Monday 16 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Evaluate $\int_0^{\frac{1}{6}\pi}(1-\sin 3x)\,dx$, giving your answer in exact form. **[3]**

2 Find the exact gradient of the curve $y=\ln(1-\cos 2x)$ at the point with x-coordinate $\frac{1}{6}\pi$. **[5]**

3 Solve the equation $|3-2x|=4|x|$. **[4]**

4 Fig. 4 shows the curve $y=f(x)$, where

$$f(x)=a+\cos bx,\ 0\leqslant x\leqslant 2\pi,$$

and a and b are positive constants. The curve has stationary points at (0, 3) and $(2\pi, 1)$.

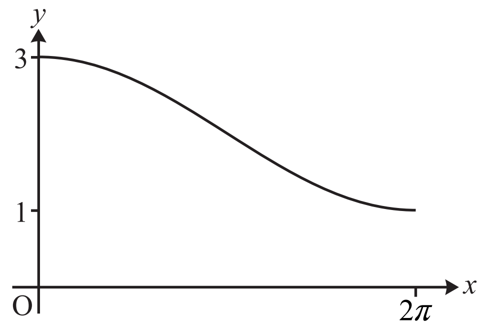

Fig. 4

(i) Find a and b. **[2]**

(ii) Find $f^{-1}(x)$, and state its domain and range. **[5]**

5 A spherical balloon of radius r cm has volume $V\,\text{cm}^3$, where $V=\frac{4}{3}\pi r^3$. The balloon is inflated at a constant rate of $10\,\text{cm}^3\,\text{s}^{-1}$. Find the rate of increase of r when $r=8$. **[5]**

6 The value £V of a car t years after it is new is modelled by the equation $V=Ae^{-kt}$, where A and k are positive constants which depend on the make and model of the car.

(i) Brian buys a new sports car. Its value is modelled by the equation

$$V=20\,000\,e^{-0.2t}.$$

Calculate how much value, to the nearest £100, this car has lost after 1 year. **[2]**

(ii) At the same time as Brian buys his car, Kate buys a new hatchback for £15000. Her car loses £2000 of its value in the first year. Show that, for Kate's car, $k=0.143$ correct to 3 significant figures. **[3]**

(iii) Find how long it is before Brian's and Kate's cars have the same value. **[3]**

7 Either prove or disprove each of the following statements.

(i) 'If m and n are consecutive odd numbers, then at least one of m and n is a prime number.' **[2]**

(ii) 'If m and n are consecutive even numbers, then mn is divisible by 8.' **[2]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{x}{\sqrt{2+x^2}}$.

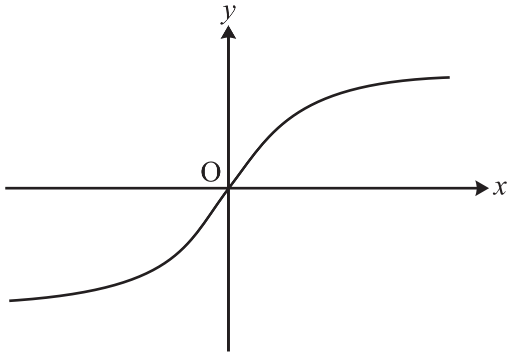

Fig. 8

(i) Show algebraically that $\mathrm{f}(x)$ is an odd function. Interpret this result geometrically. **[3]**

(ii) Show that $\mathrm{f}'(x) = \dfrac{2}{(2+x^2)^{\frac{3}{2}}}$. Hence find the exact gradient of the curve at the origin. **[5]**

(iii) Find the exact area of the region bounded by the curve, the x-axis and the line $x = 1$. **[4]**

(iv) (A) Show that if $y = \dfrac{x}{\sqrt{2+x^2}}$, then $\dfrac{1}{y^2} = \dfrac{2}{x^2} + 1$. **[2]**

(B) Differentiate $\dfrac{1}{y^2} = \dfrac{2}{x^2} + 1$ implicitly to show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2y^3}{x^3}$. Explain why this expression cannot be used to find the gradient of the curve at the origin. **[4]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = xe^{-2x}$ together with the straight line $y = mx$, where m is a constant, with $0 < m < 1$. The curve and the line meet at O and P. The dashed line is the tangent at P.

