MEI Casio Tasks for Mechanics

Task 1: Kinematics – Constant Acceleration Equations and Graphs

Example

A particle acceleration has $a = -0.8\text{ms}^{-2}$ and initial velocity $u = 3.6\text{ms}^{-1}$. Sketch the graphs of displacement, velocity and acceleration against time.
(Note that you have to use x instead of t, when graphing on the calculator.)

1. Select the Run/Matrix Mode: [MENU] [1]

2. Enter the following values, $a = -0.8$ and $u = 3.6$: [(-)] [0] [·] [8] [→] [ALPHA] [X,θ,T] [EXE]
   [3] [·] [6] [EXE] [→] [ALPHA] [1] [EXE]

3. Add a new Graphs screen: [MENU] [5]

4. Enter the equation **Y1=A**: [ALPHA] [X,θ,T] [EXE]

5. Enter the equation **Y2=U+Ax**: [ALPHA] [1] [+] [ALPHA] [X,θ,T] [X,θ,T] [EXE]

6. Enter the equation **Y3=Ux+0.5Ax^2**: [ALPHA] [1] [X,θ,T] [+] [0] [·] [5] [ALPHA] [X,θ,T] [X,θ,T] [x^2]

7. Draw the graphs: [F6]

8. Use VWINDOW to set the following values for the scales:
   Xmin=-1.5, Xmax=11.1, Ymin=-2, Ymax=10
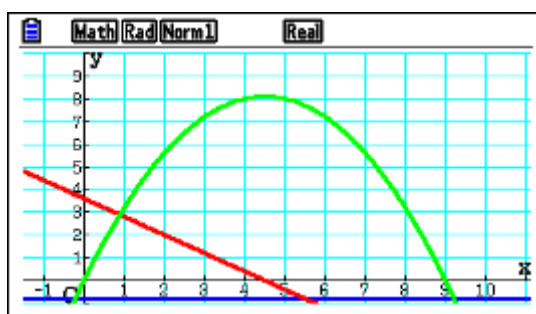
Questions

- What motion is represented by these graphs?

- What is the relationship between:
    - the area under the red velocity graph and the height of the green displacement graph?
    - the gradient of the green displacement graph and the red velocity graph?
    - the gradient of the red velocity graph and the blue acceleration graph?
    - the gradient of the green displacement graph and the red velocity graph?

- How is the area between the red line and the horizontal axis connected to the displacement?

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Problem (*Try the question with pen and paper first then check it on your calculator*)

A particle has initial velocity $u = 1\text{ms}^{-1}$ and acceleration $a = 0.4\text{ms}^{-2}$. Find the time that it takes to reach a velocity of 2.4ms^{-1} and the displacement at that time.

Further Tasks

- Find values of *u* and *a*, which produce a graph where the displacement is negative for the first 5 seconds of the motion.

- Find values of *u* and *a*, which produce a graph where the maximum displacement is 10.

- Explain the meaning of the value that you obtain if you integrate the acceleration between two specific values.

- Enter the formula **Y4=0.5(U+Y2(X))x** and explain what this produces.

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Task 2: Forces – Forces In Equilibrium

1. Open the Program List: [MENU] [ALPHA] [log] .

2. Select the program FORCE2D and press [F1] to start the program.

3. Press [EXE] and a screen similar to the one below will appear. Calculate the values of *P* and *Q*.
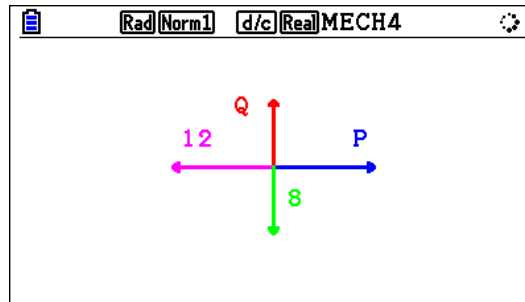
4. Press [EXE] and enter your values for *P* and *Q*. Press [EXE] after each value.
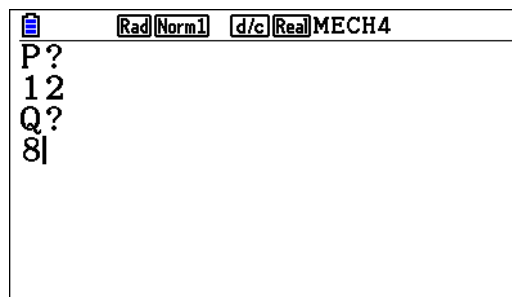

5. You will then see the correct values and have the opportunity to try four more before giving you a score out of five.

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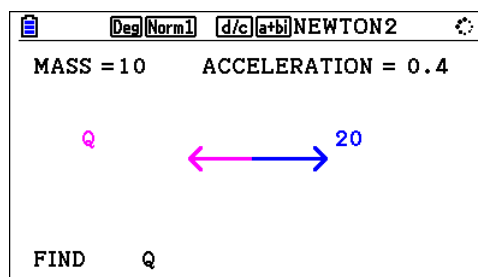## Task 3: Newton's Laws – The Second Law in One-Dimension

1. Open the Program List: [MENU] [ALPHA] [log] .

2. Select the program NEWTON1 and press [F1] to start the program.

3. Press [EXE] and a screen similar to the one of those below will appear. Calculate the acceleration, taking either upwards or to the right as positive.
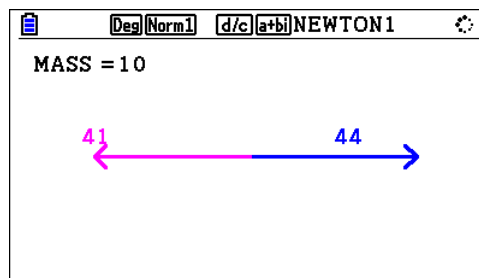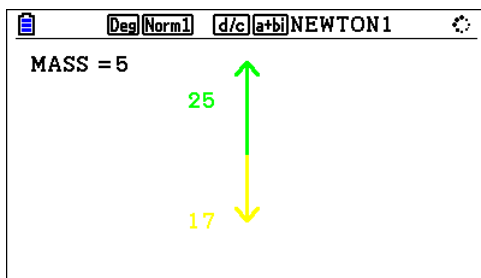
4. Press [EXE] to see the correct acceleration.
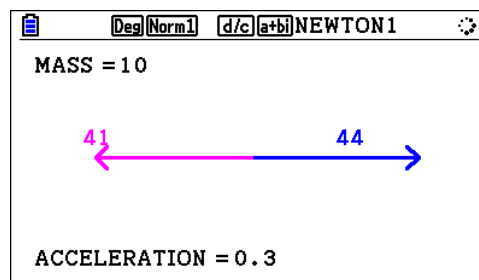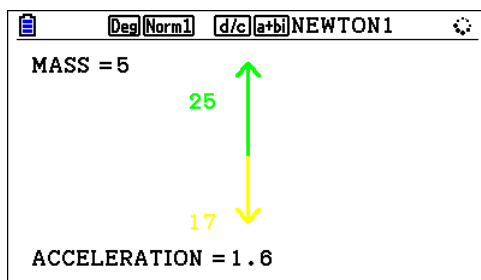
5. Press [EXE] to see another problem or press [EXIT] when the acceleration is displayed to leave the program.

