OCR
RECOGNISING ACHIEVEMENT

Friday 13 January 2012 – Morning

AS GCE MATHEMATICS (MEI)

4751 Introduction to Advanced Mathematics (C1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4751
- MEI Examination Formulae and Tables (MF2)

Other materials required:
None

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (36 marks)

1 Find the equation of the line which is perpendicular to the line $y = 5x + 2$ and which passes through the point $(1, 6)$. Give your answer in the form $y = ax + b$. **[3]**

2 **(i)** Evaluate $9^{-\frac{1}{2}}$. **[2]**

(ii) Simplify $\dfrac{(4x^4)^3\, y^2}{2x^2 y^5}$. **[3]**

3 Expand and simplify $(n + 2)^3 - n^3$. **[3]**

4 **(i)** Expand and simplify $(7 + 3\sqrt{2})(5 - 2\sqrt{2})$. **[3]**

(ii) Simplify $\sqrt{54} + \dfrac{12}{\sqrt{6}}$. **[2]**

5 Solve the following inequality.

$$\frac{2x + 1}{5} < \frac{3x + 4}{6}$$ **[4]**

6 Rearrange the following equation to make h the subject.

$$4h + 5 = 9a - ha^2$$ **[3]**

7

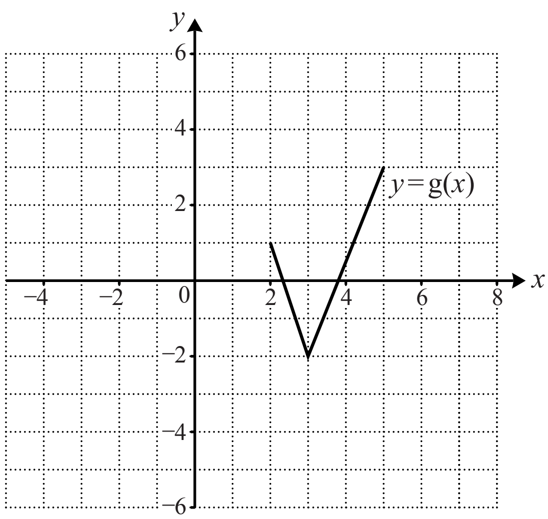

Fig. 7

Fig. 7 shows the graph of $y = \mathrm{g}(x)$. Draw the graphs of the following.

(i) $y = \mathrm{g}(x) + 3$ **[2]**

(ii) $y = \mathrm{g}(x + 2)$ **[2]**

8 Express $5x^2 + 15x + 12$ in the form $a(x + b)^2 + c$.

Hence state the minimum value of y on the curve $y = 5x^2 + 15x + 12$. **[5]**

9 Complete each of the following by putting the best connecting symbol ($\Leftrightarrow$, $\Leftarrow$ or $\Rightarrow$) in the box. Explain your choice, giving full reasons.

(i) $n^3 + 1$ is an odd integer ☐ n is an even integer **[2]**

(ii) $(x - 3)(x - 2) > 0$ ☐ $x > 3$ **[2]**

Section B (36 marks)

10 Point A has coordinates (4, 7) and point B has coordinates (2, 1).

(i) Find the equation of the line through A and B. **[3]**

(ii) Point C has coordinates (–1, 2). Show that angle ABC = 90° and calculate the area of triangle ABC. **[5]**

(iii) Find the coordinates of D, the midpoint of AC.

Explain also how you can tell, without having to work it out, that A, B and C are all the same distance from D. **[3]**

11 You are given that $f(x) = 2x^3 - 3x^2 - 23x + 12$.

(i) Show that $x = -3$ is a root of $f(x) = 0$ and hence factorise $f(x)$ fully. **[6]**

(ii) Sketch the curve $y = f(x)$. **[3]**

(iii) Find the x-coordinates of the points where the line $y = 4x + 12$ intersects $y = f(x)$. **[4]**

12 A circle has equation $(x - 2)^2 + y^2 = 20$.

(i) Write down the radius of the circle and the coordinates of its centre. **[2]**

(ii) Find the points of intersection of the circle with the y-axis and sketch the circle. **[3]**

(iii) Show that, where the line $y = 2x + k$ intersects the circle,

$$5x^2 + (4k - 4)x + k^2 - 16 = 0.$$ **[3]**

(iv) Hence find the values of k for which the line $y = 2x + k$ is a tangent to the circle. **[4]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.

