasks for A2 Core

## Teacher guidance

### Notes on using the Modify function

It is useful for students to be familiar with this mode first.

When in Modify mode the parameters and the step size can be changed with the cursor keys or values can be directly typed in.

When in Modify mode the cursor keys are used to change the parameters and cannot be used to move the axes. All moving of the axes and zooming is disabled in Modify mode. To move the axes or zoom press EXIT to come out of Modify mode. The axes can then be set to the appropriate values. To re-enter modify mode press EXIT again to return to the list of functions then F5 to go back into Modify mode.

### Task 1: The Modulus Function

The modulus function,  $abs(x)$ , is found using **OPTN > NUMERIC > Abs**

This task uses the Modify mode – see the notes above.

Problem solution:  $x = -\frac{5}{3}, \frac{1}{3}$   $y = -1$

Students might need some help structuring the investigation into  $|x + a| + b > 0$ . One strategy is to fix either  $a$  or  $b$  and investigate changing the other parameter.

### Task 2: Inverse functions

The aim of this task is to reinforce the link between the reflection in the line  $y = x$  and rearranging  $y = f(x)$  to express  $x$  in terms of  $y$ .

Problem solutions:

$$f^{-1}(x) = \sqrt{x-2}$$

$$g^{-1}(x) = x^2 - 1$$

$$h^{-1}(x) = \frac{1}{x} + 2$$

It is important to emphasise that the domain of the original function needs to be restricted so that it is one-to-one for the inverse to be a function.

### Task 3: Trigonometry – Double Angle formulae

This task uses the Modify mode – see the notes above.

To set the Angle to Degrees use SHIFT > SET UP and scroll down to the Angle option (F1: Degrees).

To set the View-Window to trigonometric scales use SHIFT > F3 and then select option F2: TRIG. NB Degrees must be set first or the calculators will set the scales to appropriate values for radians.

Students might need some help structuring the investigation into  $\sin x \cos x = a \sin (bx)$ . One strategy is to fix  $b$  and investigate changing  $a$  first to find a curve with the correct amplitude.

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Use of the compound angle formulae for  $\sin(a+b)$  and  $\cos(a+b)$  might be useful for some students to verify their results.

Problem solution:  $\theta = 30^\circ, 90^\circ, 150^\circ, 270^\circ$

### Task 4: Trigonometry: $R\cos(\theta-\alpha)$

This task uses the Modify mode – see the notes above.

Students should be familiar with setting the range of the axes before attempting this task.

Students are expected to be able to relate their findings to the expansion of  $R\cos(x-A) = R\cos x\cos A + R\sin x\sin A$ .

Problem solution:

$$4\cos\theta + 3\sin\theta = 5\cos(\theta - 36.87^\circ).$$

### Task 5: Differentiation – Trigonometric functions

It is essential that the students have their calculators set to radians for this task.

When considering how the gradient of the tangent varies one strategy would be to try and sketch a curve of this. Students should be able to observe that this has the same shape as  $\cos(x)$ .

Problem solution:

$$y = \frac{x}{2} + \frac{\sqrt{3}}{2} - \frac{\pi}{6} \text{ or } y = 0.5x + 0.342$$

### Task 6: Solutions of Trigonometric Equations (Radians)

This task encourages students to think about the symmetries of the trigonometric graphs and use these in finding solutions to equations.

V-Window: Trig is useful to set the axes to appropriate sizes. The scale on the x-axis is dependent on whether the calculator is in degrees or radian mode.

Problem solution:

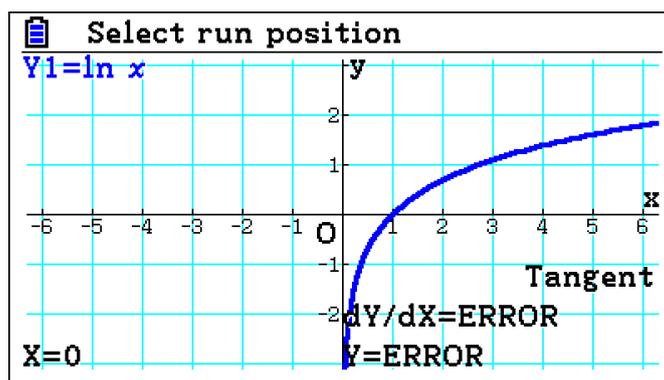
$$x = -6.082, -3.343, 0.201, 2.940, 6.485, 9.223$$

### Task 7: Gradients of tangents to the natural logarithm $y=\ln x$

The aim of this task is for students to be able to find the gradients and equations of tangents to the natural logarithm function.

