

**ADVANCED SUBSIDIARY GCE  
MATHEMATICS (MEI)**

Numerical Methods

**4776/01**

**QUESTION PAPER**

Candidates answer on the printed answer book.

**OCR supplied materials:**

- Printed answer book 4776/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Wednesday 18 May 2011  
Morning**

**Duration: 1 hour 30 minutes**



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR**

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

## Section A (36 marks)

1 The equation  $f(x) = 0$ , where  $f(x)$  is a continuous function, is known to have a single root in the interval  $[0.4, 1.8]$ .

- (i) Suppose the root is to be found using the bisection method. State the best possible estimate of the root at the start of the process. State also the maximum possible error associated with that estimate.

Determine how many iterations of the bisection process would be required to reduce the maximum possible error to less than 0.05. [4]

- (ii) Given now that  $f(0.4) = -0.2$  and  $f(1.8) = 0.5$ , find an estimate of the root using the false position method. [3]

2 The function  $g(x)$  has the values shown in the table.

$x$	1.80	2.00	2.20
$g(x)$	2.66	2.85	3.02

- (i) Taking the data to be exact, use the central difference formula to estimate  $g'(2)$ . [2]
- (ii) Suppose instead that the  $x$  values are exact but the values of  $g(x)$  are rounded to 2 decimal places. Find an appropriate range of estimates of  $g'(2)$ . [3]
- (iii) Now suppose that all the values in the table have been rounded to 2 decimal places. Find the appropriate range of estimates of  $g'(2)$  in this case. [3]

3 The function  $Q(x)$  is known to be quadratic and it has the values shown in the table.

$x$	-1	1	5
$Q(x)$	-4	-12	20

- (i) Write down the estimate of  $Q(0)$  obtained by linear interpolation. [1]
- (ii) Use Lagrange's method to write down an expression for  $Q(x)$ . [You are not required to simplify this expression.] [5]
- (iii) Find the exact value of  $Q(0)$ . [2]

- 4 (i) Show that the equation  $x = 1 - x^4$  has a root in the interval  $[0.7, 0.8]$ . [2]
- (ii) Show, by considering the derivative of  $1 - x^4$ , that the iteration  $x_{r+1} = 1 - x_r^4$ , with a starting value in the interval  $[0.7, 0.8]$ , will diverge. [4]
- 5 (i) Find the absolute error and the relative error when  $X = 3.162$  is used as an approximation to  $x = \sqrt{10}$ . [3]
- (ii) Find the relative error if  $X^4$  is used as an approximation to  $x^4$ . [3]
- (iii) State, in terms of  $k$ , the approximate relative error if  $X^k$  is used as an approximation to  $x^k$ . [1]

**Section B (36 marks)**

- 6 The integral  $I = \int_2^{2.8} \sqrt{1+x^3} \, dx$  is to be determined numerically. You should give all your answers to 7 decimal places unless instructed otherwise.
- (i) Find mid-point rule and trapezium rule estimates of  $I$ , taking  $h = 0.8$ .
- Use these two estimates to find a second trapezium rule estimate and a Simpson's rule estimate of  $I$ . [8]
- (ii) Find the mid-point rule estimate with  $h = 0.4$ , and hence obtain a second Simpson's rule estimate of  $I$ . [3]
- (iii) You are now given that the mid-point rule estimate of  $I$  with  $h = 0.2$  is 3.091 429 8, correct to 7 decimal places.
- Find a third Simpson's rule estimate. Show by considering ratios of differences that Simpson's rule is of order  $h^4$ .
- Give the value of  $I$  to the accuracy that appears justified. [7]

**[Question 7 is printed overleaf.]**

7 The function  $f(x)$  has the exact values shown in the table.

$x$	1	3	5
$f(x)$	4	-2	10

- (i) Use Newton's forward difference interpolation formula to find the quadratic function that fits the data. (There is no need to simplify your answer.) [6]
- (ii) Hence estimate the values of  $f(2)$  and  $f(6)$ . State, with a reason, which of these estimates is likely to be more accurate. [3]
- (iii) Now suppose that  $f(7) = 11$ . Find the cubic function that fits all the data. Use this cubic to estimate  $f(2)$  and  $f(6)$ . [7]
- (iv) Comment on (A) the absolute changes and (B) the relative changes in the estimates of  $f(2)$  and  $f(6)$  from part (ii) to part (iii). [2]

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

**ADVANCED SUBSIDIARY GCE  
MATHEMATICS (MEI)**  
Numerical Methods

**4776/01**

**PRINTED ANSWER BOOK**

Candidates answer on the printed answer book.