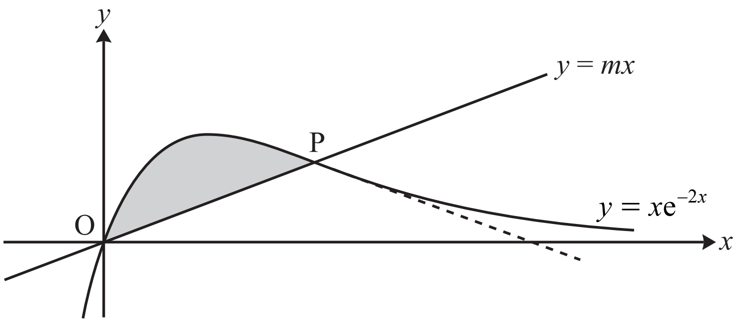

Fig. 9

(i) Show that the x-coordinate of P is $-\frac{1}{2}\ln m$. **[3]**

(ii) Find, in terms of m, the gradient of the tangent to the curve at P. **[4]**

You are given that OP and this tangent are equally inclined to the x-axis.

(iii) Show that $m = e^{-2}$, and find the exact coordinates of P. **[4]**

(iv) Find the exact area of the shaded region between the line OP and the curve. **[7]**

END OF QUESTION PAPER

OCR

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4753/01 Methods for Advanced Mathematics (C3)

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1

2

3

4 (i)

4 (ii)

5

6 (i)	
6 (ii)	

6 (iii)

7 (i)	
7 (ii)	

Section B (36 marks)

8 (i)	

8 (ii)	

8 (iii)	

8(iv)(*A*)	
8(iv)(*B*)	

9 (i)	
9 (ii)	
	(answer space continued on next page)

9 (ii)	(continued)
9 (iii)	

9 (iv)	

Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4753**: Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2014

1. Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

±

Question			Answer	Marks	Guidance	
1			$\int_0^{\pi/6}(1-\sin 3x)\,dx = \left[x+\frac{1}{3}\cos 3x\right]_0^{\pi/6}$	M1	±1/3 cos 3x seen or $\int \frac{1}{3}(1-\sin u)[du]$	i.e. condone sign error
				A1	$\left[x+\frac{1}{3}\cos 3x\right]$ or $\left[\frac{1}{3}(u+\cos u)\right]$	condone '+ c'
			= π/6 – 1/3	A1cao	o.e., must be exact	isw after correct answer seen
				[3]		
2			$y = \ln(1 - \cos 2x)$, let $u = 1 - \cos 2x$	M1	$1/(1 - \cos 2x)$ soi	
			⇒ $dy/dx = dy/du \,.\, du/dx$			
			$= (1/u).\ 2\sin 2x$	M1	$d/dx\ (1 - \cos 2x) = \pm\ 2\sin 2x$	
			$= \dfrac{2\sin 2x}{1-\cos 2x}$	A1cao		
			When $x = \pi/6$, $\dfrac{dy}{dx} = \dfrac{2\sin(\pi/3)}{1-\cos(\pi/3)}$	M1	substituting π/6 or 30° into their deriv	must be in at least two places
			$= 2\sqrt{3}$	A1cao		isw after correct answer seen
				[5]		
3			$\|3 - 2x\| = 4\|x\|$			
			⇒ $3 - 2x = 4x$, $x = ½$	M1A1		If 3 or more final answers offered, −1 for each incorrect additional answer
			or $3 - 2x = -4x$, $x = -1½$	M1A1	not 3/(−2)	−1 for final ans written as an inequality
			or			
			$(3 - 2x)^2 = 16x^2$	M1	squaring both sides	$(3 - 2x)^2 = 4x^2$ is M0
			⇒ $12x^2 + 12x - 9$ [= 0]	A1	correct quadratic o.e. but with single x^2 term	
			⇒ $x = ½, -1½$	A1 A1		
				[4]		

