OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS (MEI) 4772
Decision Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other Materials Required:
- Scientific or graphical calculator

Tuesday 22 June 2010
Afternoon

Duration: 1 hour 30 minutes

4772

OCE/23399

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

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1 **(a)** Mickey ate the last of the cheese. Minnie was put out at this. Mickey's defence was "There wasn't enough left not to eat it all".

Let "c" represent "there is enough cheese for two" and "e" represent "one person can eat all of the cheese".

(i) Which of the following best captures Mickey's argument?

$c \Rightarrow e$ $\quad$ $c \Rightarrow \sim e$ $\quad$ $\sim c \Rightarrow e$ $\quad$ $\sim c \Rightarrow \sim e$ **[1]**

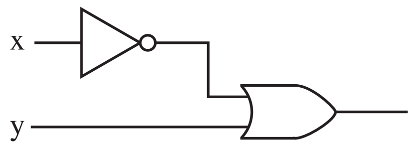
In the ensuing argument Minnie concedes that if there's a lot left then one should not eat it all, but argues that this is not an excuse for Mickey having eaten it all when there was not a lot left.

(ii) Prove that Minnie is right by writing down a line of a truth table which shows that

$(c \Rightarrow \sim e) \Leftrightarrow (\sim c \Rightarrow e)$

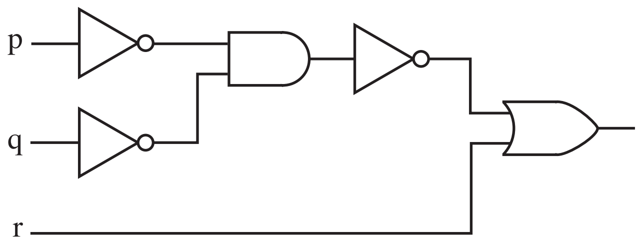
is false.
(You may produce the whole table if you wish, but you need to indicate a specific line of the table.) **[4]**

(b) **(i)** Show that the following combinatorial circuit is modelling an implication statement. Say what that statement is, and prove that the circuit and the statement are equivalent.



[5]

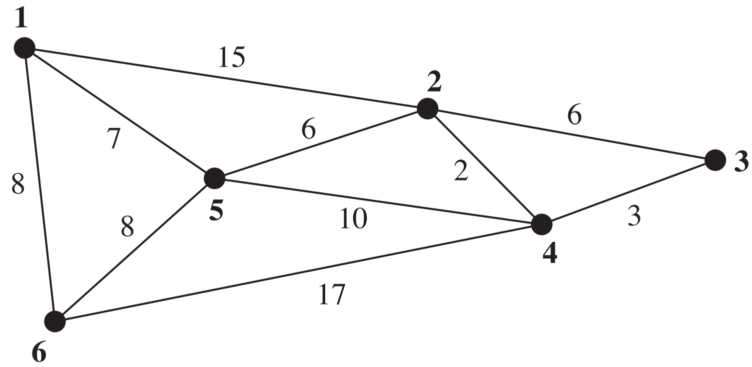
(ii) Express the following combinatorial circuit as an equivalent implication statement.



[2]

(iii) Explain why $(\sim p \wedge \sim q) \Rightarrow r$, together with $\sim r$ and $\sim q$, give p. **[4]**

2 The network is a representation of a show garden. The weights on the arcs give the **times** in minutes to walk between the six features represented by the vertices, where paths exist.



(i) Why might it be that the time taken to walk from vertex **2** to vertex **3** via vertex **4** is less than the time taken by the direct route, i.e. the route from **2** to **3** which does not pass through any other vertices? **[1]**

The matrices shown below are the results of the first iteration of Floyd's algorithm when applied to the network.

	1	**2**	**3**	**4**	**5**	**6**
1	∞	15	∞	∞	7	8
2	15	30	6	2	6	23
3	∞	6	∞	3	∞	∞
4	∞	2	3	∞	10	17
5	7	6	∞	10	14	8
6	8	23	∞	17	8	16

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	1	1	3	4	5	1
3	1	2	3	4	5	6
4	1	2	3	4	5	6
5	1	2	3	4	1	6
6	1	1	3	4	5	1

(ii) Complete the second iteration of Floyd's algorithm. **[4]**

The matrices below are the final matrices resulting from Floyd's algorithm.