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PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

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- MEI Examination Formulae and Tables (MF2)

Other materials required:
None

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1	
2 (i)	
2 (ii)	
3	

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4 (i)	
4 (ii)	
5	

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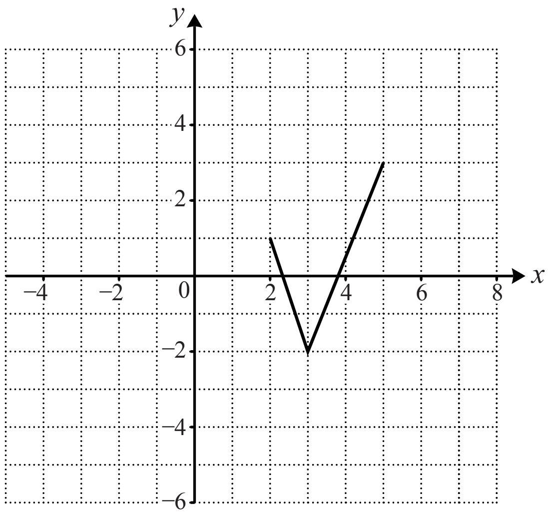
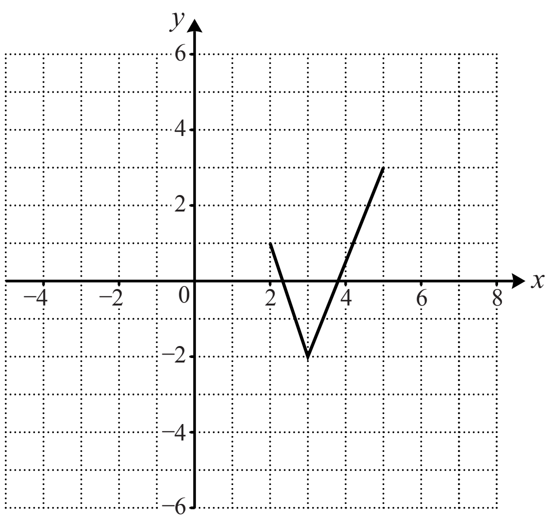
6

7 (i)



7 (ii)



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8	
9 (i)	$n^3 + 1$ is an odd integer [ ] n is an even integer
	Explanation:
9 (ii)	$(x - 3)(x - 2) > 0$ [ ] $x > 3$
	Explanation:

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Section B (36 marks)

10 (i)	
10 (ii)	

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10 (iii)	

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11 (i)

11 (ii)

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11 (iii)	

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12 (i)	
12 (ii)	

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12 (iii)	
12 (iv)	

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GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

Mark Scheme for January 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			grad = –1/5 oe	M1		allow embedded eg $5 \times -\frac{1}{5} = -1$
			$y - 6$ = their m $(x - 1)$ or 6 = their m [× 1] + c	M1		if first M1 not earned, allow second M1 for $y - 6 = k(x - 1)$ oe, k any number except 0 and 1
			$y = -0.2x + 6.2$ oe isw	A1	terms collected, with y as subject or for $a = -0.2$, $b = 6.2$ oe	allow A1 for $c = 6.2$ oe if $y = -0.2x + c$ oe already seen condone $y = \frac{-x+31}{5}$ for A1
				[3]		
2	(i)		$\frac{1}{3}$ as final answer	2	allow $\pm\frac{1}{3}$ M1 for $\frac{1}{9^{\frac{1}{2}}}$ or for $9^{\frac{1}{2}} = \sqrt{9}$ or 3 soi	eg M1 for 3^{-1}
				[2]		
2	(ii)		$32x^{10}y^{-3}$ or $\frac{32x^{10}}{y^3}$ oe as final answer	3	B1 for each element if B0, allow M1 for $(4x^4)^3 = 64x^{12}$	allow 2^5 instead of 32
				[3]		
3			$6n^2 + 12n + 8$ or $2(3n^2 + 6n + 4)$ oe as final answer	3	B2 for 2 terms correct in final answer or for $(n + 2)^3 = n^3 + 6n^2 + 12n + 8$ or B1 for 1, 3, 3, 1 soi or SC2 for final answer of $3n^2 + 6n + 4$	B1 for $n^3 + 4n^2 + 4n + 2n^2 + 8n + 8\left[-n^3\right]$, condoning one error
				[3]		