Question			Answer	Marks	Guidance	
4	(i)		$a = 2$, $b = ½$	B1B1 **[2]**		
4	(ii)		$y = 2 + \cos ½x \quad x \leftrightarrow y$ $x = 2 + \cos ½y$ $\Rightarrow \quad x - 2 = \cos ½y$ $\Rightarrow \quad \arccos(x - 2) = ½y$ $\Rightarrow \quad y = f^{-1}(x) = 2\arccos(x - 2)$ Domain $1 \le x \le 3$ Range $0 \le y \le 2\pi$	 M1 M1 A1 M1 A1 **[5]**	(may be seen later) subtracting [their] a from both sides (first) arccos(x –[their] a) = [their] $b \times y$ cao or $2\cos^{-1}(x - 2)$ domain 1 to 3, range 0 to 2π correctly specified: must be $\le$, x for domain, y or f^{-1} or $f^{-1}(x)$ for range	 need not substitute for a, b or with $x \leftrightarrow y$, need not subst for a, b may be implied by flow diagram if not stated, assume first is domain allow [1, 3], [0, 2π] not 360° (not f)
5			$dV/dr = 4\pi r^2$ $dV/dt = 10$ $dV/dt = (dV/dr)(dr/dt)$ $\Rightarrow \quad 10 = 4\pi.64.dr/dt$ $\Rightarrow \quad dr/dt = 0.0124$ cm s^{-1}	B1 B1 M1 A1 A1 **[5]**	or $12\pi r^2/3$, condone dr/dV, dV/dR a correct chain rule soi o.e. (soi) must be correct 0.012 or better or $10/256\pi$ or $5/128\pi$	 Condone use of other letters for t o.e. e.g. $dr/dt = (dr/dV)(dV/dt)$ mark final answer

Question			Answer	Marks	Guidance	
6	(i)		$V = 20000e^{-0.2t}$ when $t = 1$, $V = 16374.615…$ so car loses (£)3600	B1 B1 **[2]**	(soi) art 16400 condone no £, must be to nearest £100	 or B2 for correct answer
6	(ii)		When $t = 1$, $V = 13000$ $\Rightarrow$ $13000 = 15000\ e^{-k}$ $\Rightarrow$ $-k$ [ln e] = ln(13000/15000) $\Rightarrow k =$ 0.1431… = 0.143 (3sf) *	M1 M1 A1 **[3]**	 taking lns correctly oe e.g. ln 13000 = ln15000–k [lne] cao **NB AG** must show some working if 4th d.p. not shown	If $k = 0.143$ verified ,e.g. $15000\ e^{-0.143} = 13001[.31…]$, SCB1 need not have substituted for V and A e.g. $k = -\ln(13000/15000) = 0.143$
6	(iii)		$15000e^{-0.143t} = 20000e^{-0.2t}$ $\Rightarrow$ $(15000/20000) = e^{(0.143-0.2)t}$ $\Rightarrow$ $t = \ln 0.75 / -0.057 = 5.05$ years so after 5 years	M1* M1dep A1 **[3]**	must be correct, but could use a more accurate value for k dep * cao accept answers in the range 5 − 5.1	If M0, SCB1 for 5 – 5.1 years from correct calculations for each car, rot e.g. $t = 5$, £7358 (Brian), £7338(Kate) or (£7334 with more accurate k) o.e. e.g. ln15000–0.143t = ln 20000− 0.2t
7	(i)		False e.g. neither 25 and 27 are prime as 25 is div by 5 and 27 by 3	B1 B1 **[2]**	correct counter-example identified justified correctly	Need not explicitly say ‘false’
7	(ii)		True: one has factor of 2, the other 4, so product must have factor of 8.	B2 **[2]**	or algebraic proofs: e.g. $2n(2n+2) = 4n(n+1) = 4\times\text{even}\times\text{odd}$ no so div by 8	B1 for stating with justification div by 4 e.g. both even, or from $4(n^2 + n)$ or $4pq$