	1	**2**	**3**	**4**	**5**	**6**
1	14	13	18	15	7	8
2	13	4	5	2	6	14
3	18	5	6	3	11	19
4	15	2	3	4	8	16
5	7	6	11	8	12	8
6	8	14	19	16	8	16

	1	**2**	**3**	**4**	**5**	**6**
1	5	5	5	5	5	6
2	5	4	4	4	5	5
3	4	4	4	4	4	4
4	2	2	3	2	2	2
5	1	2	2	2	2	6
6	1	5	5	5	5	1

(iii) Explain what the algorithm has achieved.

Show how to find the shortest time and the quickest route from vertex **3** to vertex **6**.

Give the shortest time and the quickest route from vertex **3** to vertex **6**. **[5]**

A visitor to the garden wishes to visit all six features, starting from the feature represented by vertex **1**.

(iv) Use the final matrices to find an upper bound for the minimum time for which the visitor must walk, and give a route through the garden corresponding to this. **[2]**

(v) By deleting vertex **1** and its arcs construct a lower bound for the time for which the visitor must walk. You may construct a minimum connector for the reduced network without using an algorithm. **[3]**

(vi) Given that the sum of the times taken to walk the paths is 82 minutes, find the minimum time that could be taken by a member of staff to start at vertex **1**, walk along every path, and return to vertex **1**. **[1]**

3 It is Ken's 59th birthday, and he is considering whether or not to retire early. He can retire and take his pension now, when he reaches 60, or when he reaches 65.

Ken's pension is computed by taking the number of years for which he has worked, multiplying by his final salary, and dividing by 80. He has currently worked for 35 years. He is at the top of his grade and is earning £50 000 per annum. (Ignore any changes which might occur due to inflation or pay rises.)

Ken estimates that, at age 59, he has a 0.6 probability of getting a part-time (50%) contract on his current grade; at age 60 the probability will be 0.5; at age 65 the probability will be 0.25. (His part-time earnings will not affect his pension.)

Ken intends to retire completely when he reaches 70.

(i) Draw up a decision tree showing Ken's options. **[4]**

(ii) Find the EMV of Ken's gross income (before tax and other stoppages) from age 59 to age 70 in each scenario, and indicate the course of action which will maximise his EMV. **[9]**

Ken values his time and decides to apply a utility function to his gross incomes to reflect this. In each 11-year scenario he computes his utility as (gross income × 3^{-p}) where p is the proportion of working time for which he is working. Thus, in the scenario in which he retires at 65 and succeeds in securing a part-time contract thereafter, $p = \frac{6 + 2.5}{11} = \frac{17}{22}$.

(iii) Find Ken's expected utilities and indicate the course of action which will maximise his expected utility. **[7]**

4 A craft workshop produces three products, xylophones, yodellers and zithers. The times taken to make them and the total time available per week are shown in the table. Also shown are the costs and the total weekly capital available.

	xylophones	yodellers	zithers	resource availability
time (hours)	2	5	3	30
cost (£00s)	4	1	2	24

Profits are £180 per xylophone, £90 per yodeller and £110 per zither.

(i) Formulate a linear programming problem to find the weekly production plan which maximises profit within the resource constraints. **[3]**

(ii) Use the simplex algorithm to solve the problem, pivoting first on the column of your tableau containing the variable which represents the number of xylophones produced. Explain how your final tableau shows that the workshop should produce 5 xylophones and 4 yodellers. **[8]**

If, when applying the simplex algorithm, the first pivot is on the column containing the variable which represents the number of zithers produced, then the final solution produced is for the workshop to produce 1.5 xylophones and 9 zithers per week.

(iii) How can this production plan be implemented? **[1]**

(iv) Explain how the simplex algorithm can lead to different solutions. **[2]**

To satisfy demand an extra constraint has to be added to the problem – that the number of xylophones plus the number of yodellers produced per week must total exactly 7.