Question			Answer	Marks	Guidance	
4	(i)		$23+\sqrt{2}$ as final answer	3 **[3]**	B2 for 23 and B1 for $\sqrt{2}$ or $1\sqrt{2}$ <u>or</u> M2 for 3 or more terms correct of $35-14\sqrt{2}+15\sqrt{2}-12$ or M1 for 2 terms correct	mark one scheme or other, but not a mixture, to advantage of candidate eg M2 for 35 + $\sqrt{2}$ + 24
4	(ii)		$5\sqrt{6}$ isw	2 **[2]**	condone $\frac{30}{\sqrt{6}}$ for 2 marks M1 for $\left[\sqrt{54}=\right]3\sqrt{6}$ or $\left[\frac{12}{\sqrt{6}}=\right]2\sqrt{6}$	eg 2 isw for $5\sqrt{6}=\sqrt{150}$

Question			Answer	Marks	Guidance	
5			$6(2x + 1) < 5(3x + 4)$	M1	for multiplying up correctly or for correct use of a common denominator	first three Ms may be earned with an equality condone omission of brackets only if then expanded as if brackets present
			$12x + 6 < 15x + 20$ or ft	M1	for expanding brackets correctly; for combined first two steps with one error, such as $12x + 6 < 15x + 4$, allow M1M0	eg $\frac{12x+6}{30} < \frac{15x+20}{30}$ oe earns M1M1
			$-14 < 3x$ or $-3x < 14$ or ft	M1	for collecting terms correctly	ft from two x terms and two constants
			$x > -\frac{14}{3}$ oe or ft isw	M1	for final division of their inequality with ax on one side, $a \neq 1$ or 0, and non-zero number on the other allow SC3 for $-14/3$ found without correct inequality symbol(s)	allow working with equality and making correct decision at end eg allow last M1 for $x > \frac{14}{-3}$ or $\frac{-14}{3} < x$ isw reminder : $(-14/3, \infty)$ is acceptable notation
			<u>or</u> $\frac{1}{5} - \frac{4}{6} < \frac{3x}{6} - \frac{2x}{5}$ oe	<u>or</u> M1		
			$\frac{-7}{15} < \frac{3x}{30}$ oe or ft	M2	M1 for one side correct ft	
			$x > -\frac{14}{3}$ oe or ft isw	M1	as in previous method	
				[4]		

Question			Answer	Marks	Guidance	
6			$4h + ha^2 = 9a - 5$	M1	correctly collecting h terms on one side, remaining terms on other	
			$h(4 + a^2) = 9a - 5$	M1	for factorising, ft eg sign error	
			$[h =]\dfrac{9a-5}{4+a^2}$ oe as final answer	M1	for division by their factor; ft only for equiv difficulty	M0 if seen and spoilt, eg by incorrect 'cancelling'
				[3]		
7	(i)		'tick' at (2,4)(3,1)(5,6)	2	mark intent M1 for two points correct or for 'tick' at (2,–2) (3,–5) and (5,0)	overlay to be provided condone tick unruled; allow M1 for points not joined but all correct:
				[2]		
7	(ii)		'tick' at (0,1)(1,–2)(3,3)	2	mark intent M1 for two points correct or for 'tick' at (4,1) (5,–2) and (7,3)	overlay to be provided condone tick unruled; allow M1 for points not joined but all correct:
				[2]		
8			$5(x + 1.5)^2 + 0.75$ oe www	4	B1 for $a = 5$ and B1 for $b = 3/2$ oe	condone omission of square symbol
					and B2 for $c = 3/4$ oe or M1 for $12 - 5 \times (\text{their } 3/2)^2$ oe soi or for $2.4 - (\text{their } 3/2)^2$ oe [eg 0.15] soi	eg $5[(x + 7.5)^2 - 7.5^2] + 12$ oe earns B1B0M1ft
			0.75 oe or ft their c	1	0 for (–1.5, 0.75)	condone found independently eg by differentiation
				[5]		

