OCR
RECOGNISING ACHIEVEMENT

Monday 25 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4773 Decision Mathematics Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 8 page Answer Booklet (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

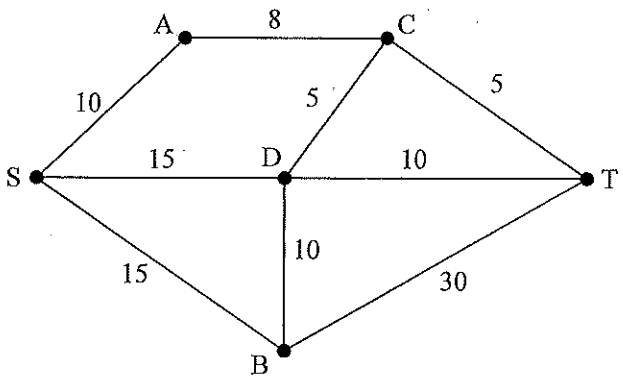
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 The diagram shows a network of pipes. Along pipes connected directly to the source, S, flows can only be away from S. Along pipes connected directly to the sink, T, flows can only be towards T. In all other pipes, flows can be in either direction. The number by each pipe is the flow capacity of that pipe.



(i) A flow has been established as follows: 15 from S to D, 5 from D to C, 5 from C to T, and 10 from D to T. Use the labelling algorithm to show this flow on a copy of the diagram, ensuring that all arcs are labelled. **[3]**

(ii) Find and identify a flow augmenting path. Augment the flow as much as is possible using this path, stating by how much you are augmenting. Show your augmented flow on a new diagram, with all arcs labelled. **[4]**

(iii) Repeat part **(ii)** as many times as is necessary to achieve a maximal flow. Produce a new, labelled, diagram for each flow augmentation. **[2]**

(iv) Prove that your final flow is maximal. **[2]**

(v) Formulate the maximal flow problem as an LP, ignoring the initial flow which was given in part **(i)**. Run your LP on LINDO, and interpret the results. **[7]**

2 The data in the table below relate to production possibilities for a mining company over a three-year planning cycle. Initially only mines A and B are open.

		Mine A	Mine B	Mine C	Mine D
Material availability (thousands of tonnes over the 3 years)		650	950	625	840
Material quality		66.0	64.1	63.0	62.1
Cost of mining (£ per tonne)	yr 1	30	28	–	–
	yr 2	33	31	27	28
	yr 3	35	33	29	30
Cost to open (£)		–	–	800 000	1 400 000

The company requires a production plan for the next 3 years satisfying the following:

- Requirements are for 600 000 tonnes of material in the first year, 850 000 tonnes of material in the second year, and 750 000 tonnes of material in the third year.
- The mean quality of the material mined in any year must not be less than 64.0.
- Mines C and/or D can be opened after the first year at the costs given. If either is opened for year 2, then it will be open for year 3 at no additional cost.
- Material cannot be stored from one year to the next.

Formulate, run and interpret a (mixed) integer LP to find the minimum cost 3-year production strategy.

[18]

3 A country's central bank has a committee which meets each month to set the bank rate, the interest rate at which the central bank lends to commercial banks. This is its tool for controlling the rate of inflation. The target is to achieve an inflation rate of 2.5%.

The committee does not realise that it takes two months for an adjustment to have an effect. Thus, if the committee puts up the bank rate by one percent in June, then the rate of inflation will rise by one percent, but not until August. By that time the committee will have had its July meeting, and will have made another change to the base rate.

(i) In June the rate of inflation was 3.74% and the committee put the bank rate down by 1.24% (since $2.5 - 3.74 = -1.24$). In May, the committee had put the bank rate down by 0.81%, so in July the rate of inflation fell from 3.74% to 2.93%. For the months up to and including December, give the rates of inflation, and the committee's actions.

Describe what will happen thereafter. **[3]**

(ii) The committee might have been better advised to be more cautious in its approach. Construct a spreadsheet column to show what would happen over a year, from the same starting values (inflation rate of 3.74% in June and 2.93% in July), if the committee subtracted the current rate of inflation from the target rate of inflation, and then applied an adjustment equal to half of this.

Produce another spreadsheet column to show what would happen over the year if, instead, the committee subtracted the current rate of inflation from the target rate of inflation, and then applied an adjustment equal to one eighth of this.