(v) Show how to adapt this problem for solution either by the two-stage simplex method or the big-M method. In either case you should show the initial tableau and describe what has to be done next. You are not required to apply your method. **[4]**

(vi) The simplex solution to the revised problem is for the workshop to produce $4\frac{1}{15}$ xylophones, $2\frac{14}{15}$ yodellers and $2\frac{2}{5}$ zithers. Ignoring the practicalities explain how this solution relates to the two solutions referred to in part **(iv)**. **[2]**

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RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **4772**

Decision Mathematics 2

Mark Scheme for June 2010

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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1.

(a)(i) $\sim c \Rightarrow e$	B1	
(ii) $(c \Rightarrow \sim e) \Leftrightarrow (\sim c \Rightarrow e)$	M1	line of a TT
1 0 01 0 01 1 1	A1	both propositions 1 or both 0
or 0 1 10 0 10 0 0	M1	an "$\Rightarrow$" correct
	A1	all OK
(b)(i) Circuit is $\sim x \vee y$. This is $x \Rightarrow y$.	B1	
10 1 0 0 1 0 10 1 1 0 1 1 01 0 0 1 0 0 01 1 1 1 1 1	B4	
(ii) $(\sim p \wedge \sim q) \Rightarrow r$	M1 A1	implication noted
(iii) $(\sim p \wedge \sim q) \Rightarrow r$ is equivalent to $\sim r \Rightarrow \sim(\sim p \wedge \sim q)$	B1	
But we have $\sim r$, so we have $\sim(\sim p \wedge \sim q)$.	B1	
$\sim(\sim p \wedge \sim q)$ is equivalent to $p \vee q$	B1	
But we have $\sim q$, so therefore p.	B1	

2.

(i) Distances longer — B1

(ii)

	1	2	3	4	5	6			1	2	3	4	5	6
1	∞	15	∞	∞	7	8		**1**	1	2	3	4	5	6
2	15	∞	6	2	6	∞		**2**	1	2	3	4	5	6
3	∞	6	∞	3	∞	∞		**3**	1	2	3	4	5	6
4	∞	2	3	∞	10	17		**4**	1	2	3	4	5	6
5	7	6	∞	10	∞	8		**5**	1	2	3	4	5	6
6	8	∞	∞	17	8	∞		**6**	1	2	3	4	5	6

not part of answer

	1	2	3	4	5	6			1	2	3	4	5	6
1	∞	15	∞	∞	7	8		**1**	1	2	3	4	5	6
2	15	30	6	2	6	23		**2**	1	1	3	4	5	1
3	∞	6	∞	3	∞	∞		**3**	1	2	3	4	5	6
4	∞	2	3	∞	10	17		**4**	1	2	3	4	5	6
5	7	6	∞	10	14	8		**5**	1	2	3	4	1	6
6	8	23	∞	17	8	16		**6**	1	1	3	4	5	1

not part of answer

	1	2	3	4	5	6			1	2	3	4	5	6
1	30	15	21	17	7	8		**1**	2	2	2	2	5	6
2	15	30	6	2	6	23		**2**	1	1	3	4	5	1
3	21	6	12	3	12	29		**3**	2	2	2	4	2	2
4	17	2	3	4	8	17		**4**	2	2	3	2	2	6
5	7	6	12	8	12	8		**5**	1	2	2	2	2	6
6	8	23	29	17	8	16		**6**	1	1	1	4	5	1

M1 30 in top left
A1 times
A1 6 to 3 route = 1
A1 rest of route

	1	2	3	4	5	6			1	2	3	4	5	6
1	30	15	21	17	7	8		**1**	2	2	2	2	5	6
2	15	12	6	2	6	23		**2**	1	3	3	4	5	1
3	21	6	12	3	12	29		**3**	2	2	2	4	2	2
4	17	2	3	4	8	17		**4**	2	2	3	2	2	6
5	7	6	12	8	12	8		**5**	1	2	2	2	2	6
6	8	23	29	17	8	16		**6**	1	1	1	4	5	1

not part of answer

	1	2	3	4	5	6			1	2	3	4	5	6
1	30	15	20	17	7	8		**1**	2	2	2	2	5	6
2	15	4	5	2	6	19		**2**	1	4	4	4	5	4
3	20	5	6	3	11	20		**3**	4	4	4	4	4	4
4	17	2	3	4	8	17		**4**	2	2	3	2	2	6
5	7	6	11	8	12	8		**5**	1	2	2	2	2	6
6	8	19	20	17	8	16		**6**	1	4	4	4	5	1

