OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS (MEI)
Decision Mathematics Computation

4773

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Friday 24 June 2011
Afternoon

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 Five friends on a skiing holiday arrive late for lunch at an alpine refuge. Only five portions of food are left:

- one portion of macaroni cheese (a pasta dish with cheese sauce – suitable for vegetarians)
- one portion of crespelle (a pancake with ham and cheese)
- one portion of vegetable lasagne (pasta – suitable for vegetarians)
- two portions of spaghetti carbonara (spaghetti with a creamy meat sauce)

Arthur and Bertie are both vegetarians. Charles will eat anything that is not vegetarian. David would like the lasagne. Edward wants spaghetti.

(i) Represent this information on a bipartite graph, and give the number of maximal matchings that are available (counting the two portions of spaghetti as different). **[4]**

(ii) Draw a separate bipartite graph showing a maximal matching in which Bertie gets no lunch. **[1]**

(iii) By considering all possible alternating paths starting with the vertex representing Bertie, prove that there is not a complete matching. **[3]**

(iv) Formulate the problem as an LP using variables such as Cc, which is to take the value 1 if Charles is allocated the crespelle, and 0 if not. Run your LP and interpret the results. **[8]**

(v) By drawing an appropriate network, show how the problem could be set up as a network flow problem. You are not required to solve your network problem. **[2]**

2 A number of investments are available to an investor. Their starting times, durations and returns are listed in the table. Thus, for example, investment number 4 requires money to be deposited in one year's time (time = 1) and will return £109 in three years' time (time = 3) for every £100 invested.

Investment number	1	2	3	4	5	6	7	8
Start	0	0	1	1	2	2	3	4
End	1	2	2	3	3	4	5	5
Return	4%	10%	4%	9%	4%	10%	12%	4%

The investor has £10 000 to invest over the next five years.

A financial analyst formulates this investment problem as an LP.

(i) Complete the following set of inequalities in the analyst's formulation (all variables are non-negative):

$x1 + x2 \leqslant 10000$
$x3 + x4 \leqslant 1.04x1$
$x5 + x6 \leqslant 1.10x2 + 1.04x3$
...
... **[5]**

(ii) Convert the inequalities into a form suitable for submission to LINDO. **[1]**

(iii) Give a suitable objective function. **[3]**

(iv) Run your LP and interpret the results. **[4]**

(v) Another analyst observes that, for this problem, there are five possible investment strategies. Identify them and pick the best. **[4]**

(vi) Why use LP to solve problems of this type? **[1]**

3 Keith has had a tooth removed and has been told that he must take an antibiotic. Each tablet contains 250 mg of antibiotic, and he should take 4 tablets a day.

The instructions supplied with the packet indicate the proportion of the antibiotic in the body which will be excreted over a given period. Keith knows that the rate of excretion will be proportional to the amount of drug in the body. From the information he deduces that, if $x(t)$ mg is the amount of antibiotic in his body at time t hours, then an appropriate model is $x(t) = 0.89^t x(0)$, provided that no tablet is taken between time 0 and time t.

Let u_n be the amount of antibiotic in mg in Keith's body immediately after he has taken the nth tablet, so that $u_1 = 250$.

(i) Produce a recurrence relation for u_{n+1} in terms of u_n, given that Keith takes a tablet every 6 hours. **[2]**

(ii) Solve your recurrence relation and find the limiting value of u_n as n increases. **[4]**

(iii) Draw a sketch graph showing the amount of antibiotic in Keith's body over a 24-hour period after he has been taking his tablets for a long time, given that he takes a tablet every 6 hours. You should accurately label relevant amounts of antibiotic on your graph. **[3]**

In fact, Keith does not disturb his sleep to take a tablet, so the intervals between him taking tablets during a day are 5 hours, 5 hours, 5 hours and 9 hours.