Question			Answer	Marks	Guidance	
8	(i)		$f(-x)=\frac{-x}{\sqrt{2+(-x)^2}}$	M1	substituting $-x$ for x in $f(x)$	$\frac{-x}{\sqrt{2+-x^2}},\frac{-x}{\sqrt{2+-(x^2)}},\frac{-x}{\sqrt{2+(-x^2)}}$ M1A0
			$=-\frac{x}{\sqrt{2+x^2}}=-f(x)$	A1	1st line must be shown, must have $f(-x)=-f(x)$ oe somewhere	$\frac{-x}{\sqrt{2-x^2}}$ M0A0
			Rotational symmetry of order 2 about O	B1	must have 'rotate' and 'O' and 'order 2 or 180 or ½ turn'	oe e.g. reflections in both x- and y-axes
				[3]		
8	(ii)		$f'(x)=\frac{\sqrt{2+x^2}.1-x.\frac{1}{2}(2+x^2)^{-1/2}.2x}{(\sqrt{2+x^2})^2}$	M1 M1 A1	quotient or product rule used ½ $u^{-1/2}$ or – ½ $v^{-3/2}$ soi correct expression	QR: condone $udv \pm vdu$, but u, v and denom must be correct $x(-1/2)(2+x^2)^{-3/2}.2x+(2+x^2)^{-1/2}$.
			$=\frac{2+x^2-x^2}{(2+x^2)^{3/2}}=\frac{2}{(2+x^2)^{3/2}}$ *	A1	**NB AG**	$=(2+x^2)^{-3/2}(-x^2+2+x^2)$
			When $x=0$, $f'(x)=2/2^{3/2}=1/\sqrt{2}$	B1	oe e.g. $\sqrt{2}/2$, $2^{-1/2}$, $1/2^{1/2}$, but not $2/2^{3/2}$	allow isw on these seen
				[5]		
8	(iii)		$A=\int_0^1\frac{x}{\sqrt{2+x^2}}[\mathrm{d}x]$	B1	correct integral and limits	limits may be inferred from subsequent working, condone no dx
			let $u=2+x^2$, $\mathrm{d}u=2x\,\mathrm{d}x$		or $v=\sqrt{(2+x^2)}$, $\mathrm{d}v=x(2+x^2)^{-1/2}\mathrm{d}x$	
			$=\int_2^3\frac{1}{2}\frac{1}{\sqrt{u}}\mathrm{d}u$	M1	$\int\frac{1}{2}\frac{1}{\sqrt{u}}[\mathrm{d}u]$ or $=\int 1[\mathrm{d}v]$ or $k(2+x^2)^{1/2}$	condone no du or dv, but not $\int\frac{1}{2}\frac{1}{\sqrt{u}}\mathrm{d}x$
			$=\left[u^{1/2}\right]_2^3$	A1	$[u^{1/2}]$ o.e. (but not $1/u^{-1/2}$) or $[v]$ or $k=1$	
			$=\sqrt{3}-\sqrt{2}$	A1cao	must be exact	isw approximations
				[4]		

