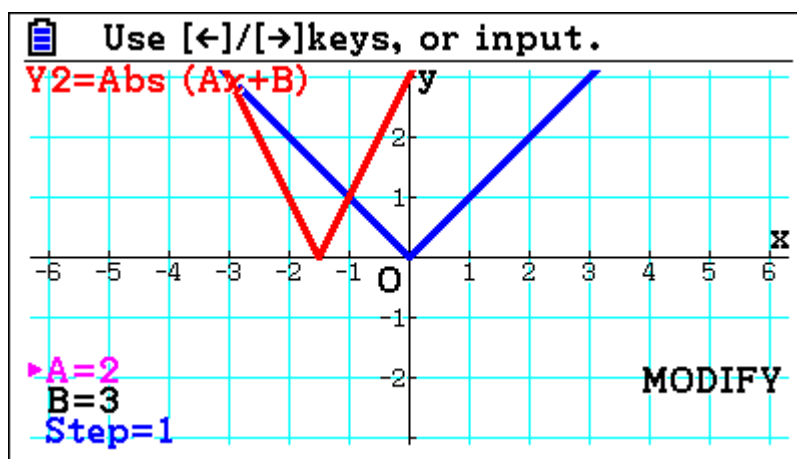


MEI Casio Tasks for A2 Core

Task 1: Functions – The Modulus Function

The modulus function, $\text{abs}(x)$, is found using **OPTN** > **NUMERIC** > **Abs**

1. Add a new Graphs screen: **MENU** **5**
2. Add the graph $y = |x|$, **Y1=Abs(x)**: **OPTN** **F5** **F1** **X,θ,T** **EXE**
3. Add the graph $y = |ax+b|$, **Y2=Abs(Ax+B)**:
OPTN **F5** **F1** **ALPHA** **X,θ,T** **X,θ,T** **+** **ALPHA** **log** **EXE**
4. Plot the curves using modify: **F5**



Questions

- What transformation maps the graph of $y = |x|$ onto the graph of $y = |ax+b|$?
- Where is the vertex on the graph of $y = |ax+b|$?
- Where does the graph of $y = |ax+b|$ intersect the y-axis?

Problem (Try the question with pen and paper first then check it on your calculator)

Sketch the graph of $y = |3x+2| - 3$ and find the points of intersection with the axes.

Find the equation of a graph that passes through the points $(-3, 1)$, $(-1, -3)$ and $(1, 1)$.

The graph $y = ||ax - b| - c|$ has vertices at $(1, 0)$, $(2, 2)$ and $(3, 0)$. Find a , b and c .

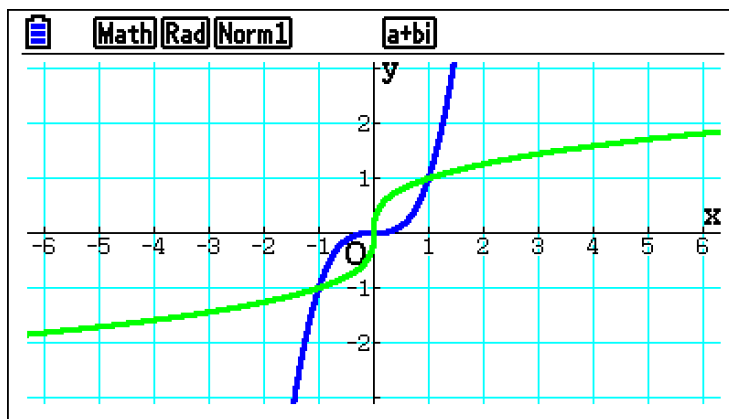
Further Tasks

- Investigate the graphs of
 - $y = |f(x)|$
 - $y = f(|x|)$for different functions $f(x)$, e.g. $y = \sin(x)$ or $y = x^3 - x^2$.
- Investigate the solutions to the inequality $|x + a| + b > 0$.

MEI Casio Tasks for A2 Core

Task 2: Inverse functions

1. Add a new Graphs screen: **MENU** **5**
2. Enter the graph $Y1 = x^3$: **X,θ,T** **^** **3** **EXE**
3. Plot the graph: **F6**
4. Plot the inverse function: Sketch > Inverse **F4** **F4**



Questions

- What graphical transformation maps the graph of the original function onto its inverse? Plot a line that is useful to illustrate this property.
- What is the equation of the graph of the inverse function?

Try finding the inverses of some other functions.