6. Press [EXIT] and select the program NEWTON2 and press [F1] to start the program.

7. Press [EXE] twice and a screen similar to the one of those below will appear. Calculate the unknown quantity, taking either upwards or to the right as positive.

8. Press [EXE] to see the correct value.

Deg Norm1 d/c a+bi NEWTON2	Deg Norm1 d/c a+bi NEWTON2
MASS = 2 ACCELERATION = -4.5	MASS = 10 ACCELERATION = 0.4
P	Q 20
24	
FIND P = 15	FIND Q = 16
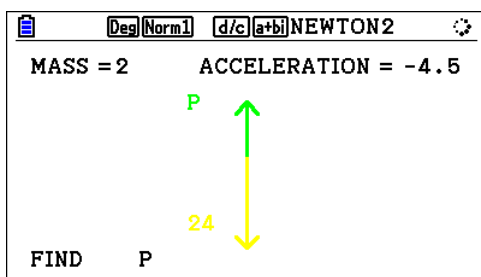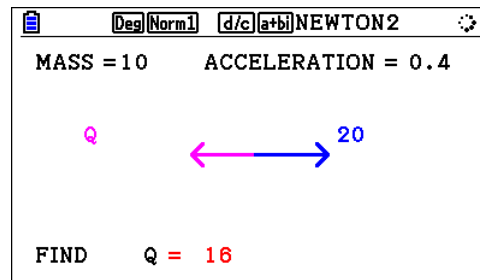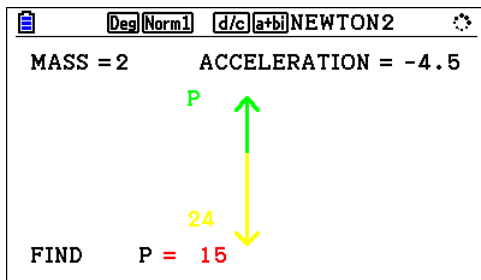
9. Press [EXE] to see another problem or press [EXIT] when the value of the unknown quantity is displayed to leave the program.

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## Task 4: Variable Acceleration – Differentiation

Example

A particle, of mass 5 kg, moves so that its displacement, s metres, at time, t seconds is modelled by $s = 8t - \dfrac{t^4}{104}$. Find the maximum displacement of the particle from its initial position and the magnitude of the resultant force on the particle when its velocity is zero.

(Note that you have to use x instead of t, when graphing on the calculator.)

1. Add a new Graphs screen: `MENU` `5`

2. Enter the equation **Y1 = 8x−x⁴/104:** `8` `X,θ,T` `−` `X,θ,T` `∧` `4` `▶` `÷` `1` `0` `4` `EXE`

3. Plot this graph. `F6`

4. Set the scales as shown below using V-Window: `F3`

    Xmin = 0,   Xmax = 5,   Xscale = 2
    Ymin = -10,   Ymax = 40,   Yscale = 5

5. Use G-Solv to find the maximum displacement of the particle from its initial position.
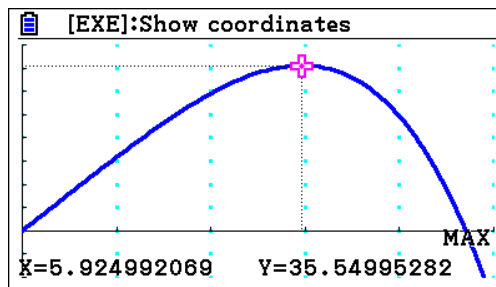
6. Return to the equation list. `EXIT`

7. Enter these expression for **Y2** and **Y3** as shown in the screen below. (Use **OPTN, CALC** to find the derivative and second derivative commands.)
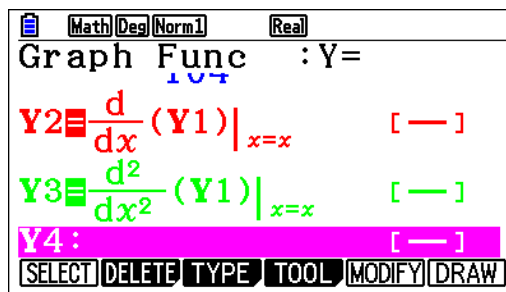
8. Plot the graphs of these functions and confirm that the velocity is zero when the displacement of the particle is a maximum.

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Problems

- A stone is projected vertically upwards. Its height above ground level, *h* metres at time *t* seconds is modelled as $h = 1.5 + 8t - 4.9t^2$.
  - Find the maximum height of the stone.
  - Find the speed at which the stone hits the ground.
  - Find the initial speed of the stone.

- A particle, of mass 4 kg, moves on a straight line so that its displacement, *s* metres, at time, *t* seconds is modelled by $s = 20t - t^3 + \dfrac{t^4}{24}$ for $0 \le t \le 8$.
  - Find the displacement of the particle when its velocity is zero.
  - Find the maximum magnitude of the resultant force on the particle.
  - Find the initial and final velocities of the particle.