not part of answer

	1	2	3	4	5	6			1	2	3	4	5	6
1	14	13	18	15	7	8		**1**	5	5	5	5	5	6
2	13	4	5	2	6	14		**2**	5	4	4	4	5	5
3	18	5	6	3	11	19		**3**	4	4	4	4	4	4
4	15	2	3	4	8	16		**4**	2	2	3	2	2	2
5	7	6	11	8	12	8		**5**	1	2	2	2	2	6
6	8	14	19	16	8	16		**6**	1	5	5	5	5	1

not part of answer

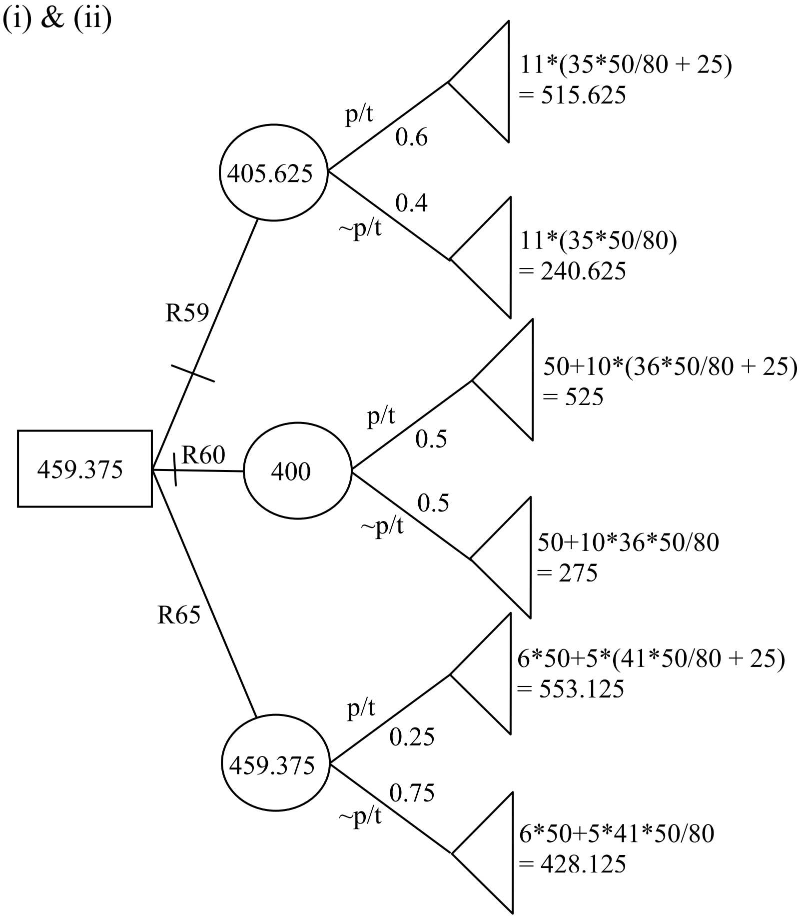
	1	2	3	4	5	6			1	2	3	4	5	6
1	14	13	18	15	7	8		**1**	5	5	5	5	5	6
2	13	4	5	2	6	14		**2**	5	4	4	4	5	5
3	18	5	6	3	11	19		**3**	4	4	4	4	4	4
4	15	2	3	4	8	16		**4**	2	2	3	2	2	2
5	7	6	11	8	12	8		**5**	1	2	2	2	2	6
6	8	14	19	16	8	16		**6**	1	5	5	5	5	1

not part of answer

(iii) cont It has found all shortest times and corresponding routes. Shortest time from x to y is in x row and y column of time matrix. For route look in x row and y column of route matrix. This gives first vertex "en route". Repeat, looking in row corresponding to the current "en route" vertex and the y column, until the "en route" vertex is y. Shortest time from 3 to 6 is 19. Corresponding route is 3 to 4 to 2 to 5 to 6.	 B1 B1 B1 B1 B1
(iv) On time matrix – 1(7)5(6)2(2)4(3)3(19)6(8)1 so 45 From route matrix – 1 5 2 4 3 4 2 5 6 1	B1 B1
(v) Lower bound = 7 + 8 + 19 = 34	M1 A1 7 + 8 A1 19
(vi) 82 + 8 = 90 minutes	B1

3.

