OCR
RECOGNISING ACHIEVEMENT

Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754 Applications of Advanced Mathematics (C4)

INSTRUCTIONS

4733120112

The examination is in two parts:

Paper A (1 hour 30 minutes)
Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B **is not issued** until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding paper A script. For Paper A only the candidates' printed answer books, in the same order as the attendance register, should be sent for marking; the question paper should be retained in the centre or destroyed. For Paper B only the question papers, on which the candidates have written their answers, should be sent for marking; the insert should be retained in the centre or destroyed. Any additional sheets used must be carefully attached to the correct paper.

<u>For Paper B (Comprehension) only.</u>

A standard English dictionary is allowed for the comprehension.
(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the Invigilator's desk at all times during the morning of Tuesday 17 January 2012.

RECOGNISING ACHIEVEMENT

Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4733130112

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Express $\dfrac{x+1}{x^2(2x-1)}$ in partial fractions. **[5]**

2 Solve, correct to 2 decimal places, the equation $\cot 2\theta = 3$ for $0° \leqslant \theta \leqslant 180°$. **[4]**

3 Express $3\sin x + 2\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence find, correct to 2 decimal places, the coordinates of the maximum point on the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 3\sin x + 2\cos x, \ 0 \leqslant x \leqslant \pi.$$ **[7]**

4 **(i)** Complete the table of values for the curve $y = \sqrt{\cos x}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y		0.9612	0.8409		

Hence use the trapezium rule with strip width $h = \frac{\pi}{8}$ to estimate the value of the integral $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\,\mathrm{d}x$, giving your answer to 3 decimal places. **[3]**

Fig. 4 shows the curve $y = \sqrt{\cos x}$ for $0 \leqslant x \leqslant \frac{\pi}{2}$.

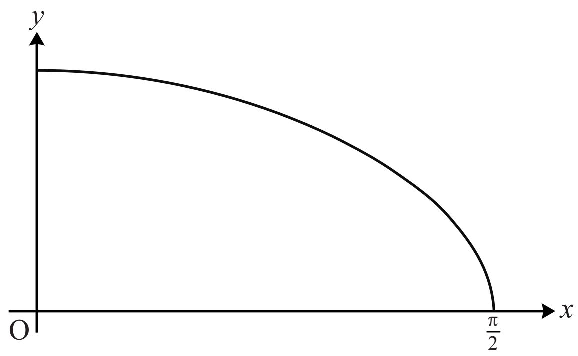

Fig. 4

(ii) State, with a reason, whether the trapezium rule with a strip width of $\frac{\pi}{16}$ would give a larger or smaller estimate of the integral. **[1]**

5 Verify that the vector $2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ is perpendicular to the plane through the points A(2, 0, 1), B(1, 2, 2) and C(0, −4, 1). Hence find the cartesian equation of the plane. **[5]**

6 Given the binomial expansion $(1 + qx)^p = 1 - x + 2x^2 + \ldots$, find the values of p and q. Hence state the set of values of x for which the expansion is valid. **[6]**

7 Show that the straight lines with equations $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ meet.

Find their point of intersection. **[5]**

Section B (36 marks)

8 Fig. 8 shows a cross-section of a car headlight whose inside reflective surface is modelled, in suitable units, by the curve

$$x = 2t^2,\ y = 4t, \quad -\sqrt{2} \leqslant t \leqslant \sqrt{2}\ .$$

$P(2t^2, 4t)$ is a point on the curve with parameter t. TS is the tangent to the curve at P, and PR is the line through P parallel to the x-axis. Q is the point (2, 0). The angles that PS and QP make with the positive x-direction are θ and ϕ respectively.

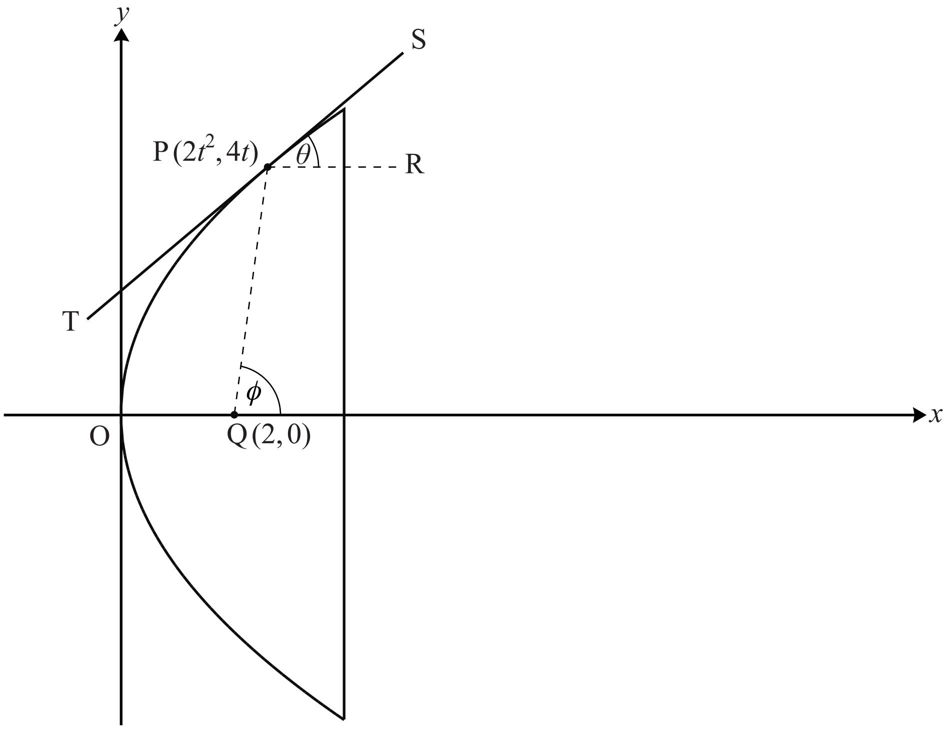

Fig. 8

(i) By considering the gradient of the tangent TS, show that $\tan\theta = \frac{1}{t}$. **[3]**