Question			Answer	Marks	Guidance	
9	(i)		'if n even then n^3 even, so $n^3 + 1$ odd' oe	B1	must mention n^3 is even or even^3 is even or even × even = even	0 for just 'if n is even, $n^3 + 1$ is odd' 0 if just examples of numbers used
			$\Leftarrow$ with if $n^3 + 1$ odd then n^3 even but if n^3 is even, n is not necessarily an integer <u>or</u> $\Leftrightarrow$ with '$n^3 + 1$ odd then n^3 even so n even', [assuming n is an integer]	B1	or '$\Leftrightarrow$ with if n is odd, n^3 is odd, so $n^3 + 1$ is even'	condone $\leftrightarrow$ instead of $\Leftrightarrow$ etc in both parts
				[2]	if 0 in question, allow SC1 for $\Leftrightarrow$ or $\Leftarrow$ and attempt at using general odd/even in explanation	must go further than restating the info in the qn; please annotate as SC
9	(ii)		showing $\Leftarrow$ is true	B1	eg when $x > 3$, +ve × +ve > 0	0 for just example(s) or for simply stating it is true
			$\Leftarrow$ chosen and showing that $\Rightarrow$ [and therefore $\Leftrightarrow$] is/ are not true	B1	stating that true when $x < 2$ or giving a counterexample such as 1, 0 or a negative number [to show quadratic inequality also true for this number]	0 for saying another solution $x > 2$
				[2]	allow B2 for $\Leftarrow$ and $x > 3$ and $x < 2$ shown/stated as soln or sketch showing two solns of $x^2 - 5x + 6 > 0$	or B1 for this argument with another symbol

Question			Answer	Marks	Guidance	
10	(i)		grad AB = $\frac{7-1}{4-2}$ oe or 3	M1		
			$y - 7$ = their m $(x - 4)$ or $y - 1$ = their m $(x - 2)$	M1	or use of y = their gradient $x + c$ with coords of A or B or M2 for $\frac{y-1}{7-1} = \frac{x-2}{4-2}$ o.e.	allow step methods used or eg M1 for $7 = 4m + c$ and $1 = 2m + c$ then M1 for correctly finding one of m and c
			$y = 3x - 5$ oe	A1 **[3]**	accept equivalents if simplified eg $3x - y = 5$ allow B3 for correct eqn www	allow A1 for $c = -5$ oe if $y = 3x + c$ oe already seen B2 for eg $y - 1 = 3(x - 2)$
10	(ii)		showing grad BC = $\frac{2-1}{-1-2} = -\frac{1}{3}$ oe	B1	may be calculation or showing on diagram	
			and $-1/3 \times 3 = -1$ or grad BC is neg reciprocal of grad AB, [so 90°]	B1	may be earned for statement / use of $m_1m_2 = -1$ oe, even if first B1 not earned for B1+B1, must be fully correct, with 3 as gradient in (i)	eg allow 2nd B1 for statement grad BC $= -1/3$ with no working if first B1 not earned
			<u>or</u> for finding AC or AC^2 independently of AB and BC	<u>or</u> B1	working needed such as $AC^2 = 5^2 + 5^2 = 50$	condone any confusion between squares and square roots etc for first B1 and for two M1s eg AC = 25 + 25 = $\sqrt{50}$
			for correctly showing $AC^2 = BC^2 + AB^2$ oe	B1	working needed using correct notation such as $BC^2 = 3^2 + 1^2 = 10$; $AB^2 = 6^2 + 2^2 = 40$, 40 + 10 = 50 [hence $AC^2 = BC^2 + AB^2$]	accept eg 3 and 1 shown on diagram and $BC^2 = 10$ etc 0 for eg $\sqrt{40} + \sqrt{10} = \sqrt{50}$