Problem (Check your answers by plotting the graphs on your calculator)

Find inverses of the following functions:

$$f(x) = x^2 + 2$$

$$g(x) = \sqrt{x+1}$$

$$h(x) = \frac{1}{x-2}$$

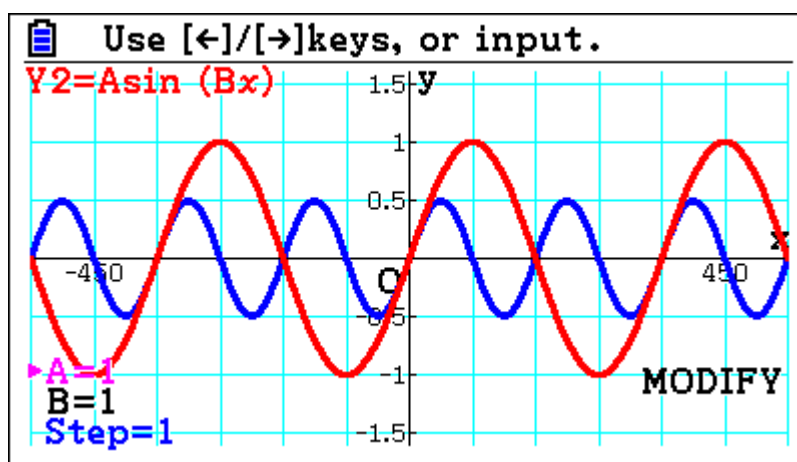
Further Tasks

- Find the inverse of the function $f(x) = x^2 + 6x + 7$. Can you always find the inverse of a quadratic function $f(x) = ax^2 + bx + c$?
- Investigate the graphs of the inverse trigonometric functions (you might find radians more convenient for this).
- Can you find a function where the inverse is the same as the original function?

MEI Casio Tasks for A2 Core

Task 3: Trigonometry – Double Angle formulae

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Angle: Degrees** and **V-Window: Trig**.
3. Add the graph $y = \sin x \cos x$, **Y1=sin x x cos x** : **sin** **X,θ,T** **X** **cos** **X,θ,T** **EXE**
4. Add the graph $y = a \sin (bx)$, **Y2=A sin(B x)**:
ALPHA **X,θ,T** **sin** **(** **ALPHA** **log** **X,θ,T** **)** **EXE**
5. Plot the curves using modify: **F5**



Questions

- For what values of a and b does $\sin x \cos x = a \sin (bx)$?
- Can you find values of a , b and c so that:
 - $\cos^2 x = a \cos (bx) + c$
 - $\sin^2 x = a \cos (bx) + c$
- How do these relationships link to the double angle formulae for sin and cos?

Problem (Try the question with pen and paper first then check it on your calculator)

Solve $\sin 2\theta - \cos \theta = 0$ in the range $0^\circ \leq \theta < 360^\circ$.

Further Tasks

- Describe the relationship $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ graphically.
- Find expressions for $\sin 3\theta$ and $\cos 3\theta$ in terms of $\sin \theta$ and $\cos \theta$.

MEI Casio Tasks for A2 Core

Task 4: Trigonometry: $R\cos(\theta-\alpha)$

1. Add a new Graphs screen: **MENU** **5**

2. In SET UP set **Angle: Degrees**

3. Set V-Window to Trig, Ymin: -3, Ymax: 3:

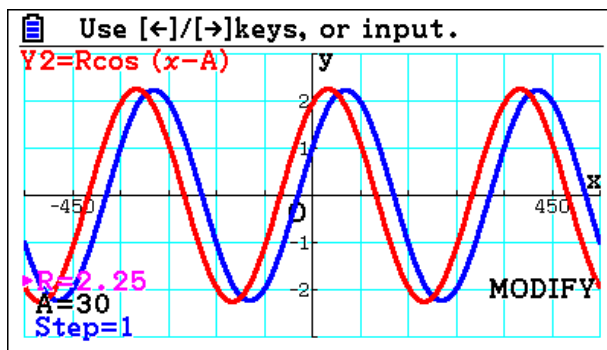
SHIFT **F3** **F2** **▼** **▼** **▼** **▼** **(-)** **3** **EXE** **3** **EXIT**

4. Enter the graphs $y = \cos x + 2\sin x$ and $y = R\cos(x - A)$:

Y1=cos x + 2 sin x, Y2=Rcos(x-A):

cos **X,θ,T** **+** **2** **sin** **X,θ,T** **EXE** **ALPHA** **6** **cos** **(** **X,θ,T** **-** **ALPHA** **X,θ,T** **)** **EXE**

5. Plot the graphs using Modify: **F5**



NB You can vary the values of **A** and **R** by typing directly in to them.