(iv) Construct a spreadsheet to show the amount of antibiotic in Keith's body immediately after he takes each of his 28 tablets. **[4]**

(v) Draw a sketch graph showing the amount of antibiotic in Keith's body over a 24-hour period after he has been taking his tablets for a long time, given that he takes tablets at intervals of 5, 5, 5 and 9 hours each day. You should accurately label relevant amounts of antibiotic on your graph. **[3]**

(vi) After he has been taking the tablets for a long time, Keith forgets to take his bedtime tablet. Find the amount of antibiotic in his body when his next tablet is due.
Should he take two tablets at this time? **[2]**

4 The management of an alpine refuge has a difficult task in planning how many meals of different types to provide at lunchtime. The number of customers is dependent on the weather. The approximate distribution of the number of customers is given in the table.

Number of customers	5	10	15	20
Probability	0.35	0.15	0.15	0.35

The refuge provides four different lunch dishes:

- macaroni cheese
- crespelle
- vegetable lasagne
- spaghetti carbonara

Customers choose these dishes in the following proportions:

Dish	macaroni	crespelle	lasagne	spaghetti
Probability	0.3	0.1	0.2	0.4

(i) Construct a spreadsheet to simulate the demand for each dish during one lunchtime. You need to simulate the number of customers, and then for each customer you need to simulate the dish chosen by that customer. You are advised to try to arrange your simulation on one line of your spreadsheet. **[9]**

(ii) Repeat your simulation 100 times and record the number of times each dish is chosen on each day. For each dish find the mean and standard deviation of the number chosen on each day. **[5]**

(iii) Explain why it is that standard deviations are relevant here rather than standard errors. **[2]**

(iv) How would you advise the management on planning the provision? **[2]**

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GCE

Mathematics (MEI)

Advanced GCE

Unit **4773:** Decision Mathematics Computation

Mark Scheme for June 2011

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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Subject-specific Marking Instructions for GCE Mathematics (MEI) Decision strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

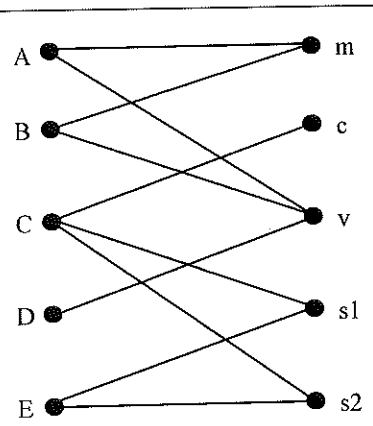
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

1.

(i)

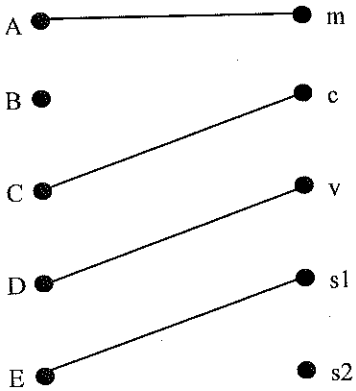


Number of maximal matchings = 4×4 = 16
(note 2 separate graphs)

M1 bipartite
A1 A B D / m v
A1 C E / c s1 s2

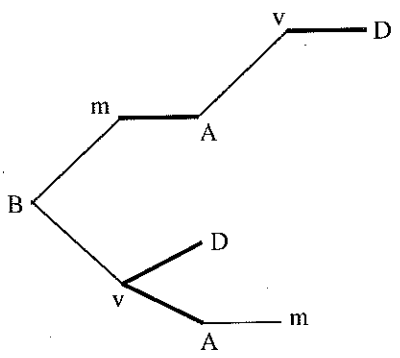
B1 cao

(ii) eg (note ... 4 solutions)



B1

(iii)



B1 first branch
B1 second branch
B1 third branch

(iv) e.g.

```
Max  Am+Av+Bm+Bv+Cc+Cs1+Cs2+Dv+Es1+Es2
st   Am+Av<1
     Bm+Bv<1
     Cc+Cs1+Cs2<1
     Dv<1
     Es1+Es2<1
     Am+Bm<1
     Cc<1
     Av+Bv+Dv<1
     Cs1+Es1<1
     Cs2+Es2<1
end
```

M1 A1

M1 people constraints
A1

M1 meal constraints
A1

e.g.