Question			Answer	Marks	Guidance	
8	(iv)	(*A*)	$y^2 = \dfrac{x^2}{2+x^2}$ $\Rightarrow$ $1/y^2 = (2 + x^2)/x^2 = 2/x^2 + 1$ *	M1 A1 **[2]**	squaring (correctly) or equivalent algebra **NB AG**	must show $\left[\sqrt{(2+x^2)}\right]^2 + 2 + x^2$ (o.e.) If argued backwards from given result without error, SCB1
8	(iv)	(*B*)	$-2y^{-3}\text{d}y/\text{d}x = -4x^{-3}$ $\Rightarrow$ $\text{d}y/\text{d}x = -4x^{-3}/-2y^{-3} = 2y^3/x^3$ * Not possible to substitute $x = 0$ and $y = 0$ into this expression	B1B1 B1 B1 **[4]**	LHS, RHS **NB AG** soi (e.g. mention of 0/0)	condone $\text{d}y/\text{d}x\ -2y^{-3}$ unless pursued Condone 'can't substitute $x = 0$' o.e. (i.e. need not mention $y = 0$). Condone also 'division by 0 is infinite'
9	(i)		$xe^{-2x} = mx$ $\Rightarrow$ $e^{-2x} = m$ $\Rightarrow$ $-2x = \ln m$ $\Rightarrow$ $x = -½ \ln m$ * **or** If $x = -½ \ln m$, $y = -½ \ln m \times e^{\ln m}$ $= -½ \ln m \times m$ so P lies on $y = mx$	M1 M1 A1 M1 A1 A1 **[3]**	may be implied from 2nd line dividing by x, or subtracting $\ln x$ **NB AG** substituting correctly	o.e. e.g. $[\ln x] - 2x = \ln m + [\ln x]$ or factorising: $x(e^{-2x} - m) = 0$
9	(ii)		let $u = x$, $u' = 1$, $v = e^{-2x}$, $v' = -2e^{-2x}$ $\text{d}y/\text{d}x = e^{-2x} - 2xe^{-2x}$ $= e^{-2.(-\frac{1}{2}\ln m)} - 2.(-\frac{1}{2}\ln m)e^{-2.(-\frac{1}{2}\ln m)}$ $= e^{\ln m} + e^{\ln m}\ln m$ $[= m + m\ln m]$	M1* A1 M1dep A1cao **[4]**	product rule consistent with their derivs o.e. correct expression subst $x = -½ \ln m$ into their deriv dep M1* condone $e^{\ln m}$ not simplified	but not $-2(-½ \ln m)$, but mark final ans

Question			Answer	Marks	Guidance	
9	(iii)		$m + m\ln m = -m$	M1	their gradient from (ii) $= -m$	
			$\Rightarrow \quad \ln m = -2$			
			$\Rightarrow \quad m = e^{-2}$ *	A1	**NB AG**	
			or			
			$y + \frac{1}{2}m\ln m = m(1 + \ln m)(x + \frac{1}{2}\ln m)$ $x = -\ln m$, $y = 0 \Rightarrow \frac{1}{2}m\ln m = m(1+\ln m)(-\frac{1}{2}\ln m)$	B2	for fully correct methods finding x-intercept of equation of tangent and equating to $-\ln m$	
			$\Rightarrow 1 + \ln m = -1$, $\ln m = -2$, $m = e^{-2}$			
			At P, $x = 1$	B1		
			$\Rightarrow \quad y = e^{-2}$	B1	isw approximations	not $e^{-2} \times 1$
				[4]		
9	(iv)		Area under curve $= \int_0^1 xe^{-2x}\,dx$			
			$u = x$, $u' = 1$, $v' = e^{-2x}$, $v = -\frac{1}{2}e^{-2x}$	M1	parts, condone $v = k\,e^{-2x}$, provided it is used consistently in their parts formula	ignore limits until 3rd A1
			$= \left[-\frac{1}{2}xe^{-2x}\right]_0^1 + \int_0^1 \frac{1}{2}e^{-2x}\,dx$	A1ft	ft their v	
			$= \left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^1$	A1	$-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$ o.e	
			$= (-\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2}) - (0 - \frac{1}{4}e^{0})$ $[= \frac{1}{4} - \frac{3}{4}e^{-2}]$	A1	correct expression	need not be simplified
			Area of triangle = ½ base × height	M1	ft their 1, e^{-2} or $[e^{-2}x^2/2]$	o.e. using isosceles triangle
			$= \frac{1}{2} \times 1 \times e^{-2}$	A1		M1 may be implied from 0.067…
			So area enclosed $= \frac{1}{4} - 5e^{-2}/4$	A1cao	o.e. must be exact, two terms only	isw
				[7]		

4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments:

This paper appeared to be broadly in line with recent papers in the level of difficulty of the questions and variability of responses. All candidates appeared to have time to complete the paper. Section A was often very well done, with plenty of straightforward tests of understanding. The two Section B questions had some more demanding parts which tested more able candidates. In general most candidates seem well prepared, and there were no obvious areas of the syllabus which were weaker than others.