Questions

- Can you find values of **A** and **R** so that the curves are the same?
- Can you find values for **A** and **R** for any a and b where $a\cos x + b\sin x = R\cos(x - A)$?
- Can you explain the relationship using $R\cos(x - A) = R\cos x \cos A + R\sin x \sin A$?

Problem (Check your answer by plotting the graphs on your calculator)

Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $0^\circ < \alpha < 90^\circ$.

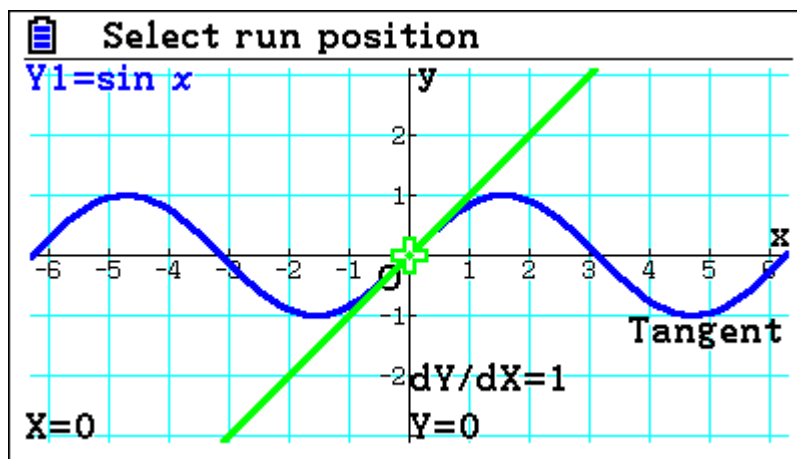
Further Tasks

- Explore how the form $R\cos(\theta - \alpha)$ can be used to find the maximum value of $a\cos\theta + b\sin\theta$ and the angles at which it occurs.
- Investigate the height of a rectangle as it is rotated through an angle θ about one of its corners.

MEI Casio Tasks for A2 Core

Task 5: Differentiation – Trigonometric functions

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Derivative: On** and **Angle:Radians**
3. Enter the graph $y = \sin x$, **Y1=sin(x)**: **sin** **X,θ,T** **EXE** **F6** **EXE**
4. Enter the graph of the derivative of Y1: **OPTN** **F2** **F1** **VARS** **F1** **1** **▶** **X,θ,T** **EXE**
5. Plot the graphs using modify:



Use **◀**/**▶** to move the position of the point on the curve.

Questions

- How does the gradient of the tangent vary as x varies:
 - What are its maximum and minimum values?
 - When is the gradient of the tangent 0?
 - For what values of x do these occur?
- Can you suggest a function for the derivative?
- Can you suggest a function for the derivative of $y = \cos(x)$?

Problem (Check your answer by plotting the graph and the tangent on your calculator)

Find the equation of the tangent to the curve $y = \sin x$ at the point $x = \frac{\pi}{3}$.

Further Tasks

- Investigate the derivatives of $y = \sin ax$ and $y = b \sin x$. Setting up the functions below will be helpful.

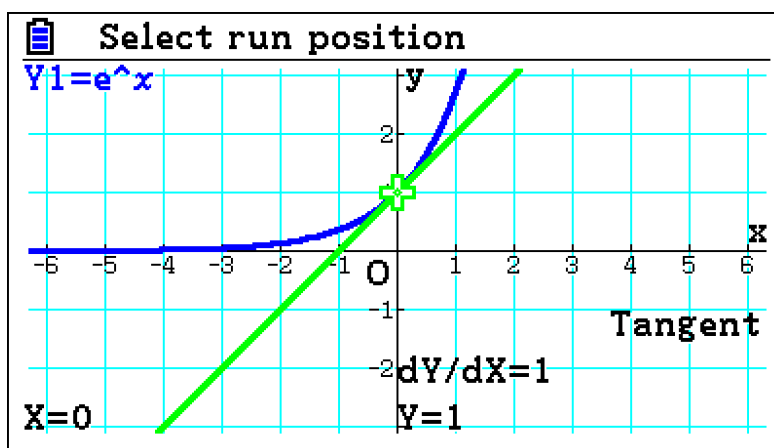
$$\begin{array}{l} \mathbf{Y1} = \sin(Ax) \quad \mathbf{[—]} \\ \mathbf{Y2} = \frac{d}{dx}(Y1) \Big|_{x=x} \quad \mathbf{[—]} \end{array}$$

- Explain why this wouldn't work as neatly if the angle was measured in degrees.