**OCR supplied materials:**

- Question paper 4776/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Wednesday 18 May 2011  
Morning**

**Duration: 1 hour 30 minutes**



Candidate forename		Candidate surname	
-----------------------	--	----------------------	--

Centre number						Candidate number				
---------------	--	--	--	--	--	------------------	--	--	--	--

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

**PLEASE DO NOT WRITE ON THIS PAGE**

Section A (36 marks)

<b>1(i)</b>	
<b>1(ii)</b>	

<b>2(i)</b>	
<b>2(ii)</b>	
<b>2(iii)</b>	



<b>3(i)</b>	
<b>3(ii)</b>	
<b>3(iii)</b>	

<b>4(i)</b>	
<b>4(ii)</b>	

<b>5(i)</b>	
<b>5(ii)</b>	
<b>5(iii)</b>	

**Section B** (36 marks)

**6(i)**


<b>6(ii)</b>	

<b>6(iii)</b>	

<b>7(i)</b>	
<b>7(ii)</b>	

<b>7(iii)</b>	
<b>7(iv)</b>	

**PLEASE DO NOT WRITE ON THIS PAGE**



**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.



**Mathematics (MEI)**

Advanced Subsidiary GCE

Unit **4776**: Numerical Methods

**Mark Scheme for June 2011**

---

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2011

Any enquiries about publications should be addressed to:

OCR Publications  
PO Box 5050  
Annesley  
NOTTINGHAM  
NG15 0DL

Telephone: 0870 770 6622  
Facsimile: 01223 552610  
E-mail: [publications@ocr.org.uk](mailto:publications@ocr.org.uk)

**Marking instructions for GCE Mathematics (MEI): Pure strand**

1. You are advised to work through the paper yourself first. Ensure you familiarise yourself with the mark scheme before you tackle the practice scripts.
2. You will be required to mark ten practice scripts. This will help you to understand the mark scheme and will not be used to assess the quality of your marking. Mark the scripts yourself first, using the annotations. Turn on the comments box and make sure you understand the comments. You must also look at the definitive marks to check your marking. If you are unsure why the marks for the practice scripts have been awarded in the way they have, please contact your Team Leader.
3. When you are confident with the mark scheme, mark the ten standardisation scripts. Your Team Leader will give you feedback on these scripts and approve you for marking. (If your marking is not of an acceptable standard your Team Leader will give you advice and you will be required to do further work. You will only be approved for marking if your Team Leader is confident that you will be able to mark candidate scripts to an acceptable standard.)
4. Mark strictly to the mark scheme. If in doubt, consult your Team Leader using the messaging system within *scoris*, by email or by telephone. Your Team Leader will be monitoring your marking and giving you feedback throughout the marking period.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

5. The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

6. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
7. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

8. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
9. **Rules for crossed out and/or replaced work**

If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

10. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

11. Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

12. For answers scoring no marks, you must either award NR (no response) or 0, as follows:

Award NR (no response) if:

- Nothing is written at all in the answer space
- There is a comment which does not in any way relate to the question being asked ("can't do", "don't know", etc.)
- There is any sort of mark that is not an attempt at the question (a dash, a question mark, etc.)

The hash key [#] on your keyboard will enter NR.

Award 0 if:

- There is an attempt that earns no credit. This could, for example, include the candidate copying all or some of the question, or any working that does not earn any marks, whether crossed out or not.

13. The following abbreviations may be used in this mark scheme.

M1	method mark (M2, etc, is also used)
A1	accuracy mark
B1	independent mark
E1	mark for explaining
U1	mark for correct units
G1	mark for a correct feature on a graph
M1 dep*	method mark dependent on a previous mark, indicated by *
cao	correct answer only
ft	follow through
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
sc	special case
soi	seen or implied
www	without wrong working

14. Annotating scripts. The following annotations are available:

✓ and ✕	
<b>BOD</b>	Benefit of doubt
<b>FT</b>	Follow through
<b>ISW</b>	Ignore subsequent working (after correct answer obtained)
<b>M0, M1</b>	Method mark awarded 0, 1
<b>A0, A1</b>	Accuracy mark awarded 0, 1
<b>B0, B1</b>	Independent mark awarded 0,1
<b>SC</b>	Special case
<b>^</b>	Omission sign
<b>MR</b>	Misread

Highlighting is also available to highlight any particular points on a script.

