OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4776/01

Numerical Methods

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Friday 14 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 **(i)** Show that the equation $1 + x = \tan x$, where x is in radians, has a root in the interval [1, 1.2]. **[2]**

(ii) Show numerically that the iteration $x_{r+1} = \tan x_r - 1$ with $x_0 = 1.1$ diverges. **[2]**

(iii) Use another iteration to find the root correct to 3 decimal places. **[4]**

2 The table shows some estimates of an integral, $\int_2^4 \mathrm{f}(x)\,\mathrm{d}x$, using the mid-point rule (M) and the trapezium rule (T), for given values of h.

h	M	T
2	1.987 467	1.354 440
1	1.830 595	
0.5		

Copy the table and fill in the additional estimates that can be found.

Obtain the Simpson's rule estimates that can be found.

Give the value of the integral to the accuracy that appears justified. **[8]**

3 The table shows values of $\mathrm{g}(x)$ correct to 4 decimal places.

x	0	0.5	1
$\mathrm{g}(x)$	1.4509	1.6799	2.0100

(i) Use the forward difference method to find two estimates of $\mathrm{g}'(0)$. State, with a reason, which of these is likely to be more accurate. **[4]**

(ii) Use the central difference method to find an estimate of $\mathrm{g}'(0.5)$. Comment on the likely accuracy of this estimate compared to those in part **(i)**. **[2]**

4 A bank's computer system calculates the interest payable on each savings account every day. A running total is kept of the daily amounts of interest, and accounts are credited with this interest at the end of each year. The bank used to *round* the daily amounts of interest payable to the nearest 0.01 of a penny, but they decide to *chop* to the nearest 0.01 of a penny instead.

(i) Find the maximum possible loss in a year to a savings account because of the chopping, and explain how this loss could occur. State, with a reason, what the average loss will be. **[4]**

(ii) The bank calculates that chopping in this way will generate an additional profit of about £150 000 per year. Estimate the number of savings accounts the bank has. **[2]**

5 The function P(x) is known to be a polynomial. Some values of P(x) are given in the table.

x	1	3	5	7	9
P(x)	–10	3	44	129	274

(i) Use a difference table to determine, with a reason, the least possible degree of polynomial that will fit all the data points. **[4]**

(ii) Assuming that P(x) is of this degree, extend your table to find the values of P(–1) and P(11). **[4]**

Section B (36 marks)

6 In this question,

$$f(x) = \frac{x}{\sin x} - \frac{\sin x}{x},$$

where x is in radians. For small non-zero values of x, f(x) may be approximated by g(x) or by h(x), where

$$g(x) = \frac{1}{3}x^2 \qquad \text{and} \qquad h(x) = \frac{2x^2}{6 - x^2}.$$

(i) Find the absolute and relative errors in g(x) and h(x) as approximations to f(x) for

(A) $x = 0.2$,

(B) $x = 0.1$ **[9]**

(ii) A third approximation to f(x) is given by $\frac{4g(x) + h(x)}{5}$. Explain by reference to part **(i)** why this would be expected to be a good approximation.
Find the absolute and relative errors when this third approximation is used to estimate f(0.2) and f(0.1). **[6]**

(iii) Use your calculator to evaluate $\frac{x}{\sin x}$ when $x = 10^{-4}$.

When $x = 10^{-4}$, a cheap calculator evaluates f(x) as zero. Use an approximate formula to find a better value for f(10^{-4}). Explain why the cheap calculator makes an error. **[3]**

7 **(i)** Show that the equation $f(x) = 0$, where

$$f(x) = x^7 + x^5 - 1, \qquad \textbf{(*)}$$

has a root in the interval [0, 1].
By considering $f'(x)$ show that there are no other roots.
Sketch the graph of $y = f(x)$ for $x \geqslant 0$. **[7]**

(ii) Obtain the Newton-Raphson iteration based on (*). Starting with $x_0 = 0.6$, find x_1 and x_2. Illustrate this iteration on your sketch of $y = f(x)$. **[7]**

(iii) Use the Newton-Raphson iteration to find x_1 and x_2 in the cases

(*A*) $x_0 = 0.3$,

(*B*) $x_0 = 0.9$.