Question			Answer	Marks	Guidance	
			<u>or</u> finding equation of line through C perpendicular to AB ($y=-\frac{1}{3}x+\frac{5}{3}$ oe)	<u>or</u> B1		eg B1 for $x+3y=5$
			showing B is on this line either by substitution or finding intersection of this line with AB	B1		or B1 for finding the equation of the line through B and C as $y=-\frac{1}{3}x+\frac{5}{3}$ oe and B1 for using condition for perp lines and showing true
			BC = $\sqrt{3^2+1^2}$ or $\sqrt{10}$	M1		
			AB= $\sqrt{6^2+2^2}$ or $\sqrt{40}$ or $2\sqrt{10}$	M1	both these Ms may be earned earlier if Pythag used to show angle ABC = 90°, but are for BC and AB, not BC^2 and AB^2	for both M1s accept unsimplified equivs
			Area = 10 [square units]	A1	must be simplified to 10	
			<u>or</u> area under AC – area under AB – area under BC	<u>or</u> M1		mark equivalently for other valid methods, eg trapezium – 2 triangles method, omitting below $y = 1$: $\frac{1}{2} \times 7 \times 5 - (\frac{1}{2} \times 3 \times 1 + \frac{1}{2} \times 2 \times 6)$ $= 17.5 - (1.5 + 6)$
			at least two of 22.5, 8 and 4.5 oe	M1		
			Area = 10 [square units]	A1	must be simplified to 10	
				[5]		

Question			Answer	Marks	Guidance	
10	(iii)		(1.5, 4.5) oe	2	B1 each coordinate	
			angle in semicircle oe is a right-angle [so B is on circle] and must mention AC as diameter or D as centre [hence A, B, C all same distance from D]	E1 **[3]**	or '[since b = 90°,] ABC are three vertices of a rectangle. D is the midpoint of one diagonal <u>and</u> so D is the centre of the rectangle <u>or</u> the diagonals of a rectangle are equal and bisect each other, [hence DA=DB=DC] or condone showing that line from D to mid point of AB is perp to AB, so DBA is isos [hence DB = DA = DC] [or equiv using DBC]	E0 for just stating 'D is midpt of the hypotenuse of a rt angled triangle ABC so DAB is isos' without showing that it is isw eg wrong calcn of radius NB some wrongly asserting that ABC is isos

Question			Answer	Marks	Guidance	
11	(i)		f(–3) used	M1		
			–54 – 27 + 69 + 12 [= 0] isw	A1	or M1 for correct division by (x + 3) or for the quadratic factor found by inspection and A1 for concluding that x = –3 [is a root] (may be earned later)	A0 for concluding that x = –3 is a factor
			attempt at division by (x + 3) as far as $2x^3 + 6x^2$ in working	M1	or inspection with at least two terms of three–term quadratic factor correct; or at least one further root found using remainder theorem	
			correctly obtaining $2x^2 - 9x + 4$	A1	or stating further factor, found from using remainder theorem again	
			factorising the correct quadratic factor	M1	for factors giving two terms of quadratic correct or for factors ft one error in quadratic formula or completing square; M0 for formula etc without factors found	allow for (x – 4) and (x – ½) given as factors eg after using remainder theorem again or quadratic formula etc
			(2x – 1)(x – 4)[(x + 3)] isw	A1	allow 2(x – ½) instead of (2x – 1), oe condone inclusion of '= 0'	isw (x – ½) as factor and/or roots found, even if stated as factors
				[6]		

