ADVANCED GCE

MATHEMATICS (MEI) 4773

Decision Mathematics Computation

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other Materials Required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Thursday 24 June 2010
Morning

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

Turn over

1 Athos puts £1000 into a deposit account. After a year, interest is added to the account, the amount of interest being 5% of the balance during the year. Athos then draws out £60. After each subsequent year, interest at 5% of the balance is added on, and Athos then withdraws £60.

(i) Let u_n be the amount (in £) in the account after n years, with $u_0 = 1000$. Construct a recurrence relation for u_n in terms of u_{n-1}. **[3]**

(ii) Solve your recurrence relation from part **(i)**, simplifying your answer as far as is possible. **[4]**

(iii) Use your answer to part **(ii)** to find how long Athos can continue to operate the account in this way. **[2]**

Porthos puts £1000 into a deposit account. Every 6 months, interest is added to the account, the amount of interest being $2\frac{1}{2}$ % of the balance over those 6 months. He draws out £60 at the end of 12 months and after each subsequent 12 months.

(iv) Construct a spreadsheet to show how the amount Porthos has in his account varies over time. **[3]**

(v) Use your spreadsheet to find for how long Porthos can operate his account in this way. **[1]**

Aramis puts £1000 into a deposit account. He draws out £30 every 6 months. Every 12 months, interest is added to the account, the amount of interest being 5% of the average balance over those 12 months.

(vi) Construct a spreadsheet to show how the amount Aramis has in his account varies over time. **[4]**

(vii) Use your spreadsheet to find for how long Aramis can operate his account in this way. **[1]**

2 The distance of the point (p, q) from the line $ax + by = c$ is given by $\left|\dfrac{c - ap - bq}{\sqrt{a^2 + b^2}}\right|$.

For example, when $a = b = 1$ and $c = 10$, the distance of (p, q) from $x + y = 10$ is given by $\left|\dfrac{10 - p - q}{\sqrt{2}}\right|$.

(i) Find the distance of $(0, 0)$ from the line $x + y = 10$.

Find the distance of $(10, 10)$ from the line $x + y = 10$. **[1]**

Consider the (minimax) LP:

Minimise m

subject to

$m \geqslant p$

$m \geqslant -p$

$m \geqslant q$

$m \geqslant -q$

$$m \geqslant \frac{10 - p - q}{\sqrt{2}}$$

$$m \geqslant \frac{p + q - 10}{\sqrt{2}}$$

(ii) Rewrite the LP in a form in which it can be submitted to LINDO.
(Approximate $\sqrt{2}$ by 1.414 214.) **[4]**

(iii) Run the LP and draw a diagram to explain what it achieves. **[5]**

(iv) Formulate an LP to find the point which is equidistant from the lines $y = 0$, $x + y = 1$ and $x - y = -1$.
(Approximate $\sqrt{2}$ by 1.414 214.) **[4]**

(v) Run your LP. **[1]**

(vi) Prove by drawing a diagram and calculating distances that your LP has achieved what was required. **[3]**

[Questions 3 and 4 are printed overleaf]

3 A logistics company has three depots, D1, D2 and D3. On a particular day it has 10 identical containers to collect from each of two supply locations, S1 and S2. These containers then have to be shipped from the depots to four customers, C1, C2, C3 and C4. C1 requires 7 containers, C2 requires 4 containers, C3 requires 6 containers and C4 requires 3 containers. Each depot can handle up to 7 containers.

The transportation costs are shown in the two tables.