- A car, of mass 1200 kg, moves on a straight line so that its displacement, *s* metres, at time, *t* seconds is modelled by $s = 10t + \dfrac{t^2}{2} - \dfrac{t^3}{40}$ for $0 \le t \le 20$
  - Find the initial speed of the car.
  - Find the maximum speed of the car.
  - Describe how the speed of the car changes.
  - Find the maximum magnitude of the resultant force on the car.
  - Find the total distance travelled by the car.
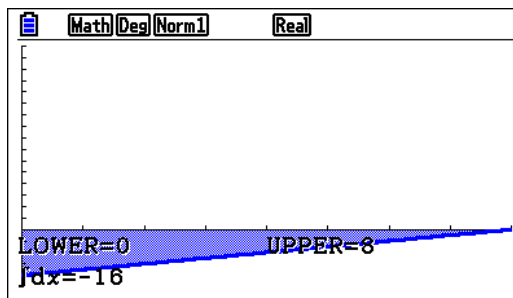
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## Task 5: Variable Acceleration – Integration

Example

The braking force on a car of mass 1000 kg decreases uniformly from 4000 N to zero as the car comes to rest, in 8 seconds on a horizontal road. Assume that no other horizontal forces act on the car. Sketch the graphs of acceleration and use this to find the initial velocity. Sketch the graph of velocity against time and use this to find the distanced covered.

(Note that you have to use *x* instead of *t*, when working with the calculator.)

1. Add a new Graphs screen: [MENU] [5]

2. Enter the equation **Y1 = 0.5x-4**: [0] [·] [5] [X,θ,T] [–] [4] [EXE]
   (The acceleration of the car changes from −4 to 0 m s⁻² and can be modelled as $a = 0.5t - 4$ for $0 \le t \le 8$).

3. Set the scales as shown below using V-Window: [SHIFT] [F3]

   Xmin = 0,   Xmax = 8,   Xscale = 1
   Ymin = -5,   Ymax = 10,   Yscale = 1

4. Plot this graph. [F6]

5. Use **G-Solv: ∫dx** to find the area of the made by the acceleration graph:
   [F5] [F6] [F3] [F1] [0] [EXE] [8] [EXE]

6. Return to the equation list. [EXIT]

7. Enter the expression for **Y2** as shown in the screen below and plot the curves.
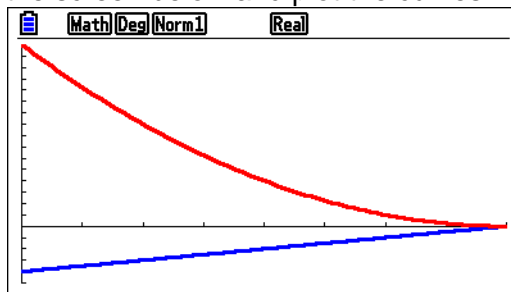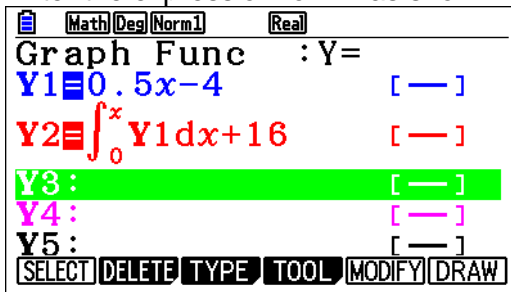
8. Use **G-Solv: ∫dx** to find the area enclosed by the axes and the velocity curve between 0 and 8.

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Problems

- A car travels between two speed humps in 10 seconds. The car is modelled as a particle that travels in a straight line. It crosses both speed humps at a speed of 1 m s^{-1}. The acceleration, a m s^{-2}, at time t seconds, is modelled as $a = 1 - 0.2t$ for $0 \leq t \leq 10$.
    - Find the maximum speed of the car between the humps.
    - Find the distance between the humps.

- A stone falls vertically from rest until it hits the ground after 6 seconds. A model attempts to account for air resistance. This model gives the acceleration, a m s^{-2}, at time t seconds, as $a = \dfrac{9.8(6-t)^2}{36}$ for $0 \leq t \leq 6$.
    - Find the speed of the stone when it hits the ground.
    - Find the distance that the stone falls.

- A ball bounces against a vertical wall. It is travelling horizontally at 5 m s^{-1} when it hits the wall. Ignore any vertical forces on the ball. A model for the acceleration, a m s^{-2}, of the ball at time t seconds is modelled as:
  $a = 6000t(t - 0.2)$ for $0 \leq t \leq 0.2$.
   - Find the speed at which the ball rebounds.

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Task 6: Kinematics – Intersecting Paths

Example

Two particles A and B have positions at time t given by $\mathbf{r}_A = (2t)\mathbf{i} + (4t - t^2)\mathbf{j}$ and $\mathbf{r}_B = (4t - 8)\mathbf{i} + (4 - t)\mathbf{j}$. Show that they pass through the point with position vector $\mathbf{i} + 2\mathbf{j}$ and find the time when they collide.

1. Add a new Graphs screen: [MENU] [5].

2. [SHIFT] [MENU] ▼ ▼ ▼ ▼ ▼ [F1]

3. Select parametric plotting mode: [F3] [F3]

4. Select simultaneous plotting: [SHIFT] [MENU] ▼ ▼ ▼ ▼ ▼ [F1]

5. Enter the path of a particle which moves with position vector $\mathbf{r}_A = (2t)\mathbf{i} + (4t - t^2)\mathbf{j}$:
   [2] [X,θ,T] [EXE] [4] [X,θ,T] [−] [X,θ,T] [x²] [EXE]

6. Add the path of a particle which moves with position vector $\mathbf{r}_B = (4t - 8)\mathbf{i} + (4 - t)\mathbf{j}$:
   [4] [X,θ,T] [−] [8] [EXE] [4] [−] [X,θ,T] [EXE]

7. Select **V-Window:** [SHIFT] [F3]

8. Enter the following values: **Tθmin:0   Tθmax:6   Tθptch:0.01**

9. Draw the paths: [F6]
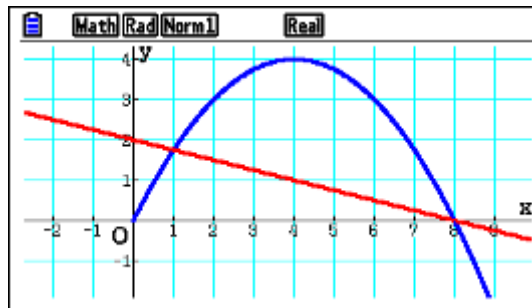
Use ◀ ▶ ▲ ▼ to adjust the screen until it looks like the version above. It can be helpful to repeat the plot using Sketch Cls [F4] [F1].

Use **Trace** to confirm that the particles pass through the point with position vector $\mathbf{i} + 2\mathbf{j}$ and find the time when they collide.

Questions

For each pairs of paths below, find the position vectors of points which both particles pass through and points where they collide.