(ii) Find the gradient of the line QP in terms of t. Hence show that $\phi = 2\theta$, and that angle TPQ is equal to θ. **[8]**

[The above result shows that if a lamp bulb is placed at Q, then the light from the bulb is reflected to produce a parallel beam of light.]

The inside surface of the headlight has the shape produced by rotating the curve about the x-axis.

(iii) Show that the curve has cartesian equation $y^2 = 8x$. Hence find the volume of revolution of the curve, giving your answer as a multiple of π. **[7]**

9

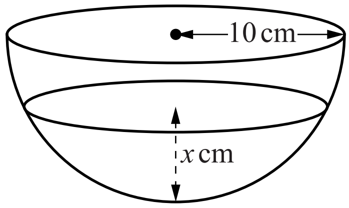

Fig. 9

Fig. 9 shows a hemispherical bowl, of radius 10 cm, filled with water to a depth of x cm. It can be shown that the volume of water, V cm^3, is given by

$$V = \pi(10x^2 - \tfrac{1}{3}x^3).$$

Water is poured into a leaking hemispherical bowl of radius 10 cm. Initially, the bowl is empty. After t seconds, the volume of water is changing at a rate, in cm^3 s^{-1}, given by the equation

$$\frac{dV}{dt} = k(20 - x),$$

where k is a constant.

(i) Find $\dfrac{dV}{dx}$, and hence show that $\pi x \dfrac{dx}{dt} = k$. **[4]**

(ii) Solve this differential equation, and hence show that the bowl fills completely after T seconds, where $T = \dfrac{50\pi}{k}$. **[5]**

Once the bowl is full, the supply of water to the bowl is switched off, and water then leaks out at a rate of kx cm^3 s^{-1}.

(iii) Show that, t seconds later, $\pi(20 - x)\dfrac{dx}{dt} = -k$. **[3]**

(iv) Solve this differential equation.

Hence show that the bowl empties in $3T$ seconds. **[6]**

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Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS MEI

4754A Applications of Advanced Mathematics (C4) Paper A

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
- Question Paper 4754A (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1	

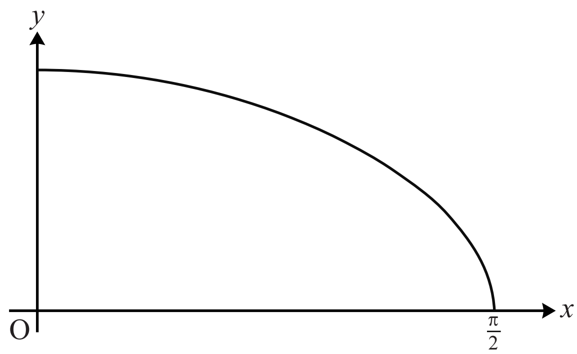
2	

3	

4 (i)

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y		0.9612	0.8409		

4 (ii)



5

6

7

Section B (36 marks)

8 (i)

8 (ii)

(answer space continued overleaf)

8 (ii)	**(continued)**
8 (iii)	
	(answer space continued on next page)

8 (iii)	(continued)

9 (i)	

9 (ii)	

9 (iii)

9 (iv)	
	(answer space continued overleaf)

(answer space continued overleaf)

9 (iv)	(continued)

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GCE

Mathematics (MEI)

Advanced GCE

Unit **4754A:** Applications of Advanced Mathematics: Paper A

Mark Scheme for January 2012

Oxford Cambridge and RSA Examinations

Any enquiries about publications should be addressed to:

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Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he / she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			$\frac{x+1}{x^2(2x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1} = \frac{Ax(2x-1)+B(2x-1)+Cx^2}{x^2(2x-1)}$	B1	correct partial fractions
			$\Rightarrow \quad x+1 = Ax(2x-1)+B(2x-1)+Cx^2$	M1	Using a correct method to find a coefficient (equating numerators and substituting oe or using cover-up) Condone omission of brackets only if brackets are implied by subsequent work. Must go as far as finding a coefficient. Not dependent on B1
			$x = 0,\ 1 = -B \Rightarrow B = -1$	A1	$B = -1$ www
			$x = ½,\ 1½ = C/4 \Rightarrow C = 6$	A1	$C = 6$ www
			x^2 coeffs: $0 = 2A + C \Rightarrow A = -3$	A1	$A = -3$ www
			$\Rightarrow \quad \frac{x+1}{x^2(2x-1)} = -\frac{3}{x} - \frac{1}{x^2} + \frac{6}{2x-1}$		isw for incorrect assembly of partial fractions following correct A,B,C
					SC $\frac{A}{x^2} + \frac{B}{2x-1}$ can get 2/5 max from B0 M1 A1 (for B=6) SC $\frac{Ax+B}{x^2} + \frac{C}{2x-1}$ can get B1 M1 A1 (C=6) and can continue for full marks if the first fraction is then split. SC $\frac{A}{x} + \frac{B}{x^2} + \frac{C+Dx}{2x-1}$ can get B1 M1 A1 A1 A1 (C=6, D=0)
				[5]	

