

Start by thinking about a situation where there are five rather than eleven scientists.

Any pair of scientists cannot open the safe but any group of three can.

There are ${}^5C_2 = 10$ pairs of scientists, and for each pair of scientists there is a lock that they cannot open. If two pairs were unable to open the same lock then combining them would form a group of at least 3 scientists who cannot open the safe and that is against the rules. Therefore there must be at least 10 locks.

Call the scientists A, B, C, D and E. Associate each pair with a lock as shown in the table below and give all scientists not associated with a particular lock the key to it. It follows that each scientist receives ${}^4C_2 = 6$ keys, the number of pairs made up from the other 4.

PAIRS	LOCKS	KEYS GIVEN				
		A	B	C	D	E
AB	1			✓	✓	✓
AC	2		✓		✓	✓
AD	3		✓	✓		✓
AE	4		✓	✓	✓	
BC	5	✓			✓	✓
BD	6	✓		✓		✓
BE	7	✓		✓	✓	
CD	8	✓	✓			✓
CE	9	✓	✓		✓	
DE	10	✓	✓	✓		

For any pair of scientists there is one lock they cannot open.

For any lock there are only 2 scientists without the key and so any group of 3 scientists must be able to open each and every lock.

Similarly, with 11 scientists, any group of 5 scientists cannot open the safe but if any one of the remaining 6 joins them then together they can. Using reasoning like that above, ${}^{11}C_5 = 462$ locks are needed and each scientist receives ${}^{10}C_5 = 252$ keys.