- $\mathbf{r}_A = (3t)\mathbf{i} + (t + 1)\mathbf{j}$ and $\mathbf{r}_B = (4t - 2)\mathbf{i} + (2t - 1)\mathbf{j}$
- $\mathbf{r}_A = (4t - t^2)\mathbf{i} + (t - 1)\mathbf{j}$ and $\mathbf{r}_B = (3.5 - 0.875t)\mathbf{i} + (0.875t - 0.5)\mathbf{j}$
- $\mathbf{r}_A = (-t^2 + 3t + 1)\mathbf{i} + (t - 2)\mathbf{j}$ and $\mathbf{r}_B = (5 - 2t)\mathbf{i} + (0.25t^2 - t + 2)\mathbf{j}$

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Problem

The paths with position vectors for two particles are given below:

$$\mathbf{r}_A = (3t)\mathbf{i} + (4t - 2)\mathbf{j} \text{ and } \mathbf{r}_B = (at)\mathbf{i} + (bt + 3)\mathbf{j}$$

Given that the particles collide when $t = 2$, find a and b.

Given that the particles collide when $t = T$, what can you deduce about a and b.

Further Tasks

What can you say about the paths given by the two position vectors below?

$$\mathbf{r}_A = (2t)\mathbf{i} + (t + 1)\mathbf{j} \text{ and } \mathbf{r}_B = (4t - 2)\mathbf{i} + (2t - 1)\mathbf{j}$$

Create other pairs of paths with the same property.

Create pairs of paths that are perpendicular and for which the particles:
- do collide;
- do not collide.

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Task 7: Forces – Connected Particles

This task concerns two masses, mass 1 and mass 2, that are connected by a light string that passes over a smooth peg, as shown in the diagram below.

Mass 1 ● ● Mass 2
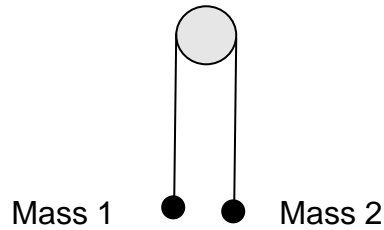
1. Open the Program List: [MENU] [ALPHA] [log] .

2. Select the program CONNECT and press [F1] to start the program.

3. Press [EXE] and enter the two masses as shown below. Press [EXE] after each value.

```
📋          Rad Norm1  d/c Real MECH5
MASS 1?
4
MASS 2?
6
```
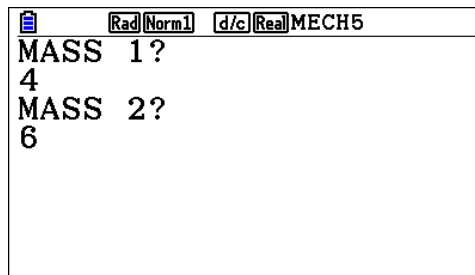
4. A screen similar to the one below will appear showing the acceleration and the tension.
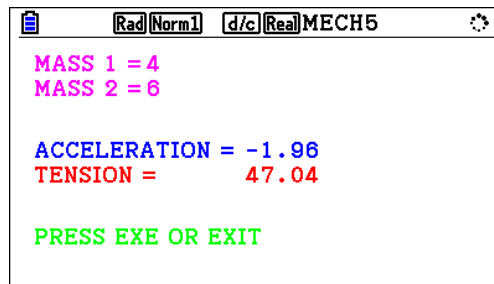
```
📋          Rad Norm1  d/c Real MECH5        ⟳
  MASS 1 = 4
  MASS 2 = 6

  ACCELERATION = -1.96
  TENSION =        47.04

  PRESS EXE OR EXIT
```

5. At this stage the keys below can be used.

Key	Action
[EXE]	Allows all new values to be entered.
[EXIT]	Ends the program

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Questions

- When is the acceleration zero?

- When is the acceleration negative?

- What is the maximum acceleration that you can obtain?

- What is the minimum tension that you can obtain?

Problems

- Can you find two mases that will give an acceleration of 4.9 m s^{-2}?

- Can you find another pair of mases that will also give an acceleration of 4.9 m s^{-2}?

- Try to find a similar way of producing the same acceleration with different masses. Can you find a general rule?

- What is the maximum tension that you can obtain, when mass 1 is 1kg?

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Task 8: Projectiles – The Maximum Range of a Projectile

1. Add a new Graphs screen: [MENU] [5].

2. Select degree mode: [SHIFT] [MENU] [▼] [▼] [▼] [▼] [▼] [▼] [▼] [▼] [▼] [F1] [EXIT]

3. Select the parametric plotting mode: [F3] [F3]

4. Add **Xt1=20cosA×T**: [2] [0] [cos] [ALPHA] [X,θ,T] [X] [X,θ,T] [EXE]

5. Add **Yt1=20sinA×T-4.9T^2**: [2] [0] [sin] [ALPHA] [X,θ,T] [X] [X,θ,T] [—] [4] [.] [9] [X,θ,T] [x²] [EXE]

6. Draw the graphs: [F6]

7. Use VWINDOW to set the following values for the scales:
   Xmin=0, Xmax=50, Xscale=10, Ymin=0, Ymax=20, Tmin=0, Tmax=5, Tptch=0.1

8. Change to MODIFY mode: [EXIT] [F5]
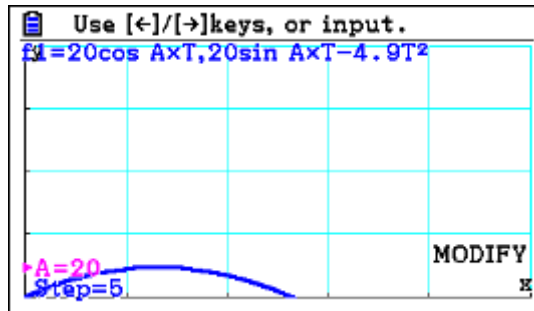
Vary the value of *A* until you can find the angle that produces the maximum range for the projectile.

Questions

- Change the initial speed of the projectile from 20 to another value. Does the maximum range still occur at the same angle of projection?

- Find two different angles that produce a range of 30. What is the relationship between these two angles? Is the same thing true for all ranges?

Problem

Derive an expression for the range of a projectile in terms of the projection speed, projection angle and *g*. Explain how this connects with the results that you have just obtained.

Further Tasks

- Compare the time taken for the projectile to reach its maximum height with the time of flight. Prove that your observation is true.

- Investigate what happens to the angle of projection that produces the maximum range if the projectile does not start and finish at the same height.