Question			Answer	Marks	Guidance
2			$\cot 2\theta = 3$ $\Rightarrow \tan 2\theta = 1/3$	M1	tan=1/ cot **used** soi
			$\Rightarrow 2\theta = 18.43°$ $\theta = 9.22°$	A1	for first correct solution (9.22 or better eg 9.217)
			$2\theta = 198.43°$	M1	for method for second solution for θ.
			$\theta = 99.22°$	A1	for second correct solution and no others in range (99.22 or better) or SC ft A1 for 90 +their first solution
			or $(2\tan\theta)/(1 - \tan^2\theta) = 1/3$	M1	use of correct double angle formula
			$\Rightarrow 6\tan\theta = 1 - \tan^2\theta$ $\Rightarrow \tan^2\theta + 6\tan\theta - 1 = 0$	M1	rearranged to a quadratic = 0 and attempt to solve by formula oe
			$\Rightarrow \tan\theta = [-6 \pm \sqrt{(36 + 4)}]/2 = 0.1623$ or -6.1623 $\Rightarrow \theta = 9.22°, 99.22°$	A1	first correct solution
				A1	second correct solution and no others in the range (9.22, 99.22 or better) or SC ft A1 for 90 + their first solution -1 MR if radians used (0.16, 1.73 or better)
				[4]	
3			$3\sin x + 2\cos x = R\sin(x + \alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$ $\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 2$	M1	Correct pairs. Condone omission of R if used correctly. Condone sign error.
			$\Rightarrow R^2 = 3^2 + 2^2 = 13,\ R = \sqrt{13}$	B1	or 3.6 or better, not $\pm\sqrt{13}$ unless $+\sqrt{13}$ chosen
			$\tan\alpha = 2/3$,	M1	ft from first M1
			$\alpha = 0.588$	A1	0.588 or better (accept 0.59), with no errors seen in method for angle (allow 33.7° or better)
			$\Rightarrow 3\sin x + 2\cos x = \sqrt{13}\sin(x + 0.588)$ maximum when $x + 0.588 = \pi/2$	M1	any valid method eg differentiating
			$\Rightarrow x = \pi/2 - 0.588 = 0.98$ rads	A1	0.98 only. Do not accept degrees or multiples of π.
			$\Rightarrow y = \sqrt{13} = 3.61$	B1	condone $\sqrt{13}$, ft their R if ,say $=\sqrt{14}$
			So coords of max point are (0.98, 3.61)	**[7]**	

Question			Answer	Marks	Guidance
4	(i)		1, 0.6186, 0 $A \approx (\pi/16)\{1 + 0+2(0.9612+0.8409+0.6186)\}$ $= 1.147$ (3 dp)	B1 M1 A1 **[3]**	4dp (or more) ft their table. Need to see trapezium rule. cao
4	(ii)		The estimate will increase, because the trapezia will be below but closer to the curve, reducing the error.	B1 **[1]**	o.e., or an illustration using the curve full answer required
5			$\overrightarrow{AB} = \begin{pmatrix}-1\\2\\1\end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix}-2\\-4\\0\end{pmatrix}$ $\mathbf{n}.\overrightarrow{AB} = \begin{pmatrix}2\\-1\\4\end{pmatrix}.\begin{pmatrix}-1\\2\\1\end{pmatrix} = 2\times(-1)+(-1)\times 2+4\times 1=0$ $\mathbf{n}.\overrightarrow{AC} = \begin{pmatrix}2\\-1\\4\end{pmatrix}.\begin{pmatrix}-2\\-4\\0\end{pmatrix} = 2\times(-2)+(-1)\times(-4)+4\times 0=0$ $\Rightarrow$ **n** is perpendicular to plane. Equation of plane is **r**.**n** = **a**.**n** $\Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\-1\\4\end{pmatrix} = \begin{pmatrix}2\\0\\1\end{pmatrix}.\begin{pmatrix}2\\-1\\4\end{pmatrix}$ $\Rightarrow 2x - y + 4z = 8$	M1 B1 B1 M1 A1 **[5]**	scalar product with any two directions in the plane (BC=$\begin{pmatrix}-1\\-6\\-1\end{pmatrix}$) evaluation needed evaluation needed thus finding the scalar product with only one direction vector is M0 B1 B0. No marks for scalar product with position vectors. or SC finding direction of normal vector by using vector cross product, M1A1 eg $4i$-$2j$+$8k$ and showing this is a multiple of $2i$-j+$4k$, A1 For any complete method leading to the cartesian equation of the plane eg from vector form and eliminating parameters (there are many possibilities eg $r=\begin{pmatrix}2\\0\\1\end{pmatrix}+\mu\begin{pmatrix}-1\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}-2\\-4\\0\end{pmatrix}$ $x=2-\mu-2\lambda, y=2\mu-4\lambda$, $z=1+\mu$, $2x-y=4-4\mu=4-4(z-1)=8-4z$, $2x-y+4z=8$ gets M1 once the parameters have been eliminated. oe **SC1** If they say the plane is of the form $2x-y+4z=c$ and then show all points satisfy $2x-y+4z=8$ they can have M1 A1 for the first point and B2 for both the others. **SC2** If they omit verification and find equation from vector form without using normal as above and then state $2i$-j+$4k$ is perpendicular they can get M1A1B2