	D1	D2	D3
S1	2	3	7
S2	1	8	4

	C1	C2	C3	C4
D1	2	3	9	1
D2	4	7	2	5
D3	1	5	3	6

(i) Formulate an LP to find the cheapest way to transport the containers from the supply locations to the depots. **[3]**

(ii) Run your LP and interpret the results. **[3]**

(iii) Using your answer to part **(ii)**, formulate an LP to find the cheapest way to transport these containers from the depots to the customers. **[3]**

(iv) Run your LP and interpret the results. **[2]**

(v) Formulate an LP to find the cheapest way to transport the containers from the supply locations to the customers via the depots. **[4]**

(vi) Run your LP and interpret and comment on your results. **[3]**

4 Each individual in a population produces either 0, 1 or 2 offspring, the probabilities being 0.1, 0.5 and 0.4 respectively.
The population starts from a single individual (generation 0), whose offspring form generation 1. In turn the offspring from members of the population in generation 1 form generation 2, etc.

(i) Show how to use a spreadsheet command to simulate the offspring produced by a member of the population. **[4]**

(ii) Build a spreadsheet to simulate generations 1 and 2.
Print out the formulae which you used in your spreadsheet. **[9]**

(iii) Run your simulation 20 times and hence estimate the probabilities of there being 0, 1, 2, 3 or 4 individuals in generation 2. **[2]**

A second population has individuals which reproduce according to the same rule, but this population starts with 2 individuals in generation 0.

(iv) Use your simulation model to produce one simulation of the number in this population at generation 2.
Explain how you produced your result. List all possible results. **[3]**

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **4773**

Decision Mathematics Computation

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

Any enquiries about publications should be addressed to:

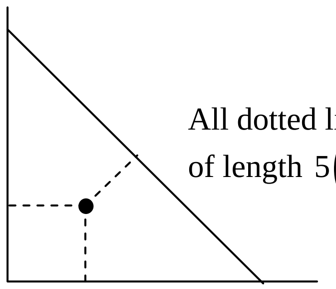
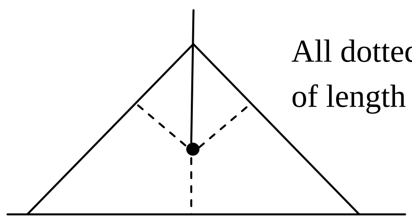
OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1.

(i) $u_n = 1.05u_{n-1} - 60$	M1 A2
(ii) $u_n = 1000\times1.05^n - 60\dfrac{(1.05^n - 1)}{0.05}$ $= 1200 - 200\times1.05^n$ or $u_n = \lambda 1.05^n + \mu$ $1000 = \lambda + \mu$ $990 = 1.05\lambda + \mu$, etc	M1 A2 A1
(iii) int(log(6)/log(1.05)) = 36 years (or spreadsheet)	M1 A1
(iv) 1000 1025 990.625 1015.391 980.7754 1005.295 etc.	M1 A1 A1
(v) 37 years (+ 6 months OK)	B1 cao
(vi) 1000 970 989.25 959.25 977.9625 947.9625 etc.	M1 A1 A1 interest OK A1
(vii) 35 years	B1 cao

2.

(i) $5\sqrt{2}$	B1
(ii) e.g: `min  m` `st   p-m<0` `     -p-m<0` `     q-m<0` `     -q-m<0` `     -p-q-1.414214m<-10` `     p+q-1.414214m<10` `end`	(negatives of these OK) M1 first 2 pairs A1 first pair A1 second pair A1 last pair
(iii) `Objective value:  2.928932` `Variable    Value          Reduced Cost` `M           2.928932       0.000000` `P           2.928932       0.000000` `Q           2.928932       0.000000`  All dotted lines are of length $5\left(2-\sqrt{2}\right)$	B1 M1 drawing A1 lines A1 point B1 equidistant
(iv) e.g: `min  m` `st   q-m<0` `     -q-m<0` `     p+q-1.414214m<1` `     -p-q-1.414214m<-1` `     p-q-1.414214m<-1` `     -p+q-1.414214m<1` `end`	M1 A1 first pair A1 second pair A1 third pair
(v) `Objective value:  0.4142135` `Variable    Value          Reduced Cost` `M           0.4142135      0.000000` `Q           0.4142135      0.000000` `P           0.000000       0.000000`	B1
(vi)  All dotted lines are of length $\sqrt{2}-1$	B1 lines B1 point B1 distances

3.