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Task 9: Projectiles – The Velocity and Acceleration of a Projectile

1. Open the Program List: [MENU] [ALPHA] [log] .

2. Select the program PROJECT and press [F1] to start the program. It draws the path of a projectile and shows the velocity and acceleration at a number of points on the path. Enter the angle 40, initial speed 20 and number of points 4. Press [EXE] after each value. You will also need to press [EXE] to add the velocity vectors and again to add the acceleration vectors. You should obtain the screen shown below:
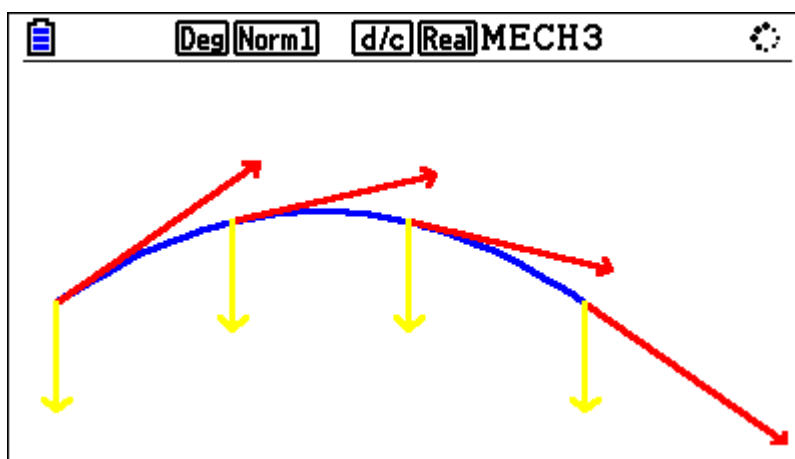
The velocity vectors are shown in red and the acceleration vectors in yellow. At this stage the keys below can be used.

Key	Action
[EXE]	Allows all new values to be entered.
[EXIT]	Ends the program

3. Use the program for different values. Note that the following values are allowed:
    -   Angles in the range 20 to 80°.
    -   Initial speeds greater than 15 m s^{-1}
    -   Number of points greater than 1.
    The points are placed so that the time to move from one point to the next is the same.

Questions

-   Why are the acceleration vectors always the same for each plot?

-   Why does the program draw a velocity and acceleration vector that are perpendicular when an odd number of points are selected?

-   Why do the velocity vectors have a smaller magnitude near to the top of the path?

-   Use a large angle, for example 80°. Why are the velocity vectors are so much smaller near to the top of the path when the angle is large (e.g. 80°)?

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Task 10: Friction – The Friction Law

1. Open the Program List: [MENU] [ALPHA] [log].

2. Select the program FRICTION and press [F1] to start the program. It concerns the forces acting on a block on a horizontal surface, when a horizontal force of magnitude *P* is applied to the block. Input initial values of:
   P = 150, *μ* = 0.4 and a mass of 50 kg.
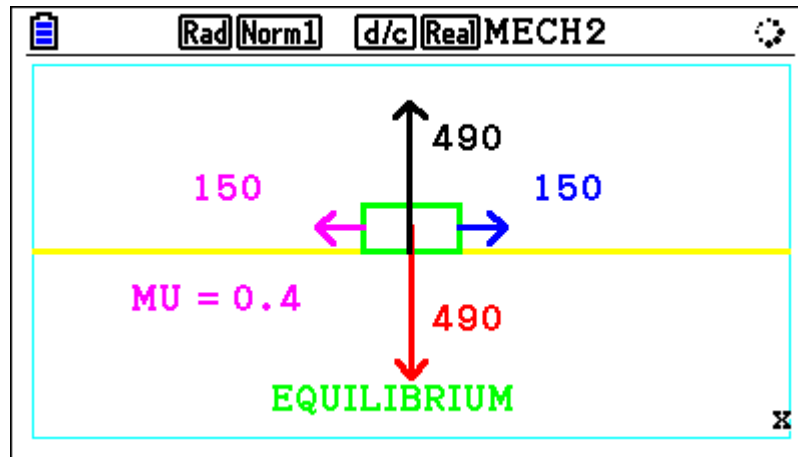
You can change the values using the commands below:

Key	Action
▶	Increases *P* by 1.
◀	Decreases *P* by 1.
▲	Increases *P* by 5.
▼	Decreases *P* by 5.
⊞	Increases mass by 1.
⊟	Decreases mass by 1.
☒	Increases coefficient of friction by 0.1.
÷	Decreases coefficient of friction by 0.1.
[EXE]	Allows all new values to be entered.
[EXIT]	Ends the program

What is the largest value of *P* for which the block remains at rest?

Find the value of *P* that produces an acceleration of 0.16 m s^{-2}.

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Questions

- Why is the normal reaction (in black) is always equal to the weight (in red)?

- Complete the table below:

Mass	Coefficient of Friction	Maximum *P* for Equilibrium
25 kg		98 N
200 kg	0.2	
150 kg		735 N
	0.8	392 N

- Can you deduce a relationship between the variables involved?

- Why is the friction force always equal to the applied force, *P*, when in equilibrium?

Problems

- Predict the maximum friction that can be obtained when a block of mass 40 kg is on a rough horizontal surface where the coefficient of friction between the block and the surface is 0.7.

- A horizontal force of magnitude 30 N is applied to a block of mass 6 kg. If the coefficient of friction between the block and the surface is 0.1, find the acceleration of the block.

- A horizontal force of 100 N produces an acceleration of 1.08 m s^{-2}, when applied to a block of mass 20 kg. Find the coefficient of friction.
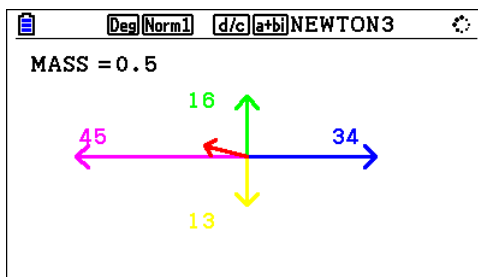
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Task 11: Newton's Laws –
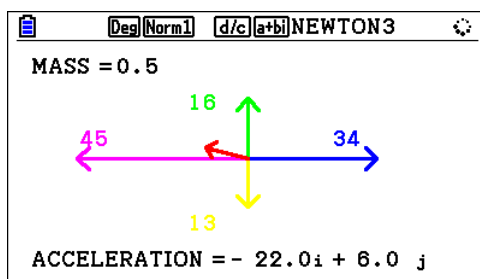## Newton's Second Law in Two-Dimensions