Question			Answer	Marks	Guidance
6			$(1 + qx)^p = 1 + pqx + ½\,p(p-1)q^2x^2 + \ldots$	B1 B1	$(1)\ldots + pqx$ $\ldots + ½\,p(p-1)q^2x^2$
			$\Rightarrow \quad pq = -1,\ q = -1/p$ $½\,p(p-1)q^2 = 2$ $\Rightarrow \quad p(p-1)/2p^2 = (p-1)/2p = 2$	M1	eliminating q (or p) from simultaneous equations involving both variables oe $\frac{1}{2}\left(\frac{-1}{q}\right)\left(\frac{-1}{q}-1\right)q^2 = 2$, $-1(-1-q)=4$, $q=3$
			$\Rightarrow \quad p - 1 = 4p,\ p = -1/3$	A1	$p = -1/3$ www (or $q = 3$)
			$\Rightarrow \quad q = 3$	A1ft	$q = 3$ (or $p = -1/3$) for second value, ft their p or q eg -1/the other , provided only a single computational error in the method and correct initial equations
			Valid for $-1 < 3x < 1 \Rightarrow -1/3 < x < 1/3$	B1 **[6]**	or $\lvert x\rvert < 1/3$ www, allow $-1/3<\lvert x\rvert<1/3$ but not say, $x<1/3$ (actually $-1/3<x\le 1/3$ is correct)
7			$\begin{pmatrix} 4+3\lambda \\ 2 \\ 4+\lambda \end{pmatrix} = \begin{pmatrix} -1-\mu \\ 4+\mu \\ 9+3\mu \end{pmatrix}$		
			$\Rightarrow \quad 4 + 3\lambda = -1 - \mu$ (1) $2 = 4 + \mu$ (2) $4 + \lambda = 9 + 3\mu$ (3)	M1	equating components
			(2) $\Rightarrow \mu = -2$	B1	$\mu = -2$
			(1) $\Rightarrow 4 + 3\lambda = -1 + 2 \Rightarrow \lambda = -1$	A1	$\lambda = -1$
			(3) $\Rightarrow 4 + (-1) = 9 + 3 \times (-2)$ so consistent	A1	checking third component
			Point of intersection is (1, 2, 3)	A1 **[5]**	dependent on all previous marks being obtained
8	(i)		$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4}{4t} = \frac{1}{t}$	M1 A1	their dy/dt /dx/dt accept $4/4t$ here
			But gradient of tangent = $\tan\theta$ * $\Rightarrow \tan\theta = 1/t$	A1 **[3]**	**ag** -need reference to gradient is $\tan\theta$

Question			Answer	Marks	Guidance
8	(ii)		Gradient of QP = $\dfrac{4t}{2t^2-2}=\dfrac{2t}{t^2-1}$	M1 A1	correct method for subtracting co-ordinates correct (does not need to be cancelled)
			$=\dfrac{2\dfrac{1}{\tan\theta}}{\dfrac{1}{\tan^2\theta}-1}$	M1	either substituting t=1/tanθ in above expression or substituting tanθ=1/t in double angle formula for tan2θ. (tan2θ=2tanθ/(1-tan²θ)=2/t/(1-1/t²)=2t/(t²-1)
			$=\dfrac{2\tan\theta}{1-\tan^2\theta}=\tan 2\theta$	A1	showing expressions are equal
			$\Rightarrow \tan\phi = \tan 2\theta$		
			$\Rightarrow \phi = 2\theta$ *	A1	**ag**
			$\Rightarrow$ Angle QPR = 180 – 2θ	M1	supplementary angles oe
			$\Rightarrow \angle$TPQ + 180 – 2θ + θ = 180	M1	angles on a straight line oe
			$\Rightarrow \angle$TPQ = θ *	A1	**ag**
				[8]	
8	(iii)		$t = y/4$		
			$\Rightarrow$ $x = 2\,y^2/16 = y^2/8$	M1	eliminating t from parametric equation
			$\Rightarrow$ $y^2 = 8x$*	A1	**ag**
			When $t = \sqrt{2}$, $x = 2\times(\sqrt{2})^2 = 4$	B1	
			So $V=\int_0^4 \pi y^2\,\mathrm{d}x=\int_0^4 8\pi x\,\mathrm{d}x$	M1 A1	for M1 allow no limits or their limits need correct limits but they may appear later
			$=\left[4\pi x^2\right]_0^4$	B1	for 4πx² (ignore incorrect or missing limits)
			$= 64\pi$	A1 **[7]**	in terms of π only allow SC B1 for omission of π throughout integral but otherwise correct