Question			Answer	Marks	Guidance	
11	(ii)		sketch of cubic right way up, with two turning points	B1	0 if stops at x-axis ignore graph of $y = 4x + 12$	must not be ruled; no curving back (except condone between $x = 0$ and $x = 0.5$); condone some 'flicking out' at ends but not approaching more turning points; must continue beyond axes; allow max on y axis or in 1st or 2nd quadrants condone some doubling / feathering
			values of intns on x axis shown, correct (–3, 0.5 and 4) or ft from their factors or roots in (i)	B1	on graph or nearby in this part mark intent for intersections with both axes	allow if no graph condone 3 on neg x axis as slip for –3; condone eg 0.5 roughly halfway between their 0 and 1 marked on x axis
			12 marked on y-axis	B1 **[3]**	or $x = 0$, $y = 12$ seen in this part if consistent with graph drawn	allow if no graph, but eg B0 for graph with intn on –ve y-axis or nowhere near their indicated 12
11	(iii)		$2x^3 - 3x^2 - 23x + 12 = 4x + 12$ oe	M1	or ft their factorised f(x)	
			$2x^3 - 3x^2 - 27x$ [= 0]	A1	after equating, allow A1 for cancelling ($x + 3$) factor on both sides and obtaining $2x^2 - 9x$ [= 0]	condone slip of '= y' instead of '= 0'
			[x]($2x - 9$)($x + 3$) [= 0]	M1	for linear factors of correct cubic, giving two terms correct or for quadratic formula or completing square used on correct quadratic $2x^2 - 3x - 27 = 0$, condoning one error in formula etc;	or after cancelling ($x + 3$) factor allow M1 for $x(2x - 9)$ oe or obtaining $x = 0$ or 9/2 oe M0 for eg quadratic formula used on cubic, unless recovery and all 3 roots given
			[x =] 0, –3 and 9/2 oe	A1 **[4]**	need not be all stated together	eg $x = 0$ may be earlier

Question			Answer	Marks	Guidance	
12	(i)		$\sqrt{20}$ isw or $2\sqrt{5}$ (2, 0)	B1 B1 **[2]**	0 for $\pm\sqrt{20}$	
12	(ii)		subst of $x = 0$ into circle eqn soi $y = \pm 4$ oe sketch of circle with centre (2, 0) or ft their centre from (i)	M1 A1 B1 **[3]**	or Pythag used on sketch of circle: $2^2 + y^2 = 20$ oe or B2 for just $y = \pm 4$ seen oe; accept both 4 and –4 shown on y axis on sketch if both values not stated if the centre is not marked, it should look roughly correct by eye – coords need not be given on sketch; condone intersections with axes not marked	M0 for just $y^2 = 20$; M1 for $y^2 = 16$ or for $y = 4$ ignore intns with x-axis also found circle should intersect both +ve and neg x- and y-axes; must be clear attempt at circle; ignore any tangents drawn
12	(iii)		$(x - 2)^2 + (2x + k)^2 = 20$ $x^2 - 4x + 4 + 4x^2 + 4kx + k^2 = 20$ $5x^2 + (4k - 4)x + k^2 - 16 = 0$	M1 M1 dep A1 **[3]**	for attempt to subst $2x + k$ for y for correct expansion of at least one set of brackets, dependent on first M1 correct completion to given answer; dependent on both Ms	allow for attempt to subst $k = y - 2x$ into given eqn similarly for those working backwards condone omission of further interim step if both sets of brackets expanded correctly, but for cands working backwards, at least one interim step is needed; if cands have made an error and tried to correct it, corrections must be complete to award this A mark

Question			Answer	Marks	Guidance	
12	(iv)		$b^2 - 4ac = 0$ seen or used	M1	need not be substituted into; may be stated after formula used or argument towards expressing eqn as a perfect square	eg M1 for $(4k - 4)^2 - 4 \times 5 \times (k^2 - 16) = 0$
			$4k^2 + 32k - 336$ [= 0] or $k^2 + 8k - 84$ [= 0]	M1	expansion and collection of terms, condoning one error ft their $b^2 - 4ac$	dep on an attempt at $b^2 - 4ac$ with at least two of a, b and c correct; may be earned with < 0 etc; may be in formula
			use of factorising or quadratic formula or completing square	M1	condone one error ft	dep on attempt at obtaining required quadratic equation in k, not for use with any eqn/inequality they have tried
			$k = 6$ or -14 <u>or</u> Grad of tgt is 2, and normal passes through centre, hence finding equation of normal as $y = -\frac{1}{2}x + 1$ oe	A1 <u>or</u> M1		
			finding x values where diameter $y = -x/2 + 1$ intersects circle as $x = 6$ or -2 (condone one error in method)	M1	oe for y values; condone one error in method	or finding intn of tgt and normal as $\left(\frac{2-2k}{5}, \frac{k+4}{5}\right)$
			finding corresponding y values on circle and subst into $y = 2x + k$ or subst their x values into $5x^2 + (4k - 4)x + k^2 - 16 = 0$	M1	intns are (6, –2) and (–2, 2), M0 for just (6, 2) and (–2, –2) used but condone used as well as correct intns this last method gives extra values for k, for the non-tangent lines $y =$ through (6, 2) and (–2, –2), but allow for the M mark	or subst their intn of tgt and normal into eqn of circle: $\left(\frac{2-2k}{5} - 2\right)^2 + \left(\frac{k+4}{5}\right)^2 = 20$ or ft
			$k = 6$ or -14	A1 **[4]**	and no other values	