15. The comments box will be used by the Principal Examiner to explain his or her marking of the practice scripts for your information. Please refer to these comments when checking your practice scripts.

**Please do not type in the comments box yourself.** Any questions or comments you have for your Team Leader should be communicated by the *scoris* messaging system, e-mail or by telephone.

16. Write a brief report on the performance of the candidates. Your Team Leader will tell you when this is required. The Assistant Examiner's Report Form (AERF) can be found on the Cambridge Assessment Support Portal. This should contain notes on particular strengths displayed, as well as common errors or weaknesses. Constructive criticisms of the question paper/mark scheme are also appreciated.
17. Link Additional Objects with work relating to a question to those questions (a chain link appears by the relevant question number) – see *scoris* assessor Quick Reference Guide page 19-20 for instructions as to how to do this – this guide is on the Cambridge Assessment Support Portal and new users may like to download it with a shortcut on your desktop so you can open it easily! For AOs containing just formulae or rough working not attributed to a question, tick at the top to indicate seen but not linked. When you submit the script, *scoris* asks you to confirm that you have looked at all the additional objects. Please ensure that you have checked all Additional Objects thoroughly.
18. The schedule of dates for the marking of this paper is displayed under 'OCR Subject Specific Details' on the Cambridge Assessment Support Portal. It is vitally important that you meet these requirements. If you experience problems that mean you may not be able to meet the deadline then you must contact your Team Leader without delay.

**1(i)** Best estimate: 1.1    Maximum possible error: 0.7 [B1B1]

Bisecting mpe: 0.7 → 0.35 → 0.175 → 0.0875 → 0.04375 so 4 iterations [M1A1]

**(ii)** False position: estimate is  $(0.4 \times 0.5 - 1.8 \times (-0.2) / (0.5 - (-0.2))) = 0.8$  BoD for 0.8 alone [M1A1]  
[A1]

[TOTAL 7]

**2(i)**  $(3.02 - 2.66) / (2.20 - 1.80) = 0.9$  [M1A1]

**(ii)** max:  $(3.025 - 2.655) / (2.20 - 1.80) = 0.925$  [M1A1]  
min:  $(3.015 - 2.665) / (2.20 - 1.80) = 0.875$  [A1]

**(iii)** max:  $(3.025 - 2.655) / (2.195 - 1.805) = 0.94872$  [M1A1]  
min:  $(3.015 - 2.665) / (2.205 - 1.795) = 0.85366$  [A1]

*[M1] either denominator correct. Max [2] if 3 dp*

[TOTAL 8]

**3(i)** Linear interpolation:  $Q(0) = -8$  write down or any method [B1]

**(ii)** Lagrange:  $Q(x) = (-4)(x - 1)(x - 5) / (-1 - 1)(-1 - 5) + (-12)(x + 1)(x - 5) / (1 + 1)(1 - 5) + 20(x + 1)(x - 1) / (5 + 1)(5 - 1)$  Lagrange form [M1]  
three terms [DM1]  
terms [A1,1,1]

**(iii)** Hence by substitution  $Q(0) = -10$  cao [M1A1]

[TOTAL 8]

**4(i)**

	x		$1 - x^4$				
	0.7	<	0.7599				
	0.8	>	0.5904	(hence root)			[M1A1]

**(ii)** Derivative of  $1 - x^4$  is  $-4x^3$  [M1]  
 $\times \text{abs}(-4x^3)$   
 0.7    1.372 > 1  
 0.8    2.048 > 1    (so for all [0.7, 0.8] abs gradient or abs RHS > 1) [M1A1E1]

*[M1] for grad anywhere in interval*

[TOTAL 6]

**5(i)**

	x	X	abs err	rel err		
	3.162278	3.162	-0.00028	-0.0000878	<i>do not insist on sign but do require consistency between parts (i) and (ii)</i>	[M1A1A1]

**(ii)**

	$x^4$	$X^4$	(abs err	rel err		
	100	99.96488	-0.03512)	-0.0003512	<i>allow multiplying answer in (i) by 4</i>	[M1A1A1]

**(iii)** Relative error will be approximately  $k \times (-)0.0000878$  [B1]

[TOTAL 7]