In each case you should describe the behaviour of your sequence of monthly inflation rates. **[4]**

(iii) Explain why, in the 'one eighth' case, the recurrence relation

$$u_{n+2} - u_{n+1} + 0.125u_n = 0.3125$$

models the sequence of inflation rates. **[1]**

(iv) Given that a particular solution is $u_n = 2.5$ (with $n=0$ for June), solve the recurrence relation

$$u_{n+2} - u_{n+1} + 0.125u_n = 0.3125.$$

(If you compute numerical values, then give 4 decimal place accuracy.) **[5]**

(v) Use your spreadsheet to check your answer to part **(iv)**, and explain why your answer shows that u_n converges to 2.5. **[2]**

(vi) In more general terms, if instead of using a half or one eighth, the committee uses α, where $0 < \alpha < 1$, then the recurrence relation for u_n is $u_{n+2} - u_{n+1} + \alpha u_n = 2.5\alpha$. By considering the auxiliary equation, explain why $\alpha = 0.25$ might be a good value to use.

Construct a spreadsheet column to show what would happen over a year using $\alpha = 0.25$, and comment. **[3]**

4 Arnold and Juan are to play a round of golf. This consists of 18 holes. At each hole they each try to get their ball in the hole in the least number of strokes.

There are two methods of scoring. In strokeplay the player with the lower total number of strokes over the 18 holes is the winner of the round.

In matchplay each hole is competed for. If one player uses fewer strokes for a hole, then he wins the hole. If the number of strokes is equal then they 'halve' the hole. At the end of the 18 holes if either player has won more than 9 holes, counting each 'half' as 0.5 of a hole, then he wins the round, otherwise it is a draw.

Each hole has associated with it a 'par', the expected number of strokes needed for the hole. At the course where Arnold and Juan are playing there are 4 holes with a par of 3, 10 holes with a par of 4 and 4 holes with a par of 5.

Playing records show that Arnold and Juan play to the same standards – they both achieve par on average – but that Juan is more consistent than Arnold. An analysis of their records gives the following probability distributions for the number of strokes taken per hole.

	Arnold					Juan				
par 3	1	2	3	4	5	1	2	3	4	5
	0.01	0.19	0.62	0.15	0.03	0.00	0.12	0.80	0.04	0.04
par 4	2	3	4	5	6	2	3	4	5	6
	0.06	0.15	0.60	0.11	0.08	0.02	0.11	0.77	0.05	0.05
par 5	3	4	5	6	7	3	4	5	6	7
	0.09	0.14	0.55	0.12	0.10	0.05	0.10	0.71	0.08	0.06

(i) Construct a spreadsheet to simulate the outcomes of each of a par 3, a par 4 and a par 5 hole when Arnold and Juan are playing each other. Use the matchplay scoring method for each of the three holes, and summarise your results in a table. **[10]**

(ii) Simulate a round of golf played between Arnold and Juan, with 4 par 3 holes, 10 par 4 holes and 4 par 5 holes. Ensure that you print out a summary of your simulated round of golf, showing the results for each hole. Determine who is the winner using each of strokeplay and matchplay. **[4]**

(iii) Simulate a further 19 rounds using both scoring methods. Summarise your results in a table. You do not need to show the results for each hole, only for each round. **[2]**

(iv) By repeating your simulations, decide whether or not one player has an advantage over the other using strokeplay, and whether or not one player has an advantage over the other using matchplay. **[2]**

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GCE

Mathematics (MEI)

Advanced GCE

Unit **4773:** Decision Mathematics Computation

Mark Scheme for June 2012

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

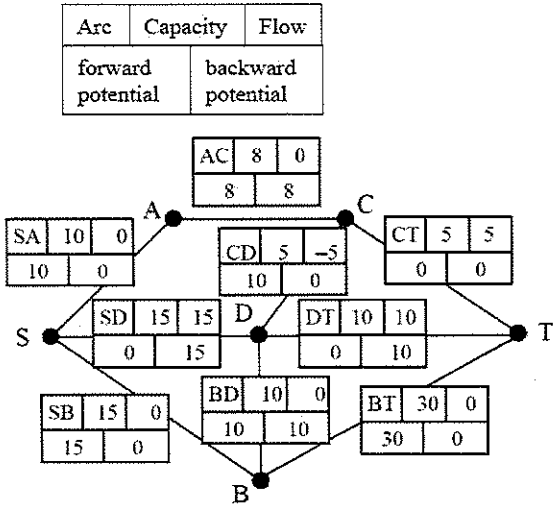
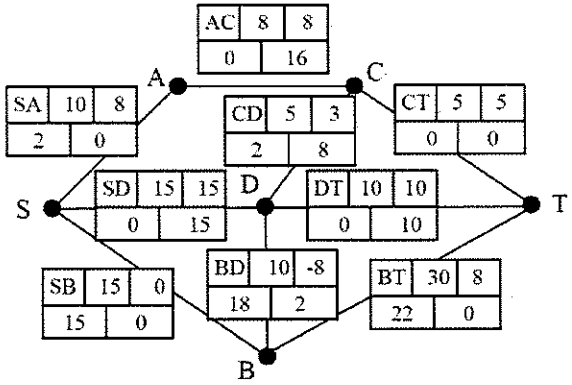
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

