# MEI Maths Item of the Month

## May 2016

## MEI Conference - Sessions about famous mathematicians

The 2015 MEI conference featured a strand of sessions about 12 famous mathematicians. The following problem is from the session about John Conway.

There are only three numbers (>1) that can be written as the sum of fourth powers of their digits:

$1634 = 1^4 + 6^4 + 3^4 + 4^4$
$8208 = 8^4 + 2^4 + 0^4 + 8^4$
$9474 = 9^4 + 4^4 + 7^4 + 4^4$

Find the smallest number (>1) that can be written as the sum of fifth powers of its digits.

## Solution

It helps to know the fifth powers of each possible digit so here they are:

| $n$ | $n^5$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 32 |
| 3 | 243 |
| 4 | 1024 |

| $n$ | $n^5$ |
|---|---|
| 5 | 3125 |
| 6 | 7776 |
| 7 | 16807 |
| 8 | 32768 |
| 9 | 59049 |

There are a few simple conclusions that can be made from this:

- There are no one digit numbers (as we don't include $0$ or $1$).
- Two digit numbers can only use $2$, $1$ ,$0$ since $3^5$ is a $3$ digit number and therefore there are none of these.
- Three digit numbers would have to contain at least one $3$ since $222$ would only give a total of $96$ and therefore there are none of these.

For four digit numbers $6$, $5$, $4$, $3$, $2$, $1$, $0$ can be used

$1000 \leq n < 2000$

The first digit is $1$ and you can't have $5$s or $6$s since both give totals $> 2000$. One of the digits must be $4$ and none of these combinations work.

$2000 \leq n < 3000$

The first digit is $2$ and you can't have $5$s or $6$s since both give totals $> 3000$. One of the digits must be $4$ and none of these combinations work.

$3000 \leq n < 4000$

The first digit is $3$ and you can't have $6$s since this gives totals $> 4000$. One of the digits must be $4$ or a $5$ and none of these combinations work.

$4000 \leq n < 5000$

The first digit is 4 and you can't have 6s since this gives totals $> 5000$. One of the other digits must be 4 or a 5.

None of the combinations with a second 4 work.

Examining numbers of the form 4 _ _ _, with one 5 gives:

$$4150 = 4^5 + 1^5 + 5^5 + 0^5$$

The next smallest solution is:

$$4151 = 4^5 + 1^5 + 5^5 + 1^5$$

The following short Python script can be used to show that these are the only 4-digit solutions:

```
def conway(n):
    for a in range(0,10):
        for b in range(0,10):
            for c in range(0,10):
                for d in range(0,10):
                    m=1000*a+100*b+10*c+d
                    if a**n+b**n+c**n+d**n==m and m>1:
                        print(m)
```