MEI Casio Tasks for A2 Core

Task 6: Gradients of tangents to the exponential function $y=e^x$

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Derivative: On**
3. Draw the graph $y = e^x$. **Y1=e^x**:
SHIFT **In** **X,θ,T** **EXE** **F6**
4. Add the tangent at the point (*Sketch > Tangent*): **F4** **F2**



Use **◀**/**▶** to move the position of the point on the curve.

Questions

- What is the relationship between the point and the gradient of the tangent on $y = e^x$?
- How does this relationship change for the graphs of $y = e^{2x}$, $y = e^{3x}$, ... ?

Problem (*Check your answer by plotting the graph and the tangent on your calculator*)

Find the equation of the tangent to the curve $y = e^{2x}$ at the point $x = 1$.

Further Tasks

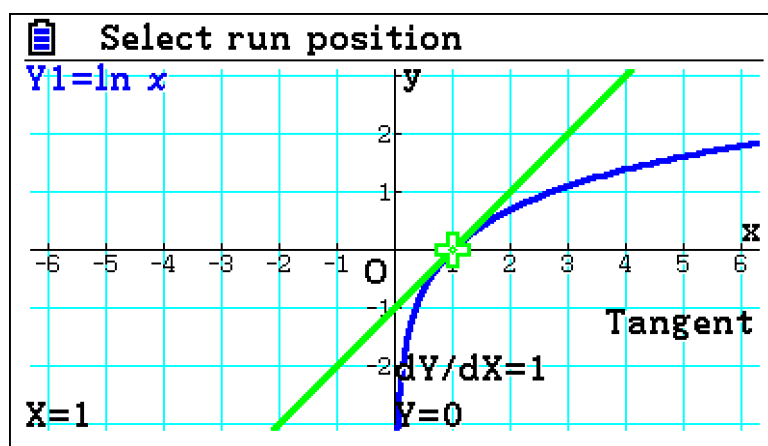
- Find the tangent to $y = e^x$ that passes through the origin.

Find the gradient of the tangent to $y = 3^x$ when $x = 0$.

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Task 7: Gradients of tangents to the natural logarithm $y = \ln x$

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Derivative: On:** **SHIFT** **MENU** **▼** **▼** **▼** **▼** **▼** **▼** **F1** **EXIT**
3. Draw the graphs $y = e^x$ and $y = \ln x$. **Y1=ln x**:
In **X,θ,T** **EXE** **F6**
4. Add the tangent at the point (*Sketch > Tangent*): **F4** **F2**



Use **◀**/**▶** to move the position of the point on the curve.

Questions

- What is the relationship between the point and the gradient of the tangent on $y = \ln x$?
- How does this relationship change for the graphs of $y = \ln 2x$, $y = \ln 3x$... ?

Problem (*Check your answer by plotting the graph and the tangent on your calculator*)

Find the equation of the tangent to the curve $y = \ln x$ at the point $x = 2$.

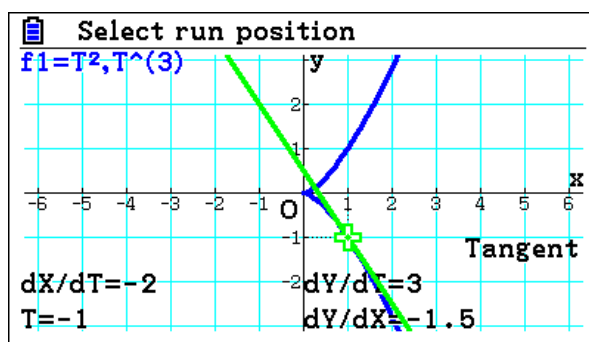
Further Tasks

- Find the tangent to $y = \ln x$ that passes through the origin.
- Explain the relationship between the derivatives of $y = e^x$ and $y = \ln x$.
Hint: consider the point (a, b) on $y = e^x$ and the reflected point (b, a) on $y = \ln x$

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Task 8: Tangents to parametric curves