The standard of presentation was as usual extremely variable, ranging from very well argued, well written solutions to others which were difficult to follow. Candidates should be advised that presenting logical mathematical arguments, rather than disjointed train-of-thought attempts, is likely to encourage accurate work. Many candidates required additional sheets to answer questions, usually caused by multiple attempts. Sometimes, their final attempt gained fewer marks than preceding solutions – candidates should be advised that their final solution, rather than the best, will be marked.

One issue worth highlighting from this paper is that of exact answers. Often the use of calculators encourages inexact solutions, but if the question asks for an exact answer (as was the case quite often in this paper) it should not be approximated. Sometimes we can ignore subsequent working after an exact answer is seen.

Comments on Individual Questions:

1 This question proved to be a straightforward starter for 3 marks. Sign errors in integrating sin 3x occurred occasionally, as did $-3\cos 3x$ instead of $-1/3 \cos 3x$.

2 Plenty of candidates scored 5 marks here with little difficulty. Some missed out the derivative of $1-\cos 2x$, and some wrote $1/2\sin 2x$ instead of $1/(1-\cos 2x)$. The substitution of $\pi/6$ into the correct derivative was usually done correctly. Some approximation of $2\sqrt{3}$ was found, but could usually be condoned by ignoring subsequent working.

3 Although plenty of candidates scored full marks with apparent ease, there were all sorts of errors as well. Some clearly do not understand the modulus function; many duplicate work by solving 4 equations from $\pm(3 - 2x) = \pm 4x$, and in the process produced additional solutions due to poor algebra. A surprisingly common error was to write $3 = 6x \Rightarrow x = 2$! Some even discounted the solution $x = -3/2$ on the grounds that answers to a modulus question need to be positive! Squaring both sides was seen occasionally, and although this method is somewhat long-winded, it does avoid conceptual errors such as $|3 - 2x| = 3 + 2x$.

4 (i) For some, this was a write-down for 2 marks. Some used transformation arguments, other substituted in particular coordinates. The most common errors were $a = 3$ and $b = 2$ or ¼ .

4 (ii) We allowed plenty of follow-through marks for the first two method marks here, which were almost always gained. Fully correct inverse functions were common. As for domain and range, most seem to know that the these are the reverse of the domain and range for f. However, the 'A' mark here demanded accurate use of notation, with '*x*' used for domain and '*y*' for range, and this mark was often lost.

5 This question proved to be accessible to the overwhelming majority of candidates, and there were many fully correct solutions. Even those who failed to get full marks usually picked up an M1 for a correctly stated chain rule, B1 for dV/dr = 4pr^2, and a B1 for dV/dt = 10. Approximate answers are perhaps preferable in a contextual question, but exact answers were also allowed.

6 Exponential growth and decay is well understood by most candidates, and this was a high scoring question.

6 (i) This was a very accessible two marks, provided candidates answered question – the loss in value rounded to the nearest £100.

6 (ii) Again, this was very well answered. Occasionally the final 'A1' was missed by skipping straight from $-k$ = ln(13/15) to k = 0.143: as this is a given answer, some additional working was required. Occasionally, the result was verified by substituting k = 0.143 and evaluating $15000e^{-0.143}$. This was treated as a special case and got 1 mark only. It is important candidates know the difference between 'show' and 'verify'.