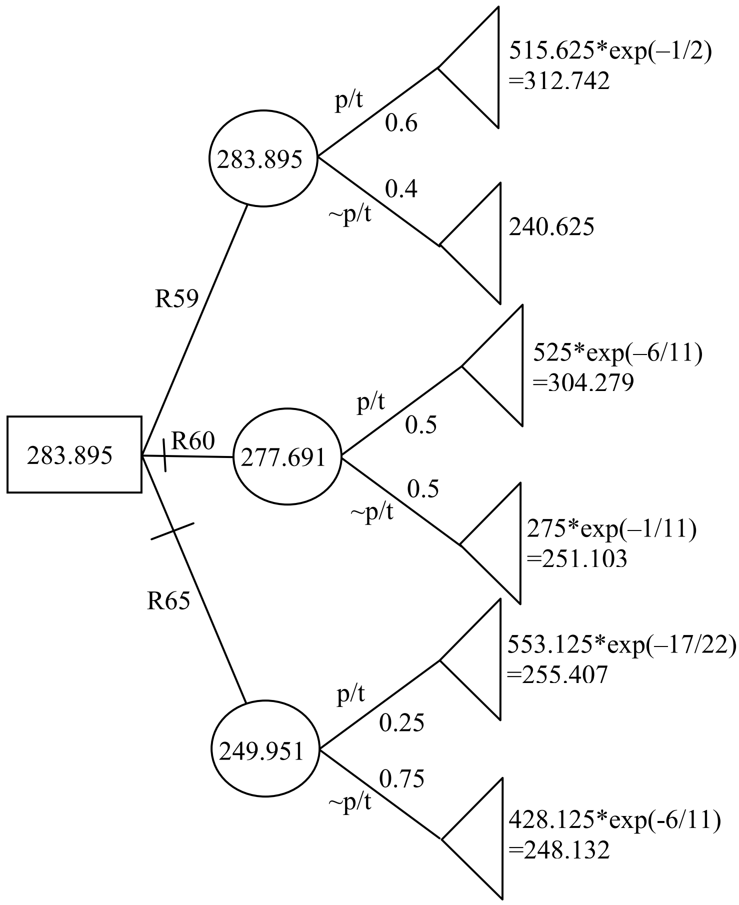
(i) & (ii)



Retire at 65, EMV = 459375

M1	3-way split
A1	choice node
M1	2-way splits
A1	chance nodes
B1	pension calculations
M1	(income + pension) × time
A1	retire at 59
A1	retire at 60
A1	retire at 65
M1	EMV's
A1√	
M1	choice
A1√	

(iii) p's (in order) $\frac{1}{2}$ 0 $\frac{6}{11}$ $\frac{1}{11}$ $\frac{17}{22}$(given) $\frac{6}{11}$ — M1 A1



M1 final utilites
A1 cao

M1 expecteds
A1 √

Retire at 59. — B1 choice

4.

(i) Max $180x + 90y + 110z$

st $2x + 5y + 3z \le 30$

$4x + \;\; y + 2z \le 24$

B1
B1
B1

(ii)

P	x	y	z	s1	s2	RHS
1	–180	–90	–110	0	0	0
0	2	5	3	1	0	30
0	4	1	2	0	1	24
1	0	–45	–20	0	45	1080
0	0	4.5	2	1	–0.5	18
0	1	0.25	0.5	0	0.25	6
1	0	0	0	10	40	1260
0	0	1	4/9	2/9	–1/9	4
0	1	0	7/18	–1/18	5/18	5

M1 initial tableau
A1

M1 first iteration
A1

M1 second iteration
A1

Identification of basic variables + values — B1 B1

(iii) Over two weeks ($x = 3$ and $z = 18$) — B1

(iv) Degeneracy (technical term not required) – objective planes are parallel to boundary line.