Question			Answer	Marks	Guidance
9	(i)		$\mathrm{d}V/\mathrm{d}x = \pi(20x - x^2)$ $\Rightarrow \quad \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}x}.\frac{\mathrm{d}x}{\mathrm{d}t}$ $= \pi x(20-x).\frac{\mathrm{d}x}{\mathrm{d}t} = k(20-x)$ $\Rightarrow \quad \pi x\frac{\mathrm{d}x}{\mathrm{d}t} = k$ *	B1 M1 A1 A1 **[4]**	 oe **ag**
9	(ii)		$\int \pi x\,\mathrm{d}x = \int k\,\mathrm{d}t$ $\Rightarrow \quad \frac{1}{2}\pi x^2 = kt + c$ When $t = 0$, $x = 0 \Rightarrow c = 0$ $\Rightarrow \quad \frac{1}{2}\pi x^2 = kt$ Full when $x = 10$, $t = T$ $\Rightarrow \quad 50\pi = kT$ $\Rightarrow \quad T = 50\pi/k$ *	M1 A1 B1 M1 A1 **[5]**	separate variables and attempt integration of both sides condone absence of c c=0 www substitute t or T=50 π/k or x=10 and rearranging for the other (dependent on first M1) oe **ag,** need to have c=0
9	(iii)		$\mathrm{d}V/\mathrm{d}t = -kx$ $\Rightarrow \pi x(20-x).\frac{\mathrm{d}x}{\mathrm{d}t} = -kx$ $\Rightarrow \pi(20-x)\frac{\mathrm{d}x}{\mathrm{d}t} = -k$ *	B1 M1 A1 **[3]**	correct $\mathrm{d}V/\mathrm{d}x.\mathrm{d}x/\mathrm{d}t = \pm kx$ ft **ag**

Question			Answer	Marks	Guidance
9	**(iv)**		$\int \pi(20-x)\,\mathrm{d}x = \int -k\,\mathrm{d}t$	M1	separate variables and intend to integrate both sides
			$\pi(20x - ½\, x^2) = -kt + c$	B1 A1	LHS (not dependent on M1) RHS ie $-kt + c$ (condone absence of c)
			When $t = 0$, $x = 10$ $\Rightarrow \quad \pi(200 - 50) = c$ $\Rightarrow \quad c = 150\pi$	A1	evaluation of c cao oe (x=10, t=0)
			$\Rightarrow \quad \pi(20x - ½\, x^2) = 150\pi - kt$ $x = 0$ when $150\pi - kt = 0$	M1	substitute x=0 and rearrange for t -dependent on first M1 and non-zero c ,oe
			$\Rightarrow \quad t = 150\pi/k = 3T$*	A1 **[6]**	**ag**

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RECOGNISING ACHIEVEMENT

Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754B Applications of Advanced Mathematics (C4) Paper B: Comprehension

QUESTION PAPER

Candidates answer on the Question Paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Rough paper

Duration: Up to 1 hour

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

- The Insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

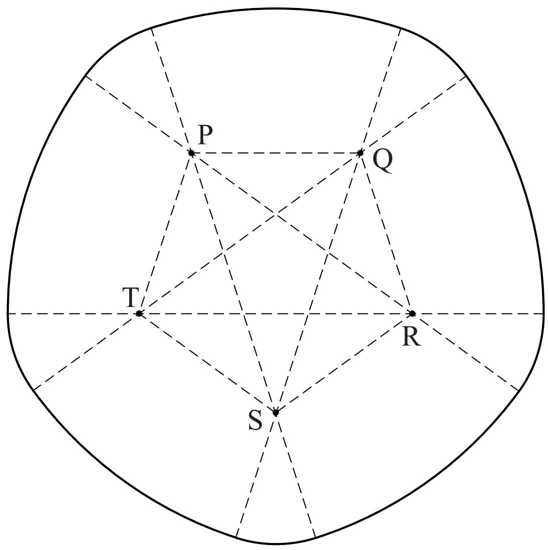
1 In lines 22 and 23 it says "arcs can be added to any regular polygon with an odd number of sides to make a curve of constant width". State why the method described cannot be applied to a regular polygon with an even number of sides. **[1]**

1	

2 **(i)** On the curve of constant width below, indicate clearly the arcs that were constructed with centre P. **[1]**

(ii) Given that this curve has perimeter 70 cm, calculate its width, correct to 3 significant figures. **[2]**

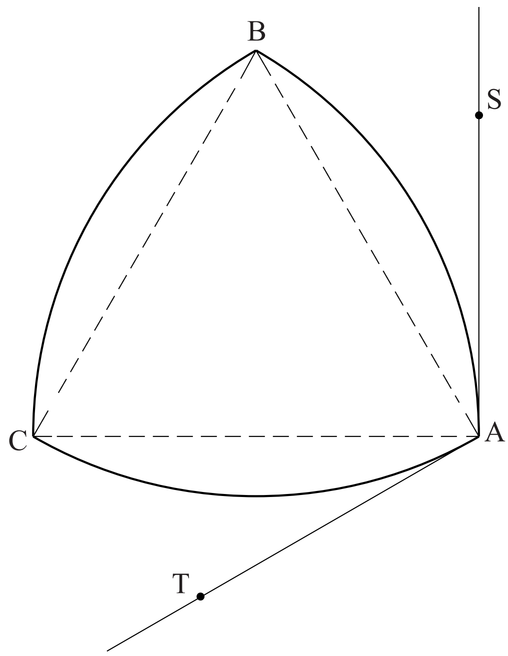
2 (i)



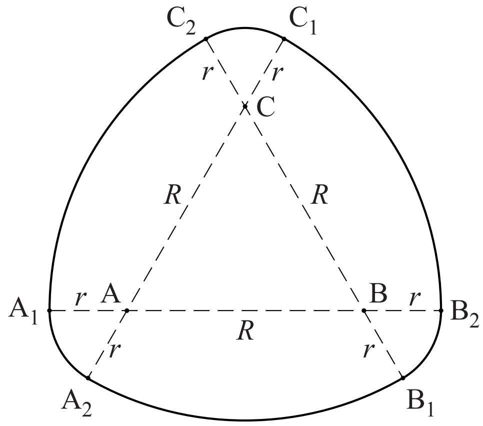
2 (ii)	

3 The diagram below shows two tangents, AS and AT, at vertex A on a Reuleaux triangle. State the angle SAT justifying your answer carefully. **[3]**

3



4 For the curve in Fig. 7b (copied below) the width, l, is $R + 2r$.



(i) Prove that the perimeter is πl. **[3]**

(ii) You are given that, in the case where $r = \frac{R}{2}$, the area enclosed by this curve is $R^2\left(\frac{5\pi - 2\sqrt{3}}{4}\right)$.