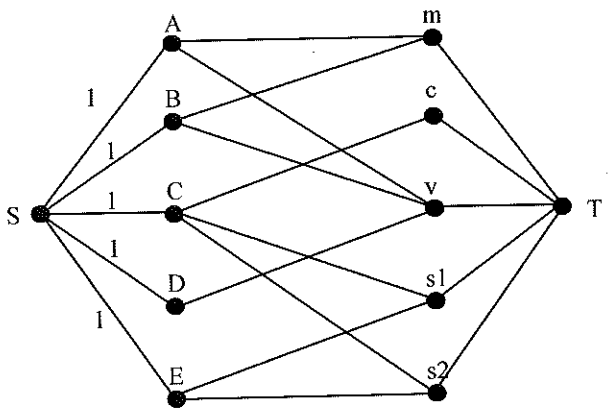
```
LP OPTIMUM FOUND AT STEP     6
OBJECTIVE FUNCTION VALUE
        1)        4.000000
VARIABLE      VALUE          REDUCED COST
      AM      1.000000       0.000000
      AV      0.000000       0.000000
      BM      0.000000       0.000000
      BV      0.000000       0.000000
      CC      0.000000       0.000000
     CS1      1.000000       0.000000
     CS2      0.000000       0.000000
      DV      1.000000       0.000000
     ES1      0.000000       0.000000
     ES2      1.000000       0.000000
```

M1 running

Arthur has the macaroni
Bertie goes hungry
Charles has the first spaghetti
David has the vegetable lasagne
Edward has the second spaghetti

A1

(v)



B1 source and sink with bipartite connection

B1 arc capacities out of S (or into T).

2.

(i)	$x1 + x2 \leq 10000$ $x3 + x4 \leq 1.04x1$ $x5 + x6 \leq 1.10x2 + 1.04x3$ x7 $\leq$ 1.09x4 + 1.04x5 x8 $\leq$ 1.10x6 (Might see extra terms for x4 and x5 feeding to x8.)	B1 x7 $\leq$ B1 1.09x4 B1 1.04x5 B1 x8 $\leq$ B1 1.10x6
(ii)	x3 + x4 –1.04x1 < 0 x5 + x6 - 1.10x2 - 1.04x3 < 0 x7 – 1.09x4 – 1.04x5 < 0 x8 – 1.10x6 < 0	B1
(iii)	max 1.12x7 + 1.04x8	B1 max B1 B1 (-1 each error)
(iv)	LP OPTIMUM FOUND AT STEP 5 OBJECTIVE FUNCTION VALUE 1) 12812.80 VARIABLE VALUE X7 11440.000000 X8 0.000000 X1 0.000000 X2 10000.000000 X3 0.000000 X4 0.000000 X5 11000.000000 X6 0.000000	B1 running
	Invest all money in investment 2, thence to investment number 5, thence to investment number 7. Gives 12812.80 at end.	M1 words! A1 strategy B1 outcome
(v)	1→3→5→7 1.04×1.04×1.04×1.12 ≈ 1.26 1→3→6→8 1.04×1.04×1.10×1.04 ≈ 1.24 1→4→7 1.04×1.09×1.12 ≈ 1.27 2→5→7 1.10×1.04×1.12 ≈ 1.28 2→6→8 1.10×1.10×1.04 ≈ 1.26	M1 A1 those starting "1" A1 those starting "2"
	Best is as above	A1
(vi)	Formulation + solution would be more easily automated for larger problems.	B1

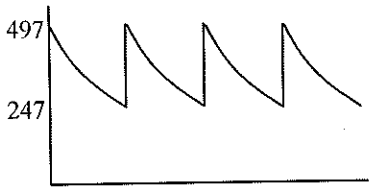
3.

