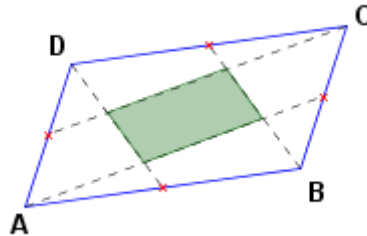


MEI Maths Item of the Month

September 2016

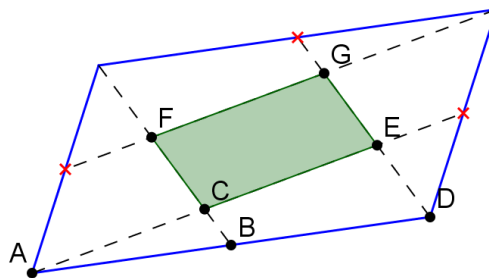
An unexpected fraction?



Start with any parallelogram. Mark the midpoint of each side. Join these midpoints to the vertex two places clockwise around the parallelogram. What fraction of the original parallelogram is the new quadrilateral?

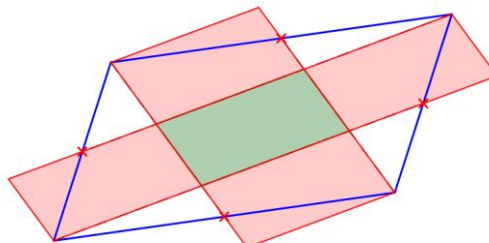
Solution

The resulting quadrilateral is $\frac{1}{5}$ of the area of the original parallelogram.



Triangle ADE is similar to triangle ABC and AD is twice AB. Therefore AC is the same length as CE. As DE is the same length as EG, by a similar argument, and CF is the same length as EG, then BC is half the length of CF.

Rotating the triangle ABC around the point B by a half-turn, and doing similar for the other three small triangles gives:



This is a cross with the same area as the original parallelogram made up of five copies of the smaller quadrilateral.