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Derivative: On** and **Angle: Radians**:
SHIFT **MENU** **▼** **▼** **▼** **▼** **▼** **▼** **F1** **▼** **▼** **▼** **▼** **F2** **EXIT**
3. Set the graph Type to Parametric: **F3** **F3**
4. Use V-Window to set the range of T to **Tmin: -4, Tmax: 4, pitch: 0.05**
SHIFT **F3** **▼** **▼** **▼** **▼** **▼** **▼** **▼** **(←)** **4** **EXE** **4** **EXE** **0** **.** **0** **5** **EXE** **EXIT**
5. Draw the graph $x = t^2, y = t^3$. **X1t=T², Y1t=T³**: **X,θ,T** **x²** **EXE** **X,θ,T** **^** **3** **EXE** **F6**
6. Add the tangent at the point (*Sketch > Tangent*): **F4** **F2**



Use **◀**/**▶** to move the position of the point on the curve.

Questions

- What is the relationship between $\frac{dy}{dx}$, $\frac{dx}{dt}$ and $\frac{dy}{dt}$?
- Does this relationship hold for other parametric curves?
 e.g. $x = 2t + 1, y = \frac{1}{t}$ or $x = \cos t, y = \sin t$.

Problem (Check your answer by plotting the graph and the tangent on your calculator)

Find the coordinates of the points on the curve $x = 2t + \cos t, y = \sin t, -2\pi < t \leq 2\pi$ for which the tangent to the curve is parallel to the x-axis.

Further Tasks

- Explore how you can find the equation of the tangent to a parametric curve at a point.
- Describe how to find the tangent to a parametric curve that passes through a specific point that is not on the curve.

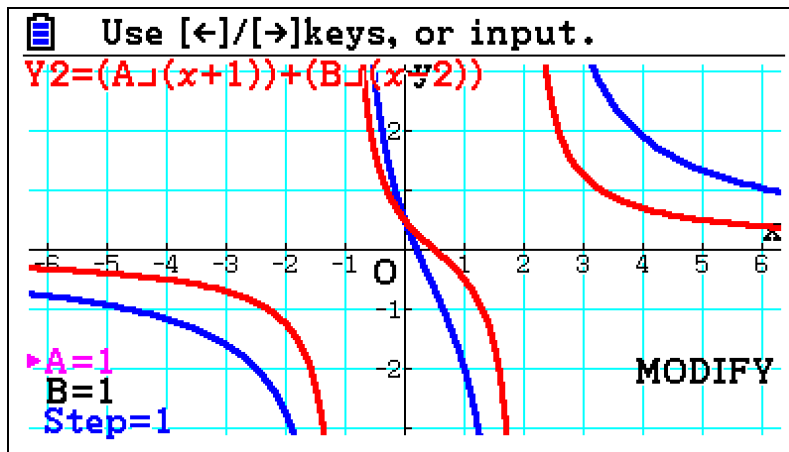
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Task 9: Partial Fractions

1. Add a new Graphs screen.

2. Enter the graphs $Y1 = \frac{5x-1}{(x+1)(x-2)}$ and $Y2 = \frac{A}{x+1} + \frac{B}{x-2}$.

3. Plot the graphs using Modify: **F5**



Use \leftarrow / \rightarrow to vary the values of **A** and **B** so that the two curves are the same.

Question

- How could you find the values of A and B using $\frac{5x-1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$?
- Does this method work for $\frac{2x+7}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$?

Problem (Check your answers by plotting the graphs on your calculator)

Find values of A and B so the following can be expressed as partial fractions:

$$\frac{7x-14}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$$

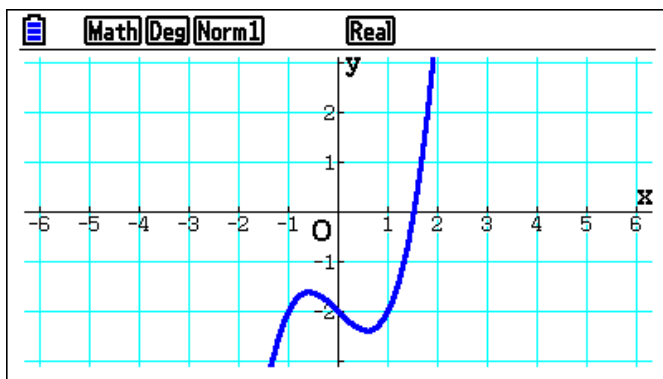
Further Tasks

- Find A , B and C such that $\frac{5x^2+3x+7}{(x+2)(x^2+3)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+3}$
- Find A , B and C such that $\frac{7x^2+29x+28}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

MEI Casio Tasks for A2 Core

Task 10 – Numerical Methods: Change of sign

1. Add a new Graphs screen: **MENU** **5**
2. Plot the graph, e.g $y = x^3 - x - 2$, **Y1 = $x^3 - x - 2$** :
X,θ,T **^** **3** **▶** **-** **X,θ,T** **-** **2** **EXE** **F6**



In this example you can see that the root lies between $x = 1$ and $x = 2$.