1. Open the Program List: [MENU] [ALPHA] [log] .

2. Select the program NEWTON3 and press [F1] to start the program.

3. Press [EXE] and a screen similar to the one below will appear.
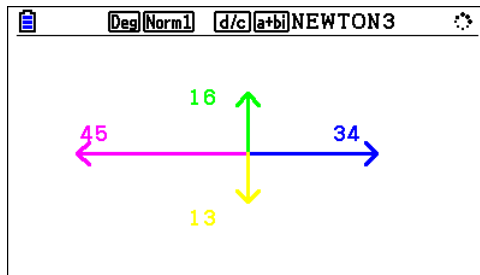
4. Press [EXE] again and you will see a screen similar to the one below. The red vector shows the resultant force. Calculate the acceleration that these forces wold produce on a particle of the given mass in the form $p\mathbf{i} + q\mathbf{j}$.
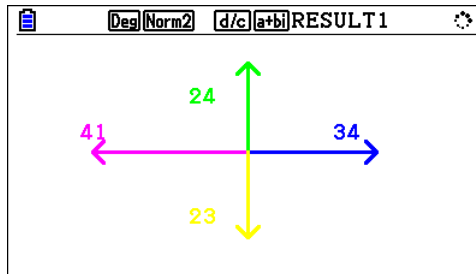

5. Press [EXE] to see the acceleration.

6. Press [EXE] to see another problem or press [EXIT] when the acceleration is displayed to leave the program.

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Task 12: Forces – Resultant Vectors 1

1. Open the Program List: [MENU] [ALPHA] [log] .

2. Select the program RESULT1 and press [F1] to start the program.

3. Press [EXE] and a screen similar to the one below will appear.

4. The red vector shows the resultant force. Calculate resultant of these forces in the form $p\mathbf{i} + q\mathbf{j}$.

5. Press [EXE] to see the resultant force.

6. Press [EXE] to see another problem or press [EXIT] when the resultant force is displayed to leave the program.
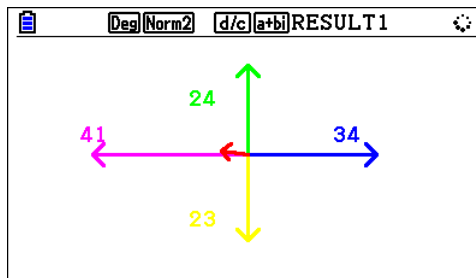
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Task 13: Forces – Resultant Vectors 2
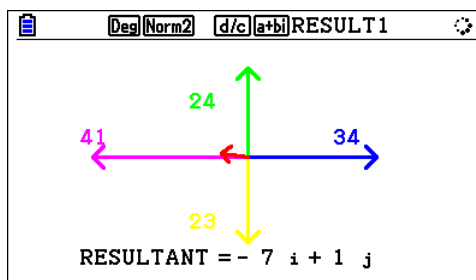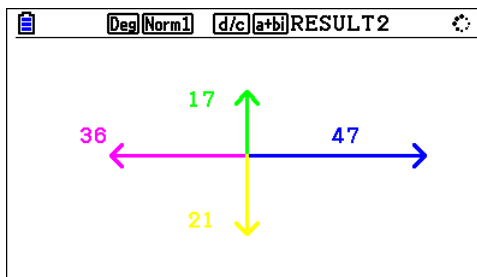
1. Open the Program List: [MENU] [ALPHA] [log] .

2. Select the program RESULT2 and press [F1] to start the program.

3. Press [EXE] and a screen similar to the one below will appear.

4. The red vector shows the resultant force. Find the magnitude and direction of the resultant.

5. Press [EXE] to see the magnitude and direction of the resultant force. Explain the convention for determining the angle. What are the maximum and minimum angles that could be obtained

6. Press [EXE] to see another problem or press [EXIT] when the magnitude and angle are displayed to leave the program.
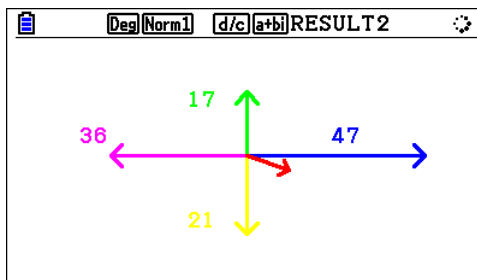
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Task 14: Vectors – Resolving Vectors 1

1. Open the Program List: [MENU] [ALPHA] [log] .

2. Select the program RESOLVE1 and press [F1] to start the program.

3. Press [EXE] and a screen similar to the one below will appear.

4. Press [EXE] and a screen similar to the one below will appear, showing the perpendicular components of the force. Calculate these components and write the force in the form $p\mathbf{i} + q\mathbf{j}$.

5. Press [EXE] to see the force expressed as a vector.

6. Press [EXE] to see another problem or press [EXIT] when the force is displayed to leave the program.

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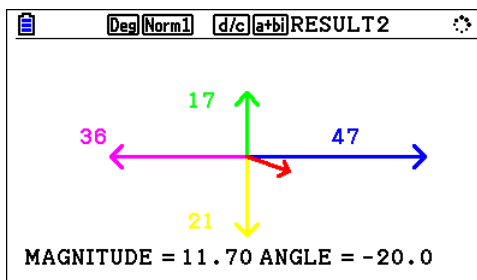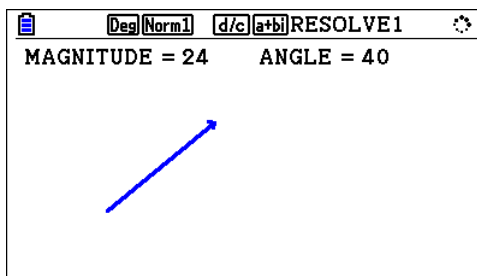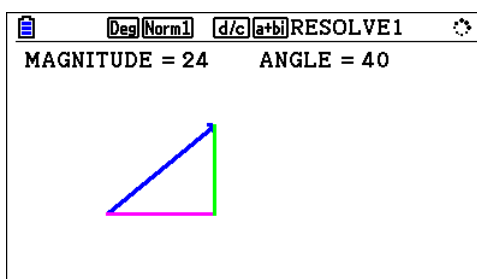
Task 15: Vectors – Resolving Vectors 2
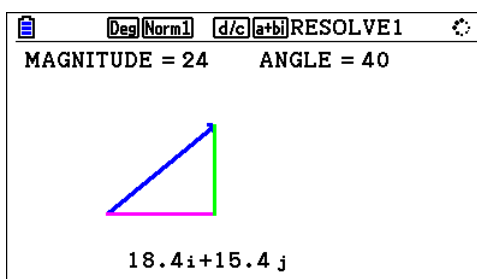
1. Open the Program List: [MENU] [ALPHA] [log] .