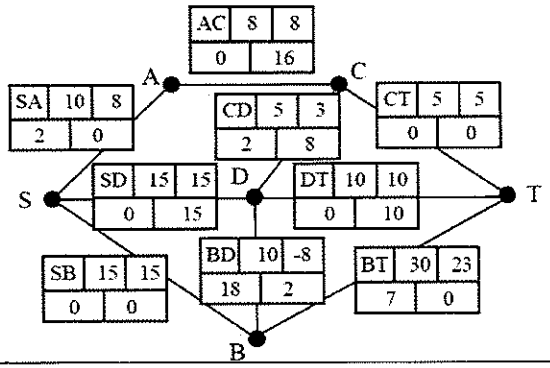
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		Key: (the forward direction is defined by the order of the letters on the arc name) 	B1 B1 FT B1 FT [3]	capacities and flows forward potentials backward potentials
1	(ii)		e.g. flow-augmenting path ... SACDBT (8) 	B1 B1 M1 A1 [4]	

Question			Answer	Marks	Guidance
1	(iii)		e.g. flow-augmenting path ... SBT (15)  	B1 B1 [2]	
1	(iv)		Flow is now 38 But cut SA/BCDT has capacity 38 So flow is maximal	M1 A1 [2]	max flow/min cut cao

Question			Answer	Marks	Guidance
1	(v)		e.g. max SA + SD + SB	B1	objective
			st SA + CA – AC = 0		
			AC + DC – CA – CD – CT = 0	M1	4 balancing equations
			SD + CD + BD – DC – DB – DT = 0	A1	
			SB + DB – BD – BT = 0		
			SA < 10		
			SD < 15		
			SB < 15		
			AC < 8		
			CA < 8	M1	capacities + back capacities
			CD < 5	A1	
			DC < 5		
			BD < 10		
			DB < 10		
			CT < 5		
			DT < 10		
			BT < 30		
			end		
			LP OPTIMUM FOUND AT STEP 2		
			OBJECTIVE FUNCTION VALUE		
			1) 38.00000		
			VARIABLE VALUE		
			SA 8.000000		
			SD 15.000000		
			SB 15.000000		
			CA 0.000000		
			AC 8.000000		
			DC 0.000000	B1	running
			CD 5.000000		
			CT 3.000000		
			BD 0.000000		
			DB 10.000000		
			DT 10.000000		
			BT 25.000000		
			Different solution ... CT down 2	B1	interpretation
			CD, DB, BT up 2 ... with same flow	[7]	

Question	Answer	Marks	Guidance
2	e.g. min 30000xa1 + 33000xa2 + 35000xa3 + 28000xb1 + 31000xb2 + 33000xb3 + 27000xc2 + 29000xc3 + 28000xd2 + 30000xd3 + 800000c + 1400000d	M1 A1 M1 A1	mining opening
	st xa – xa1 – xa2 – xa3 = 0 xb – xb1 – xb2 – xb3 = 0 xc – xc2 – xc3 = 0 xd – xd2 – xd3 = 0 xa1 + xb1 > 600 xa2 + xb2 + xc2 + xd2 > 850 xa3 + xb3 + xc3 + xd3 > 750 xc2 – 1000c2 < 0 xc3 – 1000c3 < 0 c2 – c3 < 0 xd2 – 1000d2 < 0 xd3 – 1000d3 < 0 d2 – d3 < 0 c2 + c3 – 10c < 0 d2 + d3 – 10d < 0 2xa2 + 0.1xb2 – xc2 – 1.9xd2 > 0 2xa3 + 0.1xb3 – xc3 – 1.9xd3 > 0 xa < 650 xb < 950 xc < 625 xd < 840 Note that indicator switching is over-modelled here. Costs of opening a mine do not vary from year 2 to year 3, and are payable only once. So only one indicator is needed for each of C and D. Simpler modelling allowed, see below.	 M1 A1 M1 A1 A1 M1 A1 M1 A1	 requirements indicator switching for C for D quality availabilties
	end int c2 int c3 int d2 int d3 int c int d	B1	integer variables