3. Now go into the Table mode: **MENU** **7**
4. Using SET (F5), set the Table Start to 1, End to 2 and Step to 0.1:
1 **EXE** **2** **EXE** **0** **.** **1** **EXE** **EXIT**
5. Generate the Table: **F6**

The image shows the calculator screen in table mode. The top status bar displays 'Math Deg Norm1 d/c Real'. The table has two columns: 'X' and 'Y1'. The values are as follows:

X	Y1
1.3	-1.103
1.4	-0.656
1.5	-0.125
1.6	0.496

The value 1.6 in the X column is highlighted. At the bottom right of the screen, the number 1.6 is displayed. The bottom status bar shows 'FORMULA DELETE ROW EDIT GPH-CON GPH-PLT'.

In this example there is a change of sign between $x = 1.5$ and $x = 1.6$.

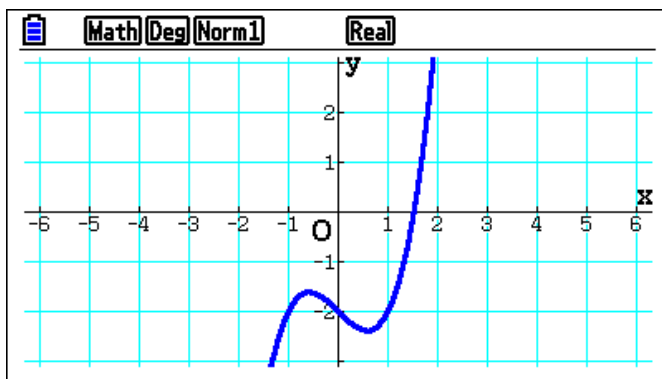
6. You can now investigate further by changing the set-up of the table so that it goes from 1.5 to 1.6 in steps of 0.01.
(Alternatively you can do this by overtyping x-values in the table or inserting new rows).
7. You can check your answer in Graphs mode using G-Solv > Root: **F5** **F1**

Try using your calculator to find the roots of other equations using the change of sign method.

MEI Casio Tasks for A2 Core

Task 11 – Numerical Methods: Fixed Point Iteration

1. Add a new Graphs screen: **MENU** **5**
2. Plot the graph, e.g. $y = x^3 - x - 2$, $Y1 = x^3 - x - 2$:
X,θ,T **^** **3** **▶** **-** **X,θ,T** **-** **2** **EXE** **F6**



In this example you can see that the root lies between $x = 1$ and $x = 2$.

3. Now go into the Recursion mode: **MENU** **8**
4. Add the recurrence relation $x_{n+1} = \sqrt[3]{x_n + 2}$, this should appear as $a_{n+1} = \sqrt[3]{a_n + 2}$:
SHIFT **^** **3** **▶** **F2** **+** **2** **EXE**
5. Using SET (F5), set the Table Start to 1, End to 5 and a_0 to 1:
6. Generate the Table: **F6**

n	a_n
2	1.5098
3	1.5197
4	1.5211
5	1.5213

1.521345368

In this example after 5 iterations the approximation is accurate to 3 decimal places.

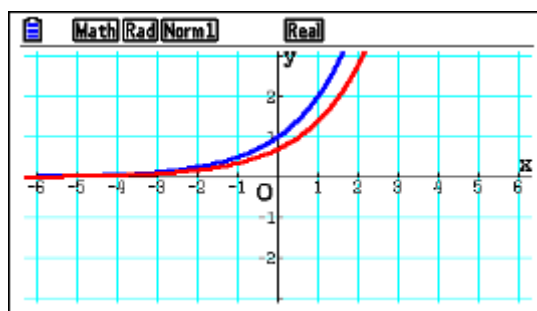
7. You can check your answer in Graphs mode using G-Solv > Root: **F5** **F1**

Try using your calculator to find the roots of other equations using a fixed-point iteration.