When students first plot a tangent the calculator will give an ERROR message:

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This is because the function isn't defined at  $x = 0$ . Using the cursor keys to move to the right will give a value where the function is defined. This is a useful discussion point in terms of the gradient of the curve as it approaches the asymptote at  $x = 0$ .

For the second discussion point students might be surprised that the result doesn't change but they should be encouraged to think of this in terms of laws of logs.

Problem solution:

$$y = 0.5x - 0.307$$

### Task 8: Tangents to parametric curves

You should be comfortable with using V-Window to set the scales on the axes and the range of the parameter prior to trying this task.

For the first question students might find it useful to make a table of values for  $\frac{dy}{dx}$ ,  $\frac{dx}{dt}$  and

$\frac{dy}{dt}$ . Students should be encouraged to explore further parametric equations and to

consider their relationship in terms of  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ .

Problem solution:

$$t = \frac{\pi}{2}, \frac{3\pi}{2} : \text{points are } (2, 1), (2, -1).$$

Pressing the EXE button when a tangent is displayed will give the coordinates at the point and the equation of the tangent at that point.

### Task 9: Partial Fractions

This task uses the Modify mode – see the notes above.

This task can be used as an introduction to partial fractions or as a consolidation exercise. Students should be encouraged to express their methods algebraically.

Solutions to partial fractions:

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$$\frac{5x-1}{(x+1)(x-2)} = \frac{2}{x+1} + \frac{3}{x-2}$$

$$\frac{2x+7}{(x+2)(x+3)} = \frac{3}{x+2} - \frac{1}{x+3}$$

$$\frac{7x-14}{(x+3)(x-4)} = \frac{5}{x+3} + \frac{2}{x-4}$$

$$\frac{5x^2+3x+7}{(x+2)(x^2+3)} = \frac{3}{x+2} + \frac{2x-1}{x^2+3}$$

$$\frac{7x^2+29x+28}{(x-1)(x+3)^2} = \frac{4}{x-1} + \frac{3}{x+3} - \frac{1}{(x+3)^2}$$

### Task 10: Numerical Methods – Change of sign

This task is a set of instructions for how to implement the change of sign method on the calculators. Students are encouraged to work through these instructions and then try solving some equations of their own.

It is useful to have some additional equations for students to be finding the roots of once they have completed this sheet.

### Task 11: Numerical Methods – Fixed Point Iteration

This task is a set of instructions for how to implement the fixed point iteration method on the calculators. Students are encouraged to work through these instructions and then try solving some equations of their own.

It is useful to have some additional equations for students to be finding the roots of once they have completed this sheet.

### Task 12 – Numerical solutions of equations: Newton-Raphson

This task is a set of instructions for how to implement the Newton-Raphson method on the calculators. Students are encouraged to work through these instructions and then try solving some equations of their own.

Students should be asked to verify that the recurrence relation in step 4 is correct. They should also consider why a starting value of 2 has been chosen for  $a_0$ . NB The “Start” value is the first value of  $n$  that the table displays. Students can also be encouraged to try other values for  $a_0$  and consider the effect this has.

It is useful to have some additional equations for students to be finding the roots of once they have completed this sheet.

### Task 13: Points of Inflection

The point (1, 2) is a (non-stationary) point of inflection on  $y = x^3 - 3x^2 + x + 2$ . Students should be encouraged to look at the geometry of the curve, its tangent and its derivative and observe some of the following:

- it is where the curve changes from concave to convex;
- it is the point where the tangent slopes down the most;
- it is at the same  $x$ -value as when the derivative has a local minimum;
- it is the only point where the tangent crosses the curve (at that point);

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- it is a point of rotational symmetry (this would not necessarily be the case for curves other than cubics).

Problem solution:

The point of inflection of  $y = x^3 + 6x^2 + 7x + 2$  is at  $(-2, 4)$ .

### Task 14: Sum of an Arithmetic Progression

This task reinforces the terms of a sequence and what the sigma notation is showing. It is important to talk students through first two screens – don't skip the table as it is key to this task.

Before moving on to the problem you should show the two forms of the sum of an AP.

Problem solution: 5

### Task 15: Sum of a Geometric Progression

This task reinforces the terms of a sequence and what the sigma notation is showing. It is important to talk students through first two screens – don't skip the table as it is key to this task.

It is unlikely that many students will be able to derive a general formula for the sum of a GP without assistance. You will need to show them the sum of a GP before they attempt the problem and you can link their observations to the formula and discuss whether they are consistent with it.

Problem solution: 3