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4751 Introduction to Advanced Mathematics

General Comments

Most candidates were able to answer most of the questions in Section A competently, with the exception of question 9 where relatively few scored both marks in either part. In section B, the questions proved to be accessible with many candidates attempting every part; however, question 12(iv) caused difficulties for most candidates as did the last part of question 10(iii). Examiners felt that those who did not attempt question 12(iv) did so because they did not know how to proceed rather than because they had run out of time.

Candidates' arithmetic with negative numbers and fractions was often poor, affecting in particular their work in questions 10(i) and 8 respectively.

A number of candidates used additional pages, with the most common questions for 'overflow' or second attempts being questions 10(ii) and 11(i).

Comments on Individual Questions

Section A

1 Most candidates knew how to find the equation of the perpendicular line. The main errors were using the same gradient of 5 or using the incorrect perpendicular gradient $\frac{1}{5}$. A few failed to write the equation in the requested form.

2 This was done well by most candidates. A minority did not deal correctly with the negative fractional index in part (i). In part (ii) numbers other than the correct 32 were sometimes seen, as was x^7 instead of x^{10}.

3 Most candidates preferred multiplying out to using the binomial expansion. The special case in the mark scheme for an answer of $3n^2 + 6n + 4$ had significant usage, from candidates treating an expression as if it was an equation. Some forgot after their cubing to subtract the n^3 as the question required. A few candidates thought that $(n + 2)^3 = n^3 + 8$.

4 Many candidates gained two of the three marks in part (i), but incorrectly dealing with $3\sqrt{2} \times -2\sqrt{2}$ was the usual error in the expansion.

In part (ii) many got as far as $\sqrt{54} = 3\sqrt{6}$ but were unable to relate the second term to this. A common error was to multiply everything by $\sqrt{6}$.

5 Solving the inequality was done well by many candidates. Some lost the final mark, usually those whose previous line was $-3x < 14$ rather than collecting the x terms on the right and obtaining $-14 < 3x$. The follow-through provisions in the mark scheme were much used and enabled almost all candidates to obtain some marks, as mistakes in handling the fractions and expanding brackets were common.

6 This rearrangement was done well by many candidates. Almost all attempted to collect terms, some making a sign error in the process. A minority did not then factorise the h terms. A few obtained the correct answer and then attempted to simplify it further and lost the final mark.

7 The translations were almost always correct. A few went down instead of up, or right instead of left, and received partial credit.

8 This question demonstrated again that most candidates have difficulties with completing the square. Most obtained a mark for 5 and many a mark for $\frac{3}{2}$, but only a minority had + 0.75 or $+\frac{3}{4}$. Arithmetic errors were common, as were lack of brackets resulting in their $\left(\frac{3}{2}\right)^2$ not being multiplied by 5; sometimes brackets were used correctly and a method mark obtained but then the candidate omitted to multiply this term by 5. The final mark was sometimes obtained as a follow through mark, but many candidates gave the coordinates of the minimum point instead of the minimum value of *y* asked for, and others omitted this part of the question.

9 Few candidates obtained full marks on this question. Some omitted crucial albeit simple step(s) in an argument and a common fault in part (i) was to write, for example, 'If *n* is an even integer $n^3 + 1$ is an odd number' without explaining why this was so. Those who solved the inequality correctly in part (ii) then had only to select the correct symbol to obtain both marks but only a minority achieved this. In both parts, counterexamples were used effectively by some candidates; however, many thought it was sufficient to use one or two numerical examples when it was not.