MEI Casio Tasks for A2 Core

Task 12: Derivatives of exponential functions $y=a^x$

1. Add a new Graphs screen: **MENU** **5**
2. In SET UP set **Derivative: On**
3. Draw the graph $y = a^x$. **Y1=A^x**: **ALPHA** **X,θ,T** **∧** **X,θ,T**
6. Enter the graph of the derivative of Y1: **OPTN** **F2** **F1** **VARΣ** **F1** **1** **▶** **X,θ,T** **EXE**
7. Plot the graphs using modify: **F5**



Find a value for A for which the graph of the function and the derivative are the same. Give your answer to at least 3 decimal places. You might find it useful to use table to compare values.

Questions

- Use table to compare the values of the function and the derivative.
- Explain why the derivative of $y = a^x$ is of the form $\frac{dy}{dx} = ka^x$ and find the value of k .

Problem (Check your answer by plotting the graph and the tangent on your calculator)

Show that the tangent to the curve $y = 2^x$ at the point where $x = \frac{1}{\ln 2}$ passes through the origin.

Further Tasks

- Investigate the derivative of functions of the form $y = a^{-x}$.
- Prove the result for the derivative $y = a^x$.

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Teacher guidance

Notes on using the Modify function

It is useful for students to be familiar with this mode first.

When in Modify mode the parameters and the step size can be changed with the cursor keys or values can be directly typed in.

When in Modify mode the cursor keys are used to change the parameters and cannot be used to move the axes. All moving of the axes and zooming is disabled in Modify mode. To move the axes or zoom press EXIT to come out of Modify mode. The axes can then be set to the appropriate values. To re-enter modify mode press EXIT again to return to the list of functions then F5 to go back into Modify mode.

Task 1: The Modulus Function

The modulus function, $\text{abs}(x)$, is found using **OPTN > NUMERIC > Abs**

This task uses the Modify mode – see the notes above.

Problem solutions:

$$x = -\frac{5}{3}, \frac{1}{3} \quad y = -1$$

$$y = |2x + 2| - 3$$

$$a = 2, b = 4, c = 2$$

Students might need some help structuring the investigation into $|x + a| + b > 0$. One strategy is to fix either a or b and investigate changing the other parameter.

Task 2: Inverse functions

The aim of this task is to reinforce the link between the reflection in the line $y = x$ and rearranging $y = f(x)$ to express x in terms of y .

Problem solutions:

$$f^{-1}(x) = \sqrt{x-2}$$

$$g^{-1}(x) = x^2 - 1$$

$$h^{-1}(x) = \frac{1}{x} + 2$$

It is important to emphasise that the domain of the original function needs to be restricted so that it is one-to-one for the inverse to be a function.

Task 3: Trigonometry – Double Angle formulae

This task uses the Modify mode – see the notes above.

To set the Angle to Degrees use **SHIFT > SET UP** and scroll down to the Angle option (F1: Degrees).

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To set the View-Window to trigonometric scales use SHIFT > F3 and then select option F2: TRIG. NB Degrees must be set first or the calculators will set the scales to appropriate values for radians.

Students might need some help structuring the investigation into $\sin x \cos x = a \sin (bx)$. One strategy is to fix b and investigate changing a first to find a curve with the correct amplitude.

Use of the compound angle formulae for $\sin(a+b)$ and $\cos(a+b)$ might be useful for some students to verify their results.

Problem solution: $\theta = 30^\circ, 90^\circ, 150^\circ, 270^\circ$

Task 4: Trigonometry: $R\cos(\theta-\alpha)$

This task uses the Modify mode – see the notes above.

Students should be familiar with setting the range of the axes before attempting this task.

Students are expected to be able to relate their findings to the expansion of $R\cos(x-A) = R\cos x \cos A + R\sin x \sin A$.

Problem solution:

$$4\cos\theta + 3\sin\theta = 5\cos(\theta - 36.87^\circ).$$

Task 5: Differentiation – Trigonometric functions

It is essential that the students have their calculators set to radians for this task.

When considering how the gradient of the tangent varies one strategy would be to try and sketch a curve of this. Students should be able to observe that this has the same shape as $\cos(x)$.

Problem solution:

$$y = \frac{x}{2} + \frac{\sqrt{3}}{2} - \frac{\pi}{6} \quad \text{or} \quad y = 0.5x + 0.342$$

Task 6: Gradients of tangents to the exponential function $y=e^x$

This task can be done on its own or with task 7. The aim of this task is for students to be able to find the gradients and equations of tangents to exponential functions.

Once students have observed that the derivative is the same as the y -coordinate they should explore other curves of the form $y = e^{ax}$.

