



In Pascal's triangle the number 1 appears infinitely many times. All other numbers will appear a finite amount of times\*.

The number 2 appears just once.

The number 3 appears twice.

What is the first number that appears 3 times?

What is the first number that appears 4 times?

Can you find a number that appears 5 times?

Can you find a number that appears 6 times?

Can you find a number that appears more than 6 times?

\*Can you prove this?

### Solution

The last row that can contain  $n$  is the  $n^{\text{th}}$  row (counting the initial 1 as the  $0^{\text{th}}$  row).  $n$  will have appeared a finite amount of times up to this point therefore all numbers other than 1 will appear a finite amount of times.

The first number to appear 3 times is 6, the first number to appear 4 times is 10.

Numbers that appear more than 4 times are much rarer. The smallest is 120 which appears 6 times. The smallest number that appears 8 times is 3003.

No examples have been found of numbers that appear exactly 5 times; however, it is not known whether any exist.

Singmaster's conjecture ([http://en.wikipedia.org/wiki/Singmaster%27s\\_conjecture](http://en.wikipedia.org/wiki/Singmaster%27s_conjecture)) states that there is a finite upper bound on the number of times a number can appear. It is thought that this upper bound is 10 or 12, though no such examples have been found therefore it could be as low as 8.