<b>6(i)</b>	<table border="0"> <tr><td>x</td><td>f(x)</td></tr> <tr><td>2</td><td>3</td></tr> <tr><td>2.4</td><td>3.850195</td></tr> <tr><td>2.8</td><td>4.790825</td></tr> </table>	x	f(x)	2	3	2.4	3.850195	2.8	4.790825	<table border="0"> <tr><td>M1 = 3.0801558</td><td><i>Lose A1, 2, 3, 4</i></td><td>[M1A1]</td></tr> <tr><td>T1 = 3.1163298</td><td><i>if 6, 5, 4, 3 dp</i></td><td>[M1A1]</td></tr> <tr><td>T2 = 3.0982428 (= (M1+T1)/2)</td><td></td><td>[M1A1]</td></tr> <tr><td>S1 = 3.0922138 (= (2M1+T1)/3)</td><td></td><td>[M1A1]</td></tr> </table>	M1 = 3.0801558	<i>Lose A1, 2, 3, 4</i>	[M1A1]	T1 = 3.1163298	<i>if 6, 5, 4, 3 dp</i>	[M1A1]	T2 = 3.0982428 (= (M1+T1)/2)		[M1A1]	S1 = 3.0922138 (= (2M1+T1)/3)		[M1A1]	[subtotal 8]
x	f(x)																						
2	3																						
2.4	3.850195																						
2.8	4.790825																						
M1 = 3.0801558	<i>Lose A1, 2, 3, 4</i>	[M1A1]																					
T1 = 3.1163298	<i>if 6, 5, 4, 3 dp</i>	[M1A1]																					
T2 = 3.0982428 (= (M1+T1)/2)		[M1A1]																					
S1 = 3.0922138 (= (2M1+T1)/3)		[M1A1]																					
<b>(ii)</b>	<table border="0"> <tr><td>x</td><td>f(x)</td></tr> <tr><td>2.2</td><td>3.412917</td></tr> <tr><td>2.6</td><td>4.309988</td></tr> </table>	x	f(x)	2.2	3.412917	2.6	4.309988	<table border="0"> <tr><td>M2 = 3.0891620</td><td>[M1A1]</td></tr> <tr><td>S2 = 3.0921890 (= (2M2+T2)/3)</td><td>[B1]</td></tr> <tr><td>or ... 889</td><td>[subtotal 3]</td></tr> </table>	M2 = 3.0891620	[M1A1]	S2 = 3.0921890 (= (2M2+T2)/3)	[B1]	or ... 889	[subtotal 3]									
x	f(x)																						
2.2	3.412917																						
2.6	4.309988																						
M2 = 3.0891620	[M1A1]																						
S2 = 3.0921890 (= (2M2+T2)/3)	[B1]																						
or ... 889	[subtotal 3]																						
<b>(iii)</b>		<table border="0"> <tr><td>M4 = 3.0914298</td><td></td></tr> <tr><td>T4 = 3.0937024</td><td>[B1]</td></tr> <tr><td>S4 = 3.0921873</td><td>[B1]</td></tr> </table>	M4 = 3.0914298		T4 = 3.0937024	[B1]	S4 = 3.0921873	[B1]															
M4 = 3.0914298																							
T4 = 3.0937024	[B1]																						
S4 = 3.0921873	[B1]																						
	<table border="0"> <tr><td>S1 = 3.0922138</td><td>diffs</td><td>ratio of</td><td></td></tr> <tr><td>S2 = 3.0921890</td><td>-2.5E-05</td><td>diffs</td><td></td></tr> <tr><td>S4 = 3.0921873</td><td>-1.6E-06</td><td>0.0648518</td><td>approx 1/16 so fourth order</td></tr> </table> <p>(FT their precision)</p>	S1 = 3.0922138	diffs	ratio of		S2 = 3.0921890	-2.5E-05	diffs		S4 = 3.0921873	-1.6E-06	0.0648518	approx 1/16 so fourth order		[M1A1E1]								
S1 = 3.0922138	diffs	ratio of																					
S2 = 3.0921890	-2.5E-05	diffs																					
S4 = 3.0921873	-1.6E-06	0.0648518	approx 1/16 so fourth order																				
	<p>Consider rate of convergence of S: conclude I = 3.09219</p>	<p>(or 3.092187 if using extrapolation) [B2] for 3.09219 alone</p>	[E1A1] [subtotal 7] [TOTAL 18]																				