Comment on your results in each case. **[4]**

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4776:** Numerical Methods

Mark Scheme for January 2011

Any enquiries about publications should be addressed to:

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Question			Answer	Marks	Guidance
1	(i)		x LHS RHS 1 2 > 1.557408 1.2 2.2 < 2.572152	M1 A1 **[2]**	no explicit explanation required
	(ii)		r 0 1 2 3 4 x_r 1.1 0.96476 0.442927 –0.52564 –1.58007	M1 A1 **[2]**	r = 3 required
	(iii)		e.g. re-arrange to $x = \arctan(1 + x)$ r 0 1 2 3 4 5 x_r 1.1 1.126377 1.131203 1.132076 1.132233 1.132261 1.132	B1 M1 A1 A1 **[4]**	
2			h / M / T: 2 / *1.987467* / *1.354440* 1 / *1.830595* / **1.670954** 0.5 / / **1.750774** Simpson's rule $(2M + T) / 3$ **1.776458** **1.777381** Reference to justification/accuracy : 1.777 or 1.78	*T*: M1A1A1 *S*: M1A1A1 E1 A1 **[8]**	Lose 1 for any additional 'answer'(s) but do not penalise extrapolation
3	(i)		$h = 1$ $g'(0) = (2.0100 – 1.4509)/1 = 0.5591$ $h = 0.5$ $g'(0) = (1.6799 – 1.4509)/0.5 = 0.458$ Estimate with smaller h (0.458) likely to be more accurate: smaller h is more accurate (provided there is no great loss of significant figures)	B1 B1 B1 E1 **[4]**	
	(ii)		$h = 0.5$ $g'(0.5) = (2.0100 – 1.4509)/1 = 0.5591$ This estimate, central diff, likely to be more accurate than either of the forward diffs	M1 E1 **[2]**	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>4</td><td>(i)</td><td></td><td>Max poss loss: 365 (or 366) times 0.01 pence: = 3.65 (or 3.66) pence
Arises if each daily amount would round up but gets chopped down
Average loss 1.825 (or 1.83) pence, because average is half of max.</td><td>B1
E1
B1 E1
[4]</td><td></td></tr>
<tr><td></td><td>(ii)</td><td></td><td>£150 000 divided by 1.825 pence: about 8.2 million (8 million) accounts</td><td>M1 A1
[2]</td><td></td></tr>
<tr><td>5</td><td></td><td></td><td>
<table>
<tr><th>x</th><th>P(x)</th><th>ΔP(x)</th><th>Δ^2P(x)</th><th>Δ^3P(x)</th><th></th></tr>
<tr><td>–1</td><td>–11</td><td></td><td></td><td></td><td>(i) bold:</td></tr>
<tr><td>1</td><td>–10</td><td>1</td><td></td><td></td><td>Diff table</td></tr>
<tr><td>3</td><td>3</td><td>13</td><td>12</td><td></td><td>3rd diffs constant</td></tr>
<tr><td>5</td><td>44</td><td>41</td><td>28</td><td>16</td><td>so cubic</td></tr>
<tr><td>7</td><td>129</td><td>85</td><td>44</td><td>16</td><td>(ii) italic:</td></tr>
<tr><td>9</td><td>274</td><td>145</td><td>60</td><td>16</td><td>working forwards</td></tr>
<tr><td>11</td><td>495</td><td>221</td><td>76</td><td>16</td><td>working backwards</td></tr>
</table>
</td><td>

M1 A1
E1
B1

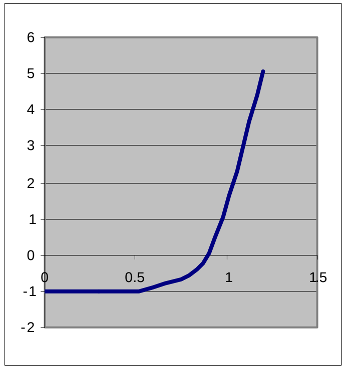
M1 A1
M1 A1
[4] + [4]</td><td></td></tr>
<tr><td>6</td><td>(i)</td><td></td><td>
<table>
<tr><th>x</th><th>f</th><th>g</th><th>h</th><th>abs err g</th><th>rel err g</th><th>abs err h</th><th>rel err h</th></tr>
<tr><td>0.2</td><td>0.013351</td><td>0.013333</td><td>0.013423</td><td>–0.0000179</td><td>–0.0013424</td><td>0.0000716</td><td>0.0053600</td></tr>
<tr><td>0.1</td><td>0.003334</td><td>0.003333</td><td>0.003339</td><td>–0.0000011</td><td>–0.0003339</td><td>0.0000045</td><td>0.0013350</td></tr>
<tr><td></td><td>A1</td><td>A1</td><td>A1</td><td>A1</td><td>A1</td><td>A1</td><td>A1</td></tr>
</table>
</td><td>abs M1

rel M1

[9]</td><td>f, g, h values may be implied</td></tr>
<tr><td></td><td>(ii)</td><td></td><td>Errors in g and h are of opposite sign; g is about 4 times as accurate as h.
<table>
<tr><th>x</th><th>f</th><th>(4g + h)/5</th><th>abs err</th><th>rel err</th></tr>
<tr><td>0.2</td><td>0.013351</td><td>0.013351</td><td>–2.5E–08</td><td>–1.9E–06</td></tr>
<tr><td>0.1</td><td>0.003334</td><td>0.003334</td><td>–4E–10</td><td>–1.2E–07</td></tr>
<tr><td></td><td></td><td>A1</td><td>A1</td><td>A1</td></tr>
</table>
</td><td>E1 E1