Show that this area falls in the range indicated in lines 28 and 29. **[3]**

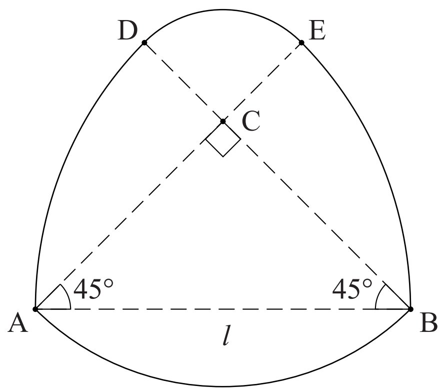
4 (i)	
4 (ii)	

5 Fig. 11b is copied below.

(i) Show that CE has length $\frac{(2-\sqrt{2})}{2}\,l$. **[2]**

(ii) Hence show that the square path traced out by point C (see line 67) has side length $(\sqrt{2}-1)\,l$. **[2]**

(iii) A square hole of side length 50 mm is to be cut in a sheet of plastic, using the method described in lines 69 to 71. Calculate the side length of the square hole needed in the guide plate, giving your answer correct to the nearest millimetre. **[1]**

5 (i)	
5 (ii)	
5 (iii)	

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OCR
RECOGNISING ACHIEVEMENT

Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754B Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

Duration: Up to 1 hour

INFORMATION FOR CANDIDATES

- This insert contains the text for use with the questions.
- This document consists of **8** pages. Any blank pages are indicated.

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Curves of Constant Width

Introduction

Imagine you need to move a heavy object over level ground. You could rest your object on a horizontal platform and use cylindrical rollers, all with the same radius, as shown in Fig. 1.

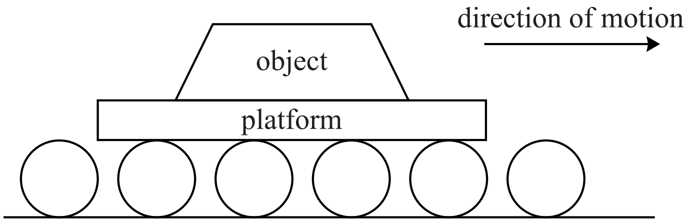

Fig. 1

Using cylindrical rollers in this way would ensure that the motion is smooth and horizontal; the object remains at the same height above the ground at all times.

If the cross-section of each roller was an ellipse, rather than a circle, then the motion would not be smooth. Fig. 2 shows different orientations of an elliptical roller.

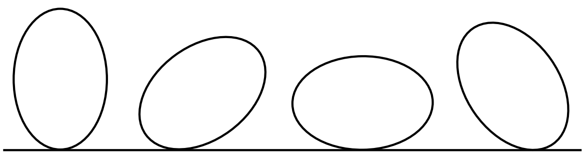

Fig. 2

You might be surprised to learn that there are infinitely many shapes which, like the circle, form the cross-section of a roller that would produce smooth horizontal motion of the object. These shapes are said to have *constant width* and the boundary of such a shape is called *a curve of constant width*. In this article you will be introduced to several types of curves of constant width.

Reuleaux triangle

The simplest non-circular curve of constant width, shown in Fig. 3, is named after Franz Reuleaux (1829–1905), a German mathematician and engineer. It is based on an equilateral triangle of side length l on which three circular arcs are drawn; each arc is centred on one vertex and passes through the other two vertices.

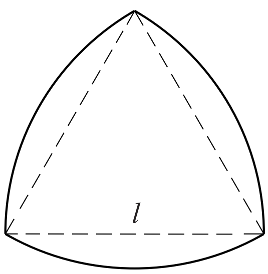

Fig. 3

If a roller with this cross-section rolled along a horizontal surface then the highest point would always be at a height l above the surface. This is illustrated in Fig. 4.

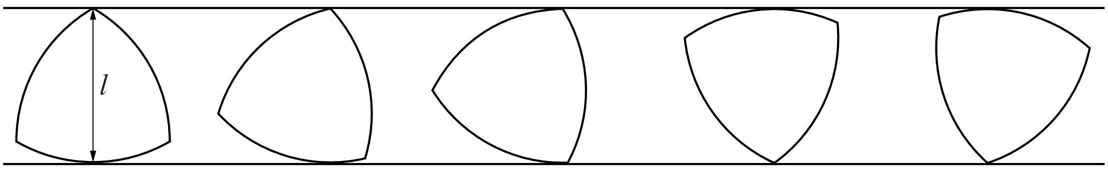

Fig. 4

In one revolution, assuming no slipping, the roller would move forward a distance πl, the same distance as for a cylindrical roller of diameter l.

In a similar way, arcs can be added to any regular polygon with an odd number of sides to make a curve of constant width. A Reuleaux pentagon, with constant width l, is shown in Fig. 5. In this curve, each arc is centred on the opposite vertex of the pentagon; for example, the arc CD has centre A and radius l.