Section B

10 (i) Finding the equation of the line through A and B was completed successfully by the majority of candidates. The main error seen was a sign error, either in working out the gradient (negative divided by negative given as negative instead of positive), or in expanding brackets and collecting terms.

10 (ii) Most tried to use gradients to show ABC was a right-angle. Many just stated 'grad BC = $-\frac{1}{3}$' without showing the calculation. The $m_1m_2 = -1$ rule was well used on the whole, although not always explicitly stated, with some just saying that 3 and $-\frac{1}{3}$ were perpendicular gradients. Those using Pythagoras to show angle ABC = 90° were often successful, but some lost a mark due to incorrect notation and / or lack of convincing steps, with $\sqrt{40}+\sqrt{10}=\sqrt{50}$ being seen on a number of scripts. A small number of candidates successfully found the equation of a line perpendicular to AB that went through C and then confirmed that B lay on this line. Some candidates worked very hard for their two marks, unnecessarily finding equations as they had not spotted the more direct methods. These more long-winded approaches were variable in terms of accuracy.

For the area, many correctly found the lengths needed but failed to simplify the surds to obtain 10. The alternative method of a rectangle minus three triangles was seen very occasionally.

10 (iii) Most found the mid-point of AC correctly but failed to score the final mark, with some omitting an explanation. Most successful explanations involved showing ABC was in a semi-circle, but many of these attempts did not mention diameter or semi-circle and were not sufficiently clear to score the mark. Some successfully showed that the right-angled triangle formed half of a rectangle with D as the centre and hence the same distance from A, B and C. The most common explanation was stating that 'D was the midpoint of the hypotenuse of a right angled triangle' (or words to that effect), which did not score. Weak attempts included the assumption that ABC was isosceles or that BD was the perpendicular bisector of AC or that A, B and C were three corners of a square.

11 (i) Factorising the cubic was generally done very well. For the first demand, some did not use f(–3) but divided successfully, although such candidates did not always conclude that finding the factor $(x + 3)$ meant that $x = -3$ was a root. The division, whether by long division or inspection, was generally done well. Candidates seem well-practised in this technique. When the quadratic had been arrived at correctly, the majority of candidates successfully found the two linear factors, although some using the formula failed to express the factors, hence losing the final two marks. The factor theorem was occasionally used to find the remaining factors, but generally did not lead to all factors being found. Some candidates confused the ideas of roots and factors.

11 (ii) Sketching the cubic was well done by most candidates. A few forgot to show the *y*-intercept but most knew the correct shape and used their roots to show intersections.

11 (iii) Most knew they had to equate the linear and cubic expressions for *y* and usually simplified to the correct cubic equation. Many were then unsure how to solve this. Many lost the $x = 0$ root by dividing by x, although these candidates often found the other two roots successfully. Some candidates started with the factorised form of $f(x)$ and divided both sides by $x + 3$; many of these lost the $x = -3$ root. Some tried to use the quadratic formula on the cubic equation.

12 (i) The majority of candidates were able to write down the centre and radius successfully. Some radii were given as 20 instead of $\sqrt{20}$, and some centres as (–2 , 0) or (0 , 2) instead of (2 , 0).

12 (ii) A significant minority of candidates forgot to find the negative square root when solving $y^2 = 16$ and so only found one intersection, but on the whole this was well done. Some also found where the circle cut the x-axis. Most sketched correctly, showing the *y*-intercepts found and their centre was correctly placed. However, a significant number of candidates took little care over their sketches, with many "circles" drawn poorly.

12 (iii) Candidates who substituted $y = 2x + k$ into the equation of the circle were generally very successful, with only a few minor slips. However, candidates who decided to work backwards from the given result usually struggled.

12 (iv) Many candidates did not know where to start, and the full four marks were rarely awarded. About a quarter of the candidates did not attempt the question and those that did make an attempt often substituted $x = 2$ or $x = 0$ at the start. Some successfully used $b^2 - 4ac = 0$ and found the correct values of k but many made errors, particularly taking c as -16 instead of $k^2 - 16$.

Some candidates found the equation of the normal, although few made further progress with this approach. A few candidates offered solutions using the gradient of the normal and finding the intersections with the circle by using a vector approach from the centre – a neat approach which usually scored full marks.