M1

[6]</td><td></td></tr>
<tr><td></td><td>(iii)</td><td></td><td>$x / \sin x \approx 1.000\ 000\ 002 \approx 1$
$g(10^{-4}) = 3.33 \times 10^{-9}$
Subtraction of nearly equal quantities</td><td>B1
B1
E1
[3]</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance
7	(i)		f(0) = –1 f(1) = 1 (hence root) $f'(x) = 7x^6 + 5x^4$ which is zero only at $x = 0$. Convincing argument that this is not a turning point No turning points implies no other roots. 	B1 M1 A1 B1 E1 G2 **[7]**	
	(ii)		NR iteration: $x_{r+1} = x_r - (x_r^7 + x_r^5 - 1) / (7x_r^6 + 5x_r^4)$ r 0 1 2 x_r 0.6 1.51756 1.289164 On graph: tangent at 0.6, intersection at 1.5, ordinate & tangent, intersection at 1.3	B1 A1 A1 G4 **[7]**	
	(iii)		r 0 1 2 x_r 0.3 22.1703 19.00128 Comment: e.g. converging but initially very slow (or difficult to tell with only 2 iter'ns) r 0 1 2 x_r 0.9 0.890174 0.889891 Comment: e.g. almost converged, root very close to 0.89	 A1 E1 A1 E1 **[4]**	

4776 Numerical Methods (Written Examination)

General Comments

There was a lot of good work in response to this paper. The standard methods were well understood and there was little evidence of unprepared candidates being entered. As ever, the layout of work was not as good as it could be. Candidates should realise that, in numerical methods in particular, laying work out systematically and compactly aids accuracy. Figures scattered wildly around the page are far less likely to be correct.

Interpretation and real world application (as required in Question 4) remain areas of relative weakness.

Comments on Individual Questions

1) Parts (i) and (ii) were done well by almost everyone. The vast majority moved immediately to the 'inverse' iteration in part (iii) and finished the question successfully. Some used other methods (bisection, secant, Newton-Raphson). These were given credit where successful, though they generally involve more work.

2) Most candidates appreciated what to do here, though some did not realise that they could calculate *two* further trapezium rule estimates. A small minority of candidates obtained a further mid-point rule estimate by extrapolation; this was not required, and its lack of accuracy makes it dubious, but it was not penalised.

3) This was a very straightforward question and almost everyone knew exactly what to do. The comments on the relative merits of the estimates were mostly correct.

4) There were a lot errors made in answering this question. Some were very basic: errors with units (pounds and pence) or with days and years. Others were more fundamental: chopping to the nearest 0.01 was often said to produce a maximum possible error of 0.009. And quite a few candidates were evidently unsure of the difference between chopping and rounding.

5) This was another very straightforward question. The only notable errors were arithmetical – usually arising when subtracting one negative number from another.

6) Part (i) was generally done well, though candidates' layout of their answers made for very difficult reading in many cases.

In part (ii) the justification for the new approximation is twofold: the errors in g(x) and h(x) are in the ratio 1:4, *and* they are of opposite sign. It was rare for candidates to make both points.

Part (ii) was often poorly answered even though it is based on a commonly examined topic, the subtraction of nearly equal quantities.

7) In part (i) the root was easily located, but the arguments to show that there are no other roots were mostly poor. Very often candidates set the first derivative to zero, identified that $x = 0$ was the only solution, and then said that the function had only one turning point and so it could have only one root. It is difficult to see what 'reasoning' lies behind that. It was refreshing to come across a script in which the candidate observed that the derivative was never negative and then made the correct deduction.

In part (ii) the numerical work was done well, but the sketches were sometimes disappointing. In quite a few cases it appeared that candidates knew what the sketch *should* show but didn't draw a graph large enough to show it. A small minority appeared not to understand that the Newton-Raphson method approximates a curve by its tangent and tried to draw staircase or cobweb diagrams.

Part (iii) was done well in most cases, though some thought that the first iteration was diverging.

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