B1 same obj value
B1 line of solutions

(v)

A	P	x	y	z	s1	s2	s3	s4	a	RHS
1	0	1	1	0	0	0	0	–1	0	7
0	1	–180	–90	–110	0	0	0	0	0	0
0	0	2	5	3	1	0	0	0	0	30
0	0	4	1	2	0	1	0	0	0	24
0	0	1	1	0	0	0	1	0	0	7
0	0	1	1	0	0	0	0	–1	1	7

B1 $= \;\rightarrow\; \le + \ge$
B1 $\le$ row
B1 $\ge$ row
B1 new objective
B1 minimise A

or

P	x	y	z	s1	s2	s3	s4	a	RHS
1	–M–180	–M–90	–110	0	0		M	0	–7M
0	2	5	3	1	0	0	0	0	30
0	4	1	2	0	1	0	0	0	24
0	1	1	0	0	0	1	0	0	7
0	1	1	0	0	0	0	–1	1	7

or

B1 $= \;\rightarrow\; \le + \ge$
B1 $\le$ row
B1 $\ge$ row
B1 new objective
B1 maximise P

(vi) Point is on the line – gives £1260 profit — B1 (either)

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4772 Decision Mathematics 2

General comments

Candidates found last year's paper relatively easy. This paper reverted to the norm. The mean mark was 42 compared to 49 in 2009 and 43 in 2008.

Comments on individual questions

1) *(Logic)*

(a) Most candidates scored 5 or 4 marks out of the 5 available in part (a). Most chose to give a full truth table for (a)(ii) rather than just one appropriate line. Not all of those candidates indicated which line/lines was/were relevant.

(b) The majority of candidates scored very badly in part (b). The modal mark was 0 out of 11. As last year, combinatorial circuits proved to be a stumbling block. Many candidates were unable to make a start.
There were many good candidates who did score well and some of the proofs seen in (b)(iii) were very good indeed.

2) *(Networks)*

(i) A very large number of flights of fancy were seen on this question, some very fanciful indeed. This comment mirrors one made on the 4771 report. Future candidates would do well to appreciate that this sort of question aims to elicit a comment on the modelling; they are not intended to test the candidates' powers of imagination. Yes, there might well have been some awful creature sitting somewhere on the direct track from vertex **2** to vertex **3**, causing delay. But what was looked for was an appreciation of the fact that times are not bound to follow the triangle inequality, any more than distances are. So prosaic answers, eg "The direct route might wend its way around flower beds" were much more satisfying.

(ii)
(iii) These parts were well done. In part (iii) some candidates used the matrices from part (i) instead of the final matrices.

(iv) Few candidates converted the route they obtained from the shortest time matrix into a route through the garden, even though nearly all had explained how to do this in part (iii).

(v) Only about 50% of candidates succeeded in this straightforward application of syllabus material.

(vi) This was also disappointing for what was a very straightforward question.

3) *(Decision Analysis)*
There have been series in which, for many candidates, this question has proved to be the bedrock of their scripts. That was seldom the case for candidates in this series. For many candidates the stumbling block was their inability to distinguish between decision nodes and chance nodes.

(i) The structure of this problem was perceived to be easy; it was defined carefully in paragraphs 1 and 3 of the question. But many candidates produced complicated decision trees which failed to model the given information.

(ii) It was recognised that the computations were intricate, and that many candidates would not get them fully correct under examination conditions. The mark scheme was carefully constructed to allow for this. The most common error was in failing to have the pension paid each year.

(iii) Again, the computations were numerically intricate, but many marks were available for candidates who made slips. However, some deserted good answers from part (ii), failing to replicate EMV calculations to produce expected utilities.
To re-emphasise the point made in paragraph 2 above, many candidates gave courses of action which indicated when Ken should retire AND whether or not he should obtain part-time work!

4) *(Linear Programming - Simplex)*

(i) Most candidates succeeded with the formulation.

(ii) The basic simplex algorithm was well understood and executed by most candidates. The “explain” part of the question was meant to provide a check as well as to test understanding. Most had obtained the correct answer, but few gave a decent *explanation of how* to interpret the final tableau, which is rather a different task from interpreting it.

(iii) A majority of candidates saw through this, and gave a fortnightly production plan.

(iv) Very few candidates noted that alternative optimal solutions could exist.

(v) Most had a good idea of 2-stage or big-M, though not all could execute the procedures accurately.

(vi) The essence of this question is to note that the given solution also gives a weekly profit of £1260, and that the solution lies on the line joining the earlier alternative optimal solutions. Very few candidates noted this.