<b>7(i)</b>	<table border="0"> <tr><td>x</td><td>f(x)</td><td><math>\Delta f(x)</math></td><td><math>\Delta^2 f(x)</math></td><td></td></tr> <tr><td>1</td><td>4</td><td></td><td></td><td></td></tr> <tr><td>3</td><td>-2</td><td>-6</td><td></td><td></td></tr> <tr><td>5</td><td>10</td><td>12</td><td>18</td><td></td></tr> </table>	x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$		1	4				3	-2	-6			5	10	12	18		[M1A1]										
x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$																													
1	4																															
3	-2	-6																														
5	10	12	18																													
	<p><math>f(x) = 4 + (-6)(x - 1) / 2 + 18(x - 1)(x - 3) / (2^2 \times 2!)</math></p>	[M1A1A1A1] [subtotal 6]																														
<b>(ii)</b>	<table border="0"> <tr><td>f(2) =</td><td>-1.25 likely to be more accurate (interpolation)</td></tr> <tr><td>f(6) =</td><td>22.75 than this (extrapolation)</td></tr> </table>	f(2) =	-1.25 likely to be more accurate (interpolation)	f(6) =	22.75 than this (extrapolation)	[A1A1E1] [subtotal 3]																										
f(2) =	-1.25 likely to be more accurate (interpolation)																															
f(6) =	22.75 than this (extrapolation)																															
<b>(iii)</b>	<table border="0"> <tr><td>x</td><td>f(x)</td><td><math>\Delta f(x)</math></td><td><math>\Delta^2 f(x)</math></td><td><math>\Delta^3 f(x)</math></td><td></td></tr> <tr><td>1</td><td>4</td><td></td><td></td><td></td><td></td></tr> <tr><td>3</td><td>-2</td><td>-6</td><td></td><td></td><td></td></tr> <tr><td>5</td><td>10</td><td>12</td><td>18</td><td></td><td></td></tr> <tr><td>7</td><td>11</td><td>1</td><td>-11</td><td>-29</td><td>← extend table</td></tr> </table>	x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$		1	4					3	-2	-6				5	10	12	18			7	11	1	-11	-29	← extend table	[M1A1]
x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$																												
1	4																															
3	-2	-6																														
5	10	12	18																													
7	11	1	-11	-29	← extend table																											
	<p>new <math>f(x) = \text{old } f(x) + (-29)(x - 1)(x - 3)(x - 5) / (2^3 \times 3!)</math></p>	<p><i>FT incorrect old f(x) here</i></p>	[M1A1]																													
	<table border="0"> <tr><td>f(2) = -1.25 + (-1.8125) =</td><td>-3.0625</td><td>[M1] for substituting</td><td>[M1A1]</td></tr> <tr><td>f(6) = 22.75 + (-9.0625) =</td><td>13.6875</td><td>cao for each [A1]</td><td>[A1]</td></tr> </table>	f(2) = -1.25 + (-1.8125) =	-3.0625	[M1] for substituting	[M1A1]	f(6) = 22.75 + (-9.0625) =	13.6875	cao for each [A1]	[A1]	[subtotal 7]																						
f(2) = -1.25 + (-1.8125) =	-3.0625	[M1] for substituting	[M1A1]																													
f(6) = 22.75 + (-9.0625) =	13.6875	cao for each [A1]	[A1]																													
<b>(iv)</b>	<p>Absolute change greater in f(6), relative change greater in f(2) [E1], [E1] for intelligent comments on absolute, relative changes</p>	<p><i>Must have all 4 values correct</i></p>	[E1E1] [subtotal 2] [TOTAL 18]																													



**OCR (Oxford Cambridge and RSA Examinations)**  
**1 Hills Road**  
**Cambridge**  
**CB1 2EU**

**OCR Customer Contact Centre**

**14 – 19 Qualifications (General)**

Telephone: 01223 553998

Facsimile: 01223 552627

Email: [general.qualifications@ocr.org.uk](mailto:general.qualifications@ocr.org.uk)

**[www.ocr.org.uk](http://www.ocr.org.uk)**

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

**Oxford Cambridge and RSA Examinations**  
**is a Company Limited by Guarantee**  
**Registered in England**  
**Registered Office; 1 Hills Road, Cambridge, CB1 2EU**  
**Registered Company Number: 3484466**  
**OCR is an exempt Charity**



**OCR (Oxford Cambridge and RSA Examinations)**  
**Head office**  
**Telephone: 01223 552552**  
**Facsimile: 01223 552553**

# 4776/01: Numerical Methods (Written Examination)

## General Comments

Most candidates found this a straightforward paper, and there were many high marks. Candidates showed a good grasp of the basic ideas of numerical mathematics, but as in previous years the standard of presentation of work was frequently not good. The point has to be made yet again that numerical mathematics is systematic and algorithmic. Setting down work in a logical order, frequently in tabular form, makes it easier to see whether a solution is correct – easier, that is, for both the candidate and the marker. Jumbles of unidentified numbers scattered across a page are unlikely to receive credit.

