OCR
Oxford Cambridge and RSA

Wednesday 24 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4773/01 Decision Mathematics Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

Turn over

1 Colin holidays in a rural mountain villa. Each night he sits on the patio with his coffee and looks out over the plain below. There is not much activity, and his attention is often drawn to a distant crossroads where there are 4 street lamps. The lamps each have the same type of fault, and each frequently switches itself off. Colin amuses himself by noting, each time he looks at them, how many lamps are on.

Each lamp, independently of the others, is randomly on for 75% of the time and off for 25% of the time. Colin glances at the lamps occasionally. When he glances he never sees a lamp change state from off to on, or vice versa, so the number of lamps that are on is always well-defined.

(i) Construct a simulation model to show the number of lamps which are on when Colin glances at them. **[3]**

(ii) Run your simulation 100 times and produce estimates of the probabilities of the number of lamps which are on when Colin glances at them. **[2]**

(iii) Repeat the run in part **(ii)** 19 more times. For each of your 20 runs, note how many glances it is until Colin sees fewer than 2 lamps on. Hence estimate his mean number of glances until he sees fewer than 2 lamps on. **[2]**

(iv) Use your 20 runs to compute an estimate of how many runs will be needed to estimate, with 95% confidence, the mean number of glances, to within an accuracy of ± half a glance. **[3]**

A villager, not an electrician, tries to make the lamps work better. This has the following consequences.

- Lamp number 1 is unaffected. It is still on, randomly, for 75% of the time, and off for 25% of the time.
- If lamp number 1 is off then lamp number 2 behaves as before: on for 75% of the time and off for 25% of the time. If lamp number 1 is on then lamp number 2 is also on.
- If either or both of lamps 1 and 2 are off then lamp number 3 behaves as before: on for 75% of the time and off for 25% of the time. If lamps 1 and 2 are both on then lamp number 3 is also on.
- If any of lamps 1, 2 and 3 are off then lamp number 4 behaves as before: on for 75% of the time and off for 25% of the time. If lamps 1, 2 and 3 are all on then lamp number 4 is also on.

(v) Construct a simulation model to show the number of lamps which are now on when Colin glances at them. **[6]**

(vi) Run your simulation 100 times and produce estimates of the probabilities of the number of lamps now on when Colin glances at them. **[2]**

2 Jean-Claude owns a clothing shop specialising in outdoor clothing. His unisex waterproof jacket is very popular. It comes in three sizes: small, medium and large. He purchases jackets twice a year for delivery to the shop, at the start of the summer season and at the start of the winter season. Because of varying demand, the prices he pays for the jackets, and the prices at which he sells the jackets, vary from season to season. They are shown below.

£	summer		winter	
	buy	sell	buy	sell
small	25	48	30	50
medium	30	54	35	57
large	35	58	40	62

Jean-Claude can keep up to 10 of each size of jacket on display. In addition he can keep up to 100 more jackets (of any size) in his storeroom.

Jean-Claude is planning his purchasing policy for the next year, starting with summer. He anticipates that his sales will be the same as this year. These are shown in the table.

number of jackets	summer	winter
small	20	30
medium	25	35
large	20	25

Jean-Claude has 7 small jackets, 0 medium jackets and 10 large jackets in stock at the end of this winter. He aims to have 10 of each size in stock at the end of next winter.

Jean-Claude wants to maximise his profit from buying and selling jackets during the next year.

(i) Formulate Jean-Claude's problem as an LP problem. **[12]**

(ii) Run your LP and interpret the results. Give Jean-Claude's optimal profit on jackets. **[5]**

(iii) Using only information in the question, criticise Jean-Claude's assumption that sales will be the same next year as this year. **[1]**

3 Angela is the stock controller for a local supplier of grain. Each week she buys grain from a wholesaler in units of 1000 tonnes, and stores it in her company's storage facility. Customers are supplied from these company stores.

She places an order at the end of each week, using a formula developed by consultants. It defines the order quantity to be the number of units sold in the week plus 0.144×(20 − stock at the end of the week). This aims for a stock level of 20.

Orders are delivered 3 weeks after they are placed. So, for example, the order placed at the end of week 0 arrives at the end of week 3, and the order placed at the end of week 3 is calculated after this delivery arrives.

(i) Assuming sales are constant, explain why this system can be modelled by the recurrence relation $u_n = u_{n-1} + 0.144(20 - u_{n-3})$ for $n \geqslant 3$. **[3]**

(ii) At the end of week 0 ($n = 0$), stock was 19 (after sales and after the delivery of the earlier order from three weeks earlier). When $n = 1$ stock was 18, and when $n = 2$ it was 23. Assuming that sales are constant, build a spreadsheet to show how stock levels vary over a period of 52 weeks. **[2]**

(iii) Extend your spreadsheet to demonstrate that the formula below models the stock levels.

$$u_n = -33.63\left(\frac{3}{5}\right)^n + 27.32\left(\frac{1+\sqrt{7}}{5}\right)^n + 5.31\left(\frac{1-\sqrt{7}}{5}\right)^n + 20$$ **[3]**

(iv) Use a spreadsheet to investigate parameter values for Angela to use other than the consultants' value of 0.144. Find a better value. **[4]**

(v) In fact, sales are not constant. Produce a spreadsheet to show how stock varies over a 10-week period when using the consultants' formula with parameter value 0.144, with the same starting values, and with sales as shown in the table below. **[2]**

week	0	1	2	3	4	5	6	7	8	9
sales (000 tonnes)	5.2	7.6	5.1	5.2	10.7	8.1	4.0	6.8	5.8	6.3

(vi) Repeat part **(v)** using your parameter value from part **(iv)**, and compare the two outcomes. Explain the basis of your comparison. **[4]**

4 A farmer has four fields, labelled 1, 2, 3 and 4, on which he grows three crops, labelled A, B and C. He plans his production over a four-year period. Each year he chooses which crop to grow in each of three of the fields, and he leaves the other field to lie fallow, i.e. without any crop. This fallow period allows a field to regain its fertility.

This year field 1 will lie fallow. Next year field 2 will lie fallow, as it is currently three years since it was last fallow. Field 3 will follow, as it is currently two years since it was last fallow. Field 4 will lie fallow in the fourth year.

Fertility varies from field to field, and with the time since the field was last fallow. Also different crops respond differently to different levels of fertility. The effects of this are summarised in the tables below. They show profits in units of £1000.

The first year after lying fallow			
	crop A	crop B	crop C
field 1	10	7	8
field 2	5	6	4
field 3	5	7	6
field 4	8	8	9

The second year after lying fallow			
	crop A	crop B	crop C
field 1	9	7	7
field 2	5	5	4
field 3	5	6	5
field 4	7	7	8

The third year after lying fallow			
	crop A	crop B	crop C
field 1	6	5	5
field 2	4	4	4
field 3	3	3	3
field 4	5	6	5

(i) Formulate an LP to maximise the farmer's profits over a four-year period, given that

- he wishes to grow each crop in each year, and
- he wishes to grow each crop in each field over the four years. **[10]**

(ii) Run your LP. **[1]**

(iii) Give the most profitable planting plan, and the associated total profit. **[3]**

(iv) How much profit will he make if he removes the constraint on growing each crop in each field over the four years? **[2]**

(v) How much profit will he make if he also removes the constraint on growing each crop in each year? **[2]**

END OF QUESTION PAPER