Problem solution:

$$y = 14.778x - 7.389$$

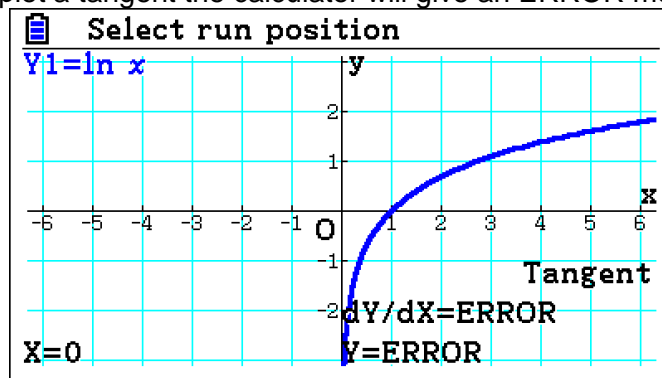
The second of the further tasks requires students to rewrite $y = 3^x$ as $y = e^{(\ln 3)x}$.

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Task 7: Gradients of tangents to the exponential function $y=e^x$

This task can be done on its own or with task 6. The aim of this task is for students to be able to find the gradients and equations of tangents to the natural logarithm function.

When students first plot a tangent the calculator will give an ERROR message:



This is because the function isn't defined at $x=0$. Using the cursor keys to move to the right will give a value where the function is defined. This is a useful discussion point in terms of the gradient of the curve as it approaches the asymptote at $x=0$.

For the second discussion point students might be surprised that the result doesn't change but they should be encouraged to think of this in terms of laws of logs.

Problem solution:

$$y = 0.5x - 0.307$$

Task 8: Tangents to parametric curves

You should be comfortable with using V-Window to set the scales on the axes and the range of the parameter prior to trying this task.

For the first question students might find it useful to make a table of values for $\frac{dy}{dx}$, $\frac{dx}{dt}$ and

$\frac{dy}{dt}$. Students should be encouraged to explore further parametric equations and to

consider their relationship in terms of $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

Problem solution:

$$t = \frac{\pi}{2}, \frac{3\pi}{2} : \text{points are } (2,1), (2,-1).$$

Pressing the EXE button when a tangent is displayed will give the coordinates at the point and the equation of the tangent at that point.

MEI Casio Tasks for A2 Core

Task 9: Partial Fractions

This task uses the Modify mode – see the notes above.

This task can be used as an introduction to partial fractions or as a consolidation exercise. Students should be encouraged to express their methods algebraically.

Solutions to partial fractions:

$$\frac{5x-1}{(x+1)(x-2)} = \frac{2}{x+1} + \frac{3}{x-2}$$

$$\frac{2x+7}{(x+2)(x+3)} = \frac{3}{x+2} - \frac{1}{x+3}$$

$$\frac{7x-14}{(x+3)(x-4)} = \frac{5}{x+3} + \frac{2}{x-4}$$

$$\frac{5x^2+3x+7}{(x+2)(x^2+3)} = \frac{3}{x+2} + \frac{2x-1}{x^2+3}$$

$$\frac{7x^2+29x+28}{(x-1)(x+3)^2} = \frac{4}{x-1} + \frac{3}{x+3} - \frac{1}{(x+3)^2}$$

Task 10: Numerical Methods – Change of sign

This task is a set of instructions for how to implement the change of sign method on the calculators. Students are encouraged to work through these instructions and then try solving some equations of their own.

It is useful to have some additional equations for students to be finding the roots of once they have completed this sheet.

Task 11: Numerical Methods – Fixed Point Iteration

This task is a set of instructions for how to implement the fixed point iteration method on the calculators. Students are encouraged to work through these instructions and then try solving some equations of their own.

It is useful to have some additional equations for students to be finding the roots of once they have completed this sheet.

Task 12: Derivatives of exponential functions $y=ax$

The aim of this task is for students to discover the value of e by looking for the case where the gradient function for an exponential curve is the same as the function of the curve. It also explores the derivative of general exponential functions.

Problem solution

For $y = 2^x$ its derivative is $\frac{dy}{dx} = 2^x \ln 2$. So at $x = \frac{1}{\ln 2}$ this gives $m = 2^{\frac{1}{\ln 2}} \ln 2$ as the gradient and hence $y = \left(2^{\frac{1}{\ln 2}} \ln 2\right)x + c$. When $x = \frac{1}{\ln 2}$, $y = 2^{\frac{1}{\ln 2}}$, and hence $c = 0$, so the tangent passes through the origin.