## Comments on Individual Questions

### 1 Solution of an equation

In part (i), the idea of halving the maximum possible error at each iteration was well understood, but many candidates counted the starting position of the bisection method as an iteration. In part (ii), false position was usually done correctly.

### 2 Numerical differentiation

Parts (i) and (ii) were generally done well, but part (iii) defeated most candidates. The idea in part (iii) is to combine the largest possible numerator with the smallest possible denominator, and vice versa.

### 3 Lagrange's interpolation formula

This was a straightforward question, but quite a number of candidates were not able to apply linear interpolation in part (i). In part (ii) there were the usual confusions between  $x$  values and function values. The question said, quite clearly, that no algebraic simplification was required; some candidates chose to simplify anyway.

### 4 Fixed point iteration

Locating the root in part (i) was very easy, but the majority of candidates were unable to use the magnitude of the derivative in a region surrounding the root to show that the iteration will diverge.

### 5 Absolute and relative errors

The numerical work in parts (i) and (ii) was done well by almost all. In part (iii), candidates were expected to know that when a number with a small relative error is raised to a power  $k$  the relative error will be increased (approximately) by a factor of  $k$ . This part was not done well.

### 6 Numerical integration

The numerical work in first two parts was done well by the vast majority of candidates. Part (iii) was more challenging. Not all candidates appeared to know how to recognise fourth order convergence. Some knew what to look for but arithmetical errors prevented them finding it. The value of  $l$  was often given to fewer decimal places than the work warranted. It is *not* correct to look for the number of figures of agreement in the two best Simpson's rule values. Thinking about differences and ratios of differences will show that the last Simpson's rule value will be very much more accurate than the one before.

**7 Newton's forward difference interpolation formula**

Almost all candidates found the quadratic in part (i) successfully, though some insisted on doing algebraic simplification which was not required. Part (i) was found easy too. In part (iii), the best approach is to add on the cubic term to the quadratic already found: this is one of the virtues of Newton's method. Some candidates worked from scratch here. The answers to part (iv) were frequently poor. Either the wrong numerical values found earlier prevented sensible comments, or what candidates said was muddled and unclear. The intended point was that the absolute change is greater in  $f(6)$  but the relative change is greater in  $f(2)$ .

<b>GCE Mathematics (MEI)</b>			<b>Max Mark</b>	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>	<b>u</b>
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	55	49	43	37	32	0	
	UMS	100	80	70	60	50	40	0	
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	53	46	39	33	27	0	
	UMS	100	80	70	60	50	40	0	
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	48	42	36	29	0	
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0	
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0	
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0	
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	63	56	50	44	38	0	
	UMS	100	80	70	60	50	40	0	
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	59	52	45	39	33	0	
	UMS	100	80	70	60	50	40	0	
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	55	48	41	34	27	0	
	UMS	100	80	70	60	50	40	0	
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	55	48	42	36	30	0	
	UMS	100	80	70	60	50	40	0	
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	39	0	
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0	
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0	
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0	
4761/01 (M1) MEI Mechanics 1	Raw	72	60	52	44	36	28	0	
	UMS	100	80	70	60	50	40	0	
4762/01 (M2) MEI Mechanics 2	Raw	72	64	57	51	45	39	0	
	UMS	100	80	70	60	50	40	0	
4763/01 (M3) MEI Mechanics 3	Raw	72	59	51	43	35	27	0	
	UMS	100	80	70	60	50	40	0	
4764/01 (M4) MEI Mechanics 4	Raw	72	54	47	40	33	26	0	
	UMS	100	80	70	60	50	40	0	
4766/01 (S1) MEI Statistics 1	Raw	72	53	45	38	31	24	0	
	UMS	100	80	70	60	50	40	0	
4767/01 (S2) MEI Statistics 2	Raw	72	60	53	46	39	33	0	
	UMS	100	80	70	60	50	40	0	
4768/01 (S3) MEI Statistics 3	Raw	72	56	49	42	35	28	0	
	UMS	100	80	70	60	50	40	0	
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0	
	UMS	100	80	70	60	50	40	0	
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	51	45	39	33	27	0	
	UMS	100	80	70	60	50	40	0	
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	58	53	48	43	39	0	
	UMS	100	80	70	60	50	40	0	
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0	
	UMS	100	80	70	60	50	40	0	
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	62	55	49	43	36	0	
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0	
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0	
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0	
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0	
	UMS	100	80	70	60	50	40	0	