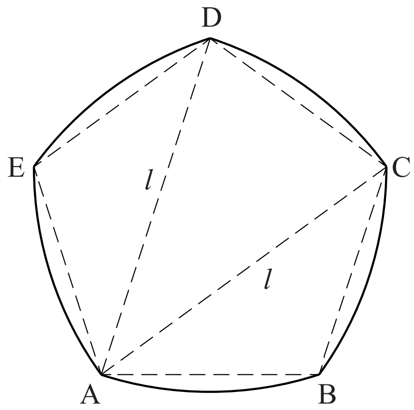

Fig. 5

Many mathematical properties of curves of constant width are known. Two of particular interest are given here.

- Every curve of constant width l has perimeter πl.
- Of all the curves of constant width l, the circle encloses the greatest area, $\frac{\pi}{4}l^2 \approx 0.785l^2$, and the Reuleaux triangle encloses the smallest area, $\left(\frac{\pi - \sqrt{3}}{2}\right) l^2 \approx 0.705l^2$.

Smooth curves of constant width

With the exception of the circle, the curves of constant width met so far have been created by constructing arcs on the sides of certain regular polygons. At each vertex of the regular polygon, two arcs meet but they do so in such a way that the curve is not smooth. For the Reuleaux pentagon in Fig. 6 there are two tangents at vertex A, one on each arc. The angle between these tangents is 144°.

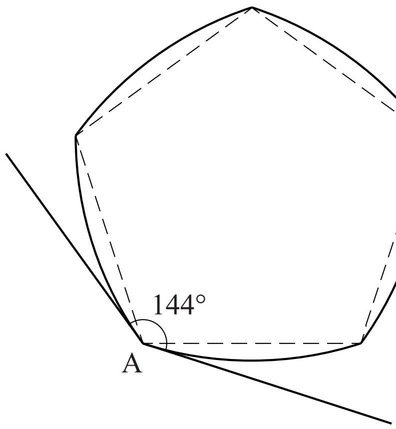

Fig. 6

It is possible to construct curves of constant width which are smooth at all points; one way of doing this is as follows.

- Draw an equilateral triangle ABC with side length R
- Extend the sides a distance r beyond each vertex, as shown in Fig. 7a
- Construct two circular arcs centred on A: arc A_1A_2 with radius r and arc B_2C_1 with radius $R + r$
- Construct similar arcs centred on B and C to give the curve shown in Fig. 7b.

This curve has constant width $R + 2r$ and is smooth. Every point on the curve has a unique tangent and the distance between parallel tangents is constant.

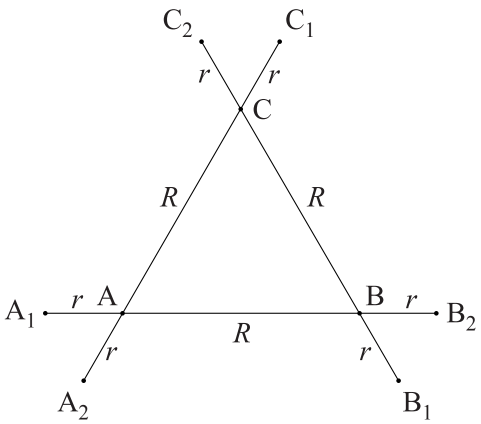

Fig. 7a

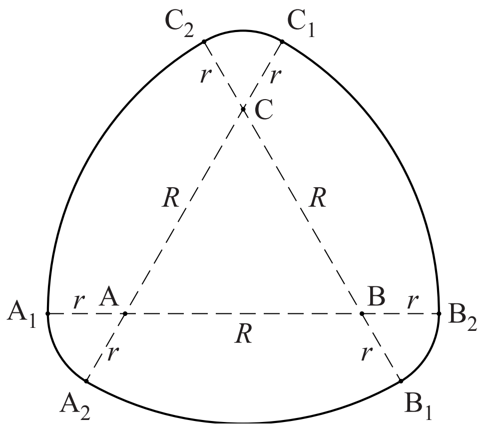

Fig. 7b

This method can be used on other regular polygons with an odd number of sides. One example of this is shown in Fig. 8. Notice that it is the longest diagonals of the heptagon that are extended.

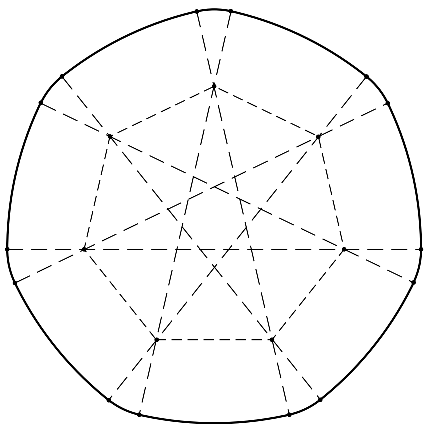

Fig. 8

Tracing out a locus

Any curve of constant width can turn inside a square; throughout the motion, the curve will always be in contact with all four sides of the square. This is illustrated for various positions of the Reuleaux triangle in Fig. 9.

