

## Maths Item of the Month July 2011

### Some sums

What numbers can be made from the sum of some (i.e. at least two) consecutive positive whole numbers?

e.g.

$$1 + 2 = 3$$

$$5 + 6 + 7 = 18$$

### Solution

Any whole number other than the powers of 2 can be made.

If a power of 2, say  $2^r$ , could be expressed as the sum of  $n$  consecutive positive integers (from  $a$  to  $a+(n-1)$ ) then this implies that  $2^r = \frac{n(2a+n-1)}{2}$ . But  $n(2a+n-1)$  has an odd factor since either  $n$  is odd or, if not,  $2a+n-1$  is odd. Therefore  $n(2a+n-1)$  cannot be equal to  $2^{r+1}$ .

To prove that all other numbers can be done, start by finding an odd factor greater than 1:

$$40 = 5 \times 8$$

$$= 8 + 8 + 8 + 8 + 8$$

$$= 6 + 7 + 8 + 9 + 10$$

$$14 = 7 \times 2 = 2 + 2 + 2 + 2 + 2 + 2 + 2$$

$$= -1 + 0 + 1 + 2 + 3 + 4 + 5$$

$$= 2 + 3 + 4 + 5$$