2. Select the program RESOLVE2 and press [F1] to start the program.

3. Press [EXE] and you will be asked the enter the magnitude and direction of a force, for example, 20 and 60°. Press [EXE] after each value.

```
📱    Deg Norm1  d/c a+bi RESOLVE2
MAGNITUDE?
20
ANGLE?
60|
```
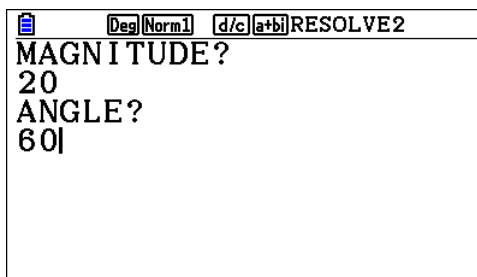
4. You will then see a screen similar to the one below showing the force that you have specified.

```
📱    Deg Norm1  d/c a+bi RESOLVE2    ⟳
MAGNITUDE = 20    ANGLE = 60
```
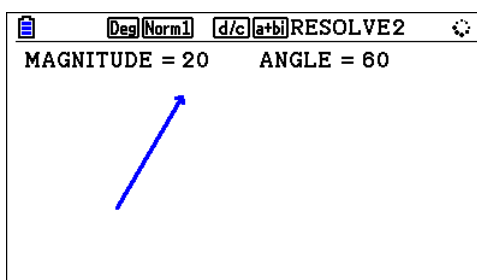
5. Press [EXE] and a screen similar to the one below will appear, showing the components of the force.

```
📱    Deg Norm1  d/c a+bi RESOLVE2    ⟳
MAGNITUDE = 20    ANGLE = 60
```
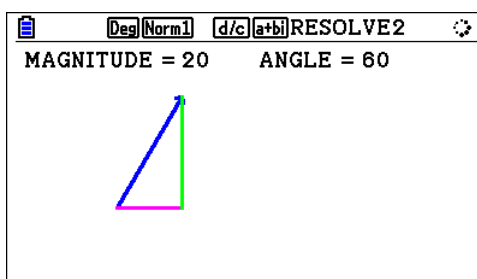
6. Press [EXE] and a screen similar to the one below will appear, showing the force in the form $p\mathbf{i} + q\mathbf{j}$.

```
📱    Deg Norm1  d/c a+bi RESOLVE2    ⟳
MAGNITUDE = 20    ANGLE = 60




        10.0i+17.3j
```
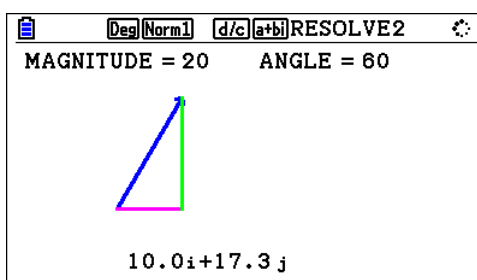
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Questions

1. What angle will produce forces of the form $k\mathbf{i} - k\mathbf{j}$?

2. Find the magnitude and angle that produce the vector $4\mathbf{i} - 3\mathbf{j}$.

3. Find the angle that produce forces of the form $k\mathbf{i} + 2k\mathbf{j}$

4. What can be deduced about the angle between forces of the form $a\mathbf{i} + b\mathbf{j}$ and $-b\mathbf{i} + a\mathbf{j}$?

5. If a force has direction θ, what can you deduce about the direction of a force with the opposite direction if:

   (a) $0 \le \theta \le 90$ (b) $90 \le \theta \le 180$ (c) $-90 \le \theta \le 0$ (d) $-180 \le \theta \le -90$

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Teacher guidance

Task 1: Constant Acceleration Equations and Graphs

This task encourages students to think graphically about the constant acceleration equations.

Note that x is used instead of t, when working with the calculator.

Problem Solution
Time = 3.5 seconds, Displacement = 5.95

Further Tasks

- Requires $u = -\dfrac{5}{2}a$, for example $u = -5$ and $a = 2$.

- Requires $u^2 = -20a$, for example $u = 5$ and $a = -1.25$.
- The integral gives the change in velocity in this period.

- This gives the displacement and relates to the formula $s = \dfrac{1}{2}(u + v)t$.

Task 2: Forces in Equilibrium

This task uses a program to provide a quick opportunity for students to recognise the magnitudes of forces when four perpendicular forces are in equilibrium.

Task 3: Newton's Second Law in One Dimension

This task uses a program to provide a quick opportunity for students to consolidate their ability to use Newton's second Law in simple one-dimensional contexts.

Task 4: Variable Acceleration - Differentiation (AS)

This task is designed to encourage students to use the facilities of the graphics calculator to solve variable acceleration problems in one dimension using differentiation.

Problem solutions

- Maximum height = 4.77 metres
  Speed at Ground Level = 9.66 m s^{-1}
  Initial Speed = 8 m s^{-1}

- Maximum Displacement = 36.6 metres
  Maximum Magnitude of the Resultant Force = 72 N
  Initial Velocity = 20 m s^{-1}
  Final Velocity = -86.7 m s^{-1}

- Initial Speed = 10 m s^{-1}
  Maximum Speed = 13.3 m s^{-1}
  Speed increases until $t = 6.67$ and then decreases to zero when $t = 20$.
  Maximum Magnitude of the Resultant Force = 2400 N
  Maximum Displacement = 200 metres

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Task 5: Variable Acceleration - Integration (AS)

This task is designed to encourage students to use the facilities of the graphics calculator to solve variable acceleration problems in one dimension using integration.

Problem solutions

- Maximum speed = 3.5 m s^{-1}
  Distance between humps = 26.7 metres

- The stone has speed 19.6 m s^{-1} when it hits the ground.
  The stone falls a distance of 88.2 metres.

- The ball rebounds at 3 m s^{-1}

Task 6: Intersecting Paths

The aim of this task is to get students to think about the difference between paths that pass through the same points and points where objects would collide.