6 (iii) This was a little more demanding, requiring candidates to combine the $e^{-0.2}$ and $e^{-0.143}$ terms. This defeated quite a few candidates. Some listed the values of each car for t = 1, 2, 3, etc years, and if successful picked out t = 5 as when they were closest. This was judged to be worth 1 mark only.

7 This proved to be quite an effective test of understanding of proof and disproof, albeit for only 4 marks.

7 (i) Most candidates stated this was false and looked for a counter-example, usually 25 and 27. We did require them to show their counter-examples were composite for a second B mark.

7 (ii) This was less successful. Most candidates could see it was true, but then failed to come up with a coherent argument. Some wrote $2n(2n+2)= 4n^2 + 4n$ or equivalent, but then failed to explain why this is then divisible by 8 (rather than just 4, which got 1 out of 2). Most successful candidates got the idea that alternate even numbers are divisible by 4 and hence the product of this with another even number is divisible by 8.

8 (i) Most candidates stated that for an odd function f(-x) = −f(x) or equivalent. It is important when writing f(−x) that brackets are placed round the $-x$ terms: if these were missing, the 'A' mark was lost. The structure of this 'show' was often a bit 'muddy': f(−x) = … = … = −f(x) is clear, but writing f(−x) = −f(x) and then writing expressions for each side of this equation below and showing they are equal is less so, as the direction of the argument, or implications, is not clear. The geometrical description of an odd function required three elements: 'rotational', 'order 2' and 'centre O' or equivalent; reflection in Ox followed by Oy was also allowed.

8 (ii) The difficulty with this sort of product or quotient rule question lies in factorising and hence simplifying the expression, and this was the case here. Many wrote down correct expressions, but then failed to 'show' the printed answer. This difficulty often encouraged multiple attempts, sometimes using a quotient rule, followed by a product rule, etc. A surprising number of candidates muddled up their 'u' and 'v' and quotient and product rule, for example using $v = (2+x^2)^{-1/2}$ in their quotient rule. Often the final answer failed to score because we insisted on this being simplified to $1/\sqrt{2}$ or equivalent.

8 (iii) A substantial minority of candidates thought this integral should be done by parts, and therefore scored nothing after the first B1. Those who tried substituting often got muddled before arriving at $\int 1/2\sqrt{u}\, du$, and some then integrated this incorrectly, e.g as $\ln\sqrt{u}$.

8 (iv) This simple piece of algebra was often over-complicated by round-the-houses methods. An all- too-commonly seen mistake was $x^2/(2+x^2) = x^2/2 + 1$.

8 (v) The implicit differentiation was usually correct, as was the algebra to arrive at the printed result. The exact logic behind why $x = 0$ and $y = 0$ could not be substituted into the result expression was often faulty (for example many stated the result would be zero or infinite); we condoned this provided they stated the idea that division by zero is undefined or not possible.

9 (i) This was well answered by the majority of candidates.

9 (ii) The product rule here was generally well done, followed by substituting $x = -\frac{1}{2}\ln m$, where some sign errors occurred. Some left the $e^{\ln m}$ terms unresolved, which was condoned here. The main error was to get a derivative of $-2xe^{-2x}$.

9 (iii) The first two marks here were the least successfully answered, because most candidates were not familiar with the fact that lines equally inclined to the *x*-axis have gradients *m* and $-m$. Only the best candidates found the result successfully. However, many recovered to find the coordinates of P correctly.

9 (iv) This question tested the more able candidates. The integration by parts required careful control of negative signs and accurate work; the area of the triangle (or integral of the line) were quite often discernable from the working, which was often scrambled and incoherent.

Unit level raw mark and UMS grade boundaries June 2014 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	61	56	51	46	42	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	57	51	45	39	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	68	61	54	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	60	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	57	51	45	39	34	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	55	48	42	36	30	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	48	41	34	28	22	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	60	53	46	40	34	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	54	47	41	35	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	51	46	41	36	31	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	46	41	36	31	26	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	54	48	43	38	32	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0