

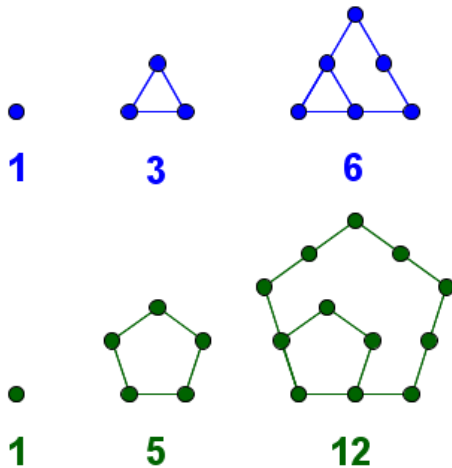
# MEI Maths Item of the Month

January 2016

## Happy 2016

2016 is a triangular number.

The first three triangular numbers are: 1, 3, 6. The first three pentagonal numbers are: 1, 5, 12.



The pentagonal numbers 1, 5 and 12 are all one third of a triangular number.

Are all pentagonal numbers one third of a triangular number?

### Algebra Solution

The formula for the  $n^{\text{th}}$  pentagonal number:

$$p_n = \frac{3n^2 - n}{2}.$$

$$\begin{aligned} 3\left(\frac{3n^2 - n}{2}\right) &= \frac{9n^2 - 3n}{2} \\ &= \frac{9n^2 - 6n + 1 - 3n + 6n - 1}{2} \\ &= \frac{(3n-1)^2 + (3n-1)}{2} \end{aligned}$$

i.e. the  $3n-1^{\text{th}}$  triangular number.

### Geometry Solution