Questions
- Collide at 6**i** + 3**j** when $t = 2$.
- Collide at 3**j** when $t = 4$. Also pass through 3**i** but not at the same time.
- Collide at −3**i** + 2**j** when $t = 4$. Also pass through **i** + **j** but not at the same time.

Problem solution
For $t = 2$, $a = 3$ and $b = 1.5$.

For $t = $ T, $a = 3$ and $b = \dfrac{4T - 5}{T}$.

Further Tasks

The paths are parallel.

Consider the two position vectors below:

$$\mathbf{r}_A = (a + bt)\mathbf{i} + (c + dt)\mathbf{j} \text{ and } \mathbf{r}_B = (e + ft)\mathbf{i} + (g + ht)\mathbf{j}$$

These paths will be parallel if $\dfrac{d}{b} = \dfrac{h}{f}$.

The paths be perpendicular if $\dfrac{d}{b} = -\dfrac{f}{h}$

The perpendicular paths will intersect if $t = \dfrac{a - e}{f - b} = \dfrac{c - g}{h - d}$ and $t \geq 0$.

Task 7: Connected Particles

This task uses a simple program to explore the relationship between the masses, acceleration and tension when two masses are connected by a light string that passes over a smooth peg.

Questions

- There is zero acceleration when both masses are equal.
- The acceleration is negative when Mass 2 > Mass 1.
- Maximum acceleration is g or 9.8 m s^{-2} when one mass is zero!
- Zero, when one mass is zero!

Problem solutions
- For example; Mass 1 = 3 and Mass 2 = 1 or Mass 1 = 6 and Mass 2 = 2.
- Whenever Mass 1 = 3 × Mass 2.
- To produce an acceleration of $\dfrac{g}{k}$ you require Mass 1 $= \left(\dfrac{k+1}{k-1} \right) \times$ Mass 2.
- As Mass 2 increases the tension tends to 2g or 19.6 N.

Task 8: The Maximum Range of a Projectile

This task aims to get students to explore the relationship between the range and the angle of projection.

Problem Solution

Projectile lands when:

$$V \sin \alpha t - \frac{1}{2} gt^2 = 0$$

$$t = \frac{2V \sin \alpha}{g}$$

$$\text{Range} = \cos \alpha t$$

$$= \cos \alpha \times \frac{2V \sin \alpha}{g}$$

$$= \frac{V}{g} \sin(2\alpha)$$

The range has a maximum when $\sin(2\alpha) = 1$ which will be for an angle of projection of 45°.

Also

$$\sin(2(90 - \alpha)) = \sin(180 - 2\alpha)$$
$$= \sin 180 \cos(2\alpha) - \cos 180 \sin(2\alpha)$$
$$= \sin(2\alpha)$$

So any pair of acute angles that sum to 90° will produce the same range.

Further Tasks
- Students will observe that the time to maximum height is half of the time of flight.

The time of flight is given by $t = \dfrac{2V \sin \alpha}{g}$ as shown above.

At the maximum height the vertical component of the velocity is zero:

$$V \sin \alpha - gt = 0$$

$$t = \frac{V \sin \alpha}{g} \quad .$$

This is half of the time of flight.

- If the projectile lands at a lower level, then an angle less than 45° produces the maximum range.

  If the projectile lands at a higher level, then an angle greater than 45° produces the maximum range.

Task 9: The Velocity and Acceleration of a Projectile

This activity aims to illustrate the velocity and acceleration vectors for a projectile.

Questions

- The acceleration is always the same as the only force acting is the weight of the projectile, which will always produce a downward acceleration of 9.8 m s^{-2}.
- The velocity and acceleration vectors will only be perpendicular when the projectile is at its maximum height and has a velocity that is horizontal. An odd number of points are needed to place a vector at this point.
- As the projectile gain height, its velocity will decrease and so the vectors will be placed closer together as the change in displacement is less.
- For large angles, the vertical component of the initial velocity is much greater than the horizontal component. As the vertical component of the velocity decreases, the velocity will be much less than the initial velocity.

Task 10: The Friction Law

This task is designed to enable students to explore the friction law. It considers a block on a rough horizontal surface when a horizontal force is applied to the block.

Questions

- The vertical forces must be in equilibrium as the block remains on the surface. Hence the weight must be equal to the normal reaction force.

- The completed table is shown below:

Mass	Coefficient of Friction	Maximum P for Equilibrium
25 kg	0.4	98 N
200 kg	0.2	392 N
150 kg	0.5	735 N
50 kg	0.8	392 N

- The students should be able to deduce that in the limiting case $P = \mu mg$. This can be extended to the friction law $F \leq \mu R$.

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- If *F* was greater than *P*, then the block would accelerate to the left! The amount of friction will always be that needed to produce equilibrium, unless this exceeds the maximum possible value of friction.

- The maximum friction force is 274.4 N

  $F \le \mu R$

  $F \le 0.7 \times 9.8 \times 40$

  $F \le 274.4$

- The acceleration is 4.02 m s^{-2}.

  $30 - 0.1 \times 6 \times 9.8 = 6a$

  $a = \dfrac{24.12}{6} = 4.02 \text{ m s}^{-2}$

- The coefficient of friction is 0.4.

  $100 - \mu \times 20 \times 9.8 = 20 \times 1.08$

  $\mu = \dfrac{100 - 21.6}{196} = 0.4$

Task 11: Newton's Second Law in Two Dimension

This task uses a program to provide a quick opportunity for students to consolidate their ability to use Newton's second Law using in two dimensions to find the acceleration of a given mass as a vector.

Task 12: Vectors - Resultant Vectors 1

This task uses a program to provide a quick opportunity for students to consolidate their ability to find resultant vectors in the form $p\mathbf{i} + q\mathbf{j}$.

Task 13: Vectors - Resultant Vectors 2

This task uses a program to provide a quick opportunity for students to consolidate their ability to find the magnitude and direction of resultant vectors.

The convention used is to measure the angle from the horizontal of the diagram. Anti-clockwise angle are given as positive and clockwise as negative.

The maximum and minimum values are 180 and -180.

Note: it would be useful for the students to have a basic scientific calculator available as they work on this activity.

Task 14: Vectors – Resolving Vectors 1

This task uses a program to provide a quick opportunity for students to consolidate their ability to express vectors in the form $p\mathbf{i} + q\mathbf{j}$ given the magnitude and direction of a vectors.

Note: it would be useful for the students to have a basic scientific calculator available as they work on this activity.