Question	Answer	Marks	Guidance
	OBJECTIVE FUNCTION VALUE 1) 0.6745000E+08 VARIABLE VALUE C2 1.000000 C3 1.000000 D2 0.000000 D3 0.000000 C 1.000000 D 0.000000 XA1 0.000000 XA2 375.000000 XA3 250.000000 XB1 600.000000 XB2 350.000000 XB3 0.000000 XC2 125.000000 XC3 500.000000 XD2 0.000000 XD3 0.000000 XA 625.000000 XB 950.000000 XC 625.000000 XD 0.000000 Interpretation needed: cost (£67.5m) what is produced where and when what is opened	B1 B1 B1 B1	submitting cost (cao) 8 production location

Question			Answer	Marks	Guidance
			Simpler LP formulation. min 30000A1 + 33000A2 + 35000A3 + 28000B1 + 31000B2 + 33000B3 + 27000C2 + 29000C3 + 28000D2 + 30000D3 + 800000XC + 1400000XD S.T. A1 + A2 + A3 <= 650 B1 + B2 + B3 <= 950 C2 + C3 <= 625 D2 + D3 <= 840 A1 + B1 >= 600 A2 + B2 + C2 + D2 >= 850 A3 + B3 + C3 + D3 >= 750 2A2 + 0.1B2 – C2 – 1.9D2 >= 0 2A3 + 0.1B3 – C3 – 1.9D3 >= 0 C2 + C3 – 1000XC <= 0 D2 + D3 – 1000XD <= 0 END INT XC INT XD	[18]	
3	(i)		Month Inflation Committee June 3.74 –1.24 July 2.93 –0.43 August 1.69 +0.81 September 1.26 +1.24 October 2.07 +0.43 November 3.31 –0.81 December 3.74 –1.24 ... thence cycling.	M1 A1 B1 [3]	

Question			Answer	Marks	Guidance
3	(ii)		3.74 | 3.74 2.93 | 2.93 2.31 | 2.775 2.095 | 2.72125 2.19 | 2.686875 2.3925 | 2.659219 2.5475 | 2.635859 2.60125 | 2.615957 2.5775 | 2.598975 2.526875 | 2.58448 2.488125 | 2.572108 2.474688 | 2.561548 2.480625 | 2.552535	B1 B1	
			(Oscillatory) convergence in the "half" case Slower, (uniform) convergence on the "one eighth" case	B1 B1 **[4]**	
3	(iii)		$u_{n+2} = u_{n+1} + 0.125 * (2.5 - u_n)$ ... plus reorganisation	B1 **[1]**	
3	(iv)		Auxiliary equation ... $x^2 - x + 0.125 = 0$ Solutions ... $(2 \pm \sqrt{2})/4$ General form ... $u_n = A((2 + \sqrt{2})/4)^n + B((2 - \sqrt{2})/4)^n + 2.5$ Simultaneous ... $A + B = 1.24$ ... $A((2 + \sqrt{2})/4) + B((2 - \sqrt{2})/4) = 0.43$ Solutions ... $A \approx 0.3513$, $B \approx 0.8887$	M1 A1 FT M1 A1 FT A1 **[5]**	
3	(v)		Spreadsheet check Because $((2 + \sqrt{2})/4)^n \to 0$ and $((2 - \sqrt{2})/4)^n \to 0$ as n increases	B1 B1 **[2]**	
3	(vi)		$\alpha = 0.25$ makes discriminant 0 ... (boundary between oscillatory and uniform convergence) spreadsheet fast (uniform) convergence	B1 B1 B1 **[3]**	

Question			Answer	Marks	Guidance
4	(i)		Lookup tables, or equivalent simulating par 3 simulating par 4 simulating par 5 modelling matchplay	M1 A3 M1 A1 A1 A1 M1 A1 **[10]**	
4	(ii)		repetition of holes, taking account of pars computing strokeplay result computing matchplay result	M1 A1 B1 B1 **[4]**	
4	(iii)		repetition of rounds collection and display of results	B1 B1 **[2]**	
	(iv)		no and no	B1 B1 **[2]**	Allow follow through