(i) $u_{n+1} = 0.89^6 u_n + 250$ — B1 structure; B1 0.89^6

(ii) $u_n = 250\dfrac{1-0.89^{6n}}{1-0.89^6}$ — M1; A1 0.89^{6n}; A1 rest OK

Tends to $\dfrac{250}{1-0.89^6} \approx 497$ — B1 limit

(iii)



M1 shape; A1 497; A1 247

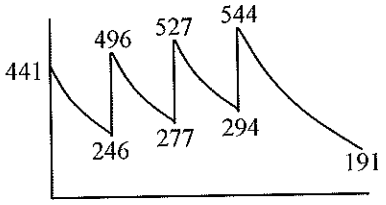
(iv)

1	250.00
2	389.60
3	467.56
4	511.09
5	429.06
6	489.59
7	523.39
8	542.26

21	440.70
22	496.09
23	527.02
24	544.29
25	440.70
26	496.09
27	527.02
28	544.29

M1 5 hour rules; A1; M1 9 hour rule; A1

(v)



B1 shape; B1 upper values; B1 lower values

(vi) Level drops to 103mg — B1 cao

No. Converges back to previous levels more quickly with single dose. — B1 analysis needed

4.

(i)	Lookup tables, or equivalent	M1 A1
	simulating ...	B1
	number of customers	B1
	a dish for a customer	B1
	simulating the correct number of dishes	M1 A1
	collecting results	M1 A1
(ii)	use of relative and absolute addressing	M1 A1
	100 repetitions	B1
	means and standard deviations	B1 B1
(iii)	Not concerned with the variation of the mean, but with the variation of an individual day's demand.	B1 B1
(iv)	Analyse the cost of waste against the cost of loss of goodwill and decide upon an appropriate level of provision, e.g. mean + 1 standard deviation.	B1
	the above would give, arguably, 6, 2, 4 and 8 respectively.	B1

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4773: Decision Mathematics Computation

General Comments

Performances were varied. The paper requires good modelling skills above all else. It also helps to have good communication skills. Candidates who produce a ream of output for the examiner are missing this point a little.

Comments on Individual Questions

1 **Networks**

This question was answered well by most candidates. They were able to draw the graph correctly in part (i). Few were able to find the correct number of maximal matchings, but that was difficult and it had no consequences.

In part (iii) not all candidates understood what a set of alternating paths should look like, and that the same vertex could appear in more than one path. Few drew all three paths.

The LP in (iv) was generally well done, although the full interpretation was often either weak or missing.

Very few candidates attempted the network flow modelling in part (v).

2 **LP Modelling**

Part (i) was generally done well. Some candidates followed the look rather than the meaning of the constraints for earlier years, and thus had x7+x8 on the left hand side of a new constraint.

Most were able to re-arrange in part (ii) and knew that they were maximising, but a significant proportion tried to maximise the sum of all of the variables. This led, in part (iv), to attempts to interpret a solution which did not make sense. In part (v) not all candidates were able to find all 5 strategies and evaluate them correctly in order to choose the best.

Very few candidates scored the final mark in part (vi).

3 **Recurrence Relations**

This was a poorly done question by most candidates. There were problems with producing the recurrence relation in part (i), and only a few candidates were able correctly to produce the solution in part (ii).

Many candidates only printed the numbers from their spreadsheet in part (iv). The rubric states that candidates should show their working, and that requires a sample of the formulae which are used.

The final advice on not taking a double dose was often given without clear analysis or justification.

4 **Simulation**

Many candidates were able to set up appropriate lookup tables and the correct simulation of customers was usually seen.

Only some followed the advice to simulate across rather than down the spreadsheet.

The choosing of dishes was sometimes not modelled by using a random variable, and the total meals prepared did not always correspond to the number of customers.

All of the above led to difficulty in creating the 100 rows needed for part (ii). Part (iii) was poorly answered and advice in (iv) rarely came with clear reasoning.