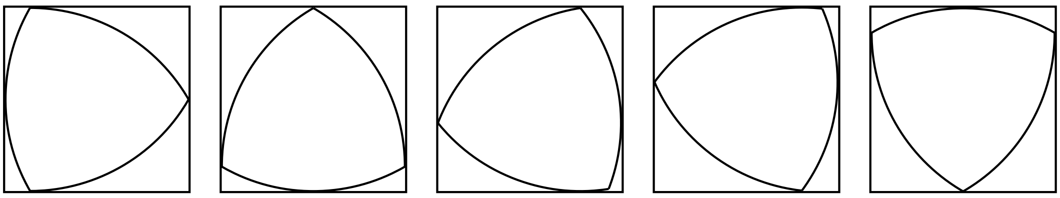

Fig. 9

If you trace the path followed by a vertex of the Reuleaux triangle, you will find that the locus is close to a square. This is shown in Fig. 10a; the locus of vertex P is made up of four straight line segments joined by rounded corners.

Fig. 10b shows the locus of the midpoint, Q, of one side of the equilateral triangle; this, too, is close to a square.

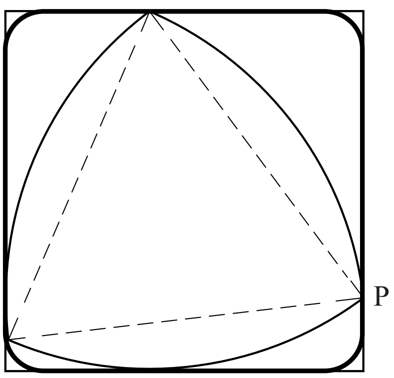

Fig. 10a

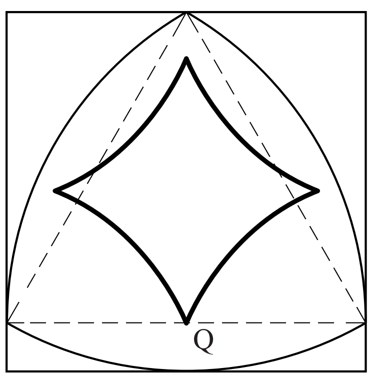

Fig. 10b

Other points in the Reuleaux triangle trace out other paths but none of these is a perfect square. This suggests the following question.

Is it possible to design a shape of constant width that contains a point which traces out a perfect square as the shape turns inside a square?

The answer to this question is 'Yes' and the curve is described below.

Drilling a square hole

Figs. 11a and 11b show how to construct a particular curve of constant width based on an isosceles triangle.

In triangle ABC, AB = l and angle CAB = angle CBA = 45°. Sides AC and BC are extended so that AE = BD = l. Arc BE has centre A, arc DA has centre B and arcs AB and ED have centre C.

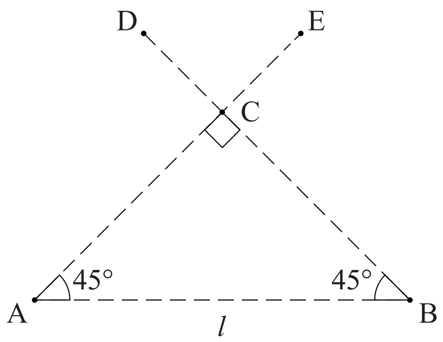

Fig. 11a

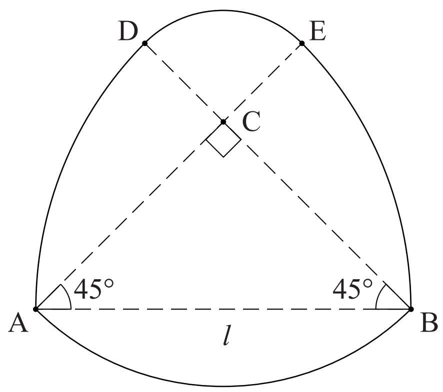

Fig. 11b

Like any curve of constant width l, the curve shown in Fig. 11b can turn inside a square of side length l. Fig. 12 shows this curve in various positions as it turns inside a square.

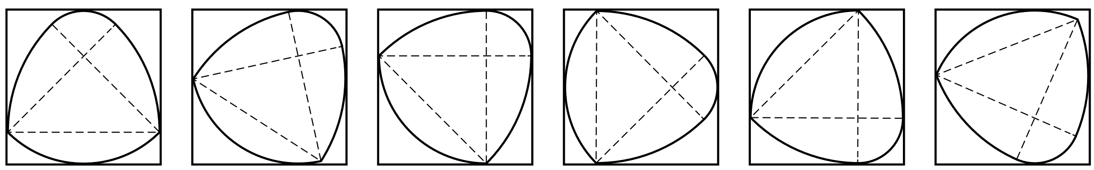

Fig. 12

The arc labelled DE in Fig. 11b remains in contact with the square throughout this motion. It follows that the locus of C is a perfect square.

This shape is used to drill a square hole.

A drill bit, with cross-section as shown in Fig. 11b, has a cutting tool at C. A metal guide plate, in which a square hole of side length l has been cut, is placed parallel to the material to be drilled. As the drill bit turns inside the guide plate, the cutting tool cuts out a square hole.

Other applications

Awareness of the existence of curves of constant width is important in engineering. Engineers rely on precision tools, many of which must be circular. To test that an object is circular, it is not sufficient simply to check that its width is constant; that would only imply that the object was one of the many shapes of constant width. Other means of checking for circularity, such as using circular templates, are needed.

It is possible that you are currently in possession of several shapes of constant width. Both the 50 pence and 20 pence coins have constant width (see Fig. 13). These coins were designed in this way so that they can easily be identified when used in machines; their width can be measured in any orientation as they move through the machine.

Fig. 13