(i)	Min 2x11+3x12+7x13+x21+8x22+4x23 st x11+x12+x13=10 x21+x22+x23=10 x11+x21<7 x12+x22<7 x13+x23<7 end	B1 B1 B1	 supplies depots
(ii)	Objective value: 55.00000 Variable Value Reduced Cost X11 3.000000 0.000000 X12 7.000000 0.000000 X13 0.000000 3.000000 X21 4.000000 0.000000 X22 0.000000 6.000000 X23 6.000000 0.000000	B1	
	3 containers from S1 to D1 7 containers from S1 to D2 4 containers from S2 to D1 6 containers from S2 to D3 total cost = 55	B1 B1	logistics cost
(iii)	Min 2y11+3y12+9y13+y14+4y21+7y22+2y23 +5y24+y31+5y32+3y33+6y34 st y11+y12+y13+y14=7 y21+y22+y23+y24=7 y31+y32+y33+y34=6 y11+y21+y31=7 y12+y22+y32=4 y13+y23+y33=6 y14+y24+y34=3 end	B1 B1 B1	 depots demands
(iv)	Objective value: 37.00000 Variable Value Reduced Cost Y11 0.000000 2.000000 Y12 4.000000 0.000000 Y13 0.000000 11.00000 Y14 3.000000 0.000000 Y21 1.000000 0.000000 Y22 0.000000 0.000000 Y23 6.000000 0.000000 Y24 0.000000 0.000000 Y31 6.000000 0.000000 Y32 0.000000 1.000000 Y33 0.000000 4.000000 Y34 0.000000 4.000000	B1	
	4 containers from D1 to C2 3 containers from D1 to C4 1 container from D2 to C1 6 containers from D2 to C3 6 containers from D3 to C1 total cost = 37	B1	logistics + cost

(v)

```
min  2x11+3x12+7x13+x21+8x22+4x23+2y11+3y12
     +9y13+y14+4y21+7y22+2y23+5y24+y31+5y32
     +3y33+6y34
```

B1

```
st   x11+x12+x13=10
     x21+x22+x23=10
```

B1 supplies

```
     y11+y21+y31=7
     y12+y22+y32=4
     y13+y23+y33=6
     y14+y24+y34=3
     x11+x21<7
     x12+x22<7
     x13+x23<7
```

B1 demands + depots

```
     y11+y12+y13+y14-x11-x21=0
     y21+y22+y23+y24–x12–x22=0
     y31+y32+y33+y34-x13-x23=0
end
```

B1 trans-shipment

(vi) Objective value: 91.00000

Variable	Value	Reduced Cost
X11	4.000000	0.000000
X12	6.000000	0.000000
X13	0.000000	2.000000
X21	3.000000	0.000000
X22	0.000000	6.000000
X23	7.000000	0.000000
Y11	0.000000	0.000000
Y12	4.000000	0.000000
Y13	0.000000	8.000000
Y14	3.000000	0.000000
Y21	0.000000	1.000000
Y22	0.000000	3.000000
Y23	6.000000	0.000000
Y24	0.000000	3.000000
Y31	7.000000	0.000000
Y32	0.000000	3.000000
Y33	0.000000	3.000000
Y34	0.000000	6.000000

B1

4 containers from S1 to D1
6 containers from S1 to D2
3 containers from S2 to D1
7 containers from S2 to D3
4 containers from D1 to C2
3 containers from D1 to C4
6 containers from D2 to C3
7 containers from D3 to C1
total cost = 91

B1

Suboptimising does not give the optimum

B1 cao

4.

(i)	e.g. = lookup(rand(),A1:A3,B1:B3) with	M1 A1
	A B	
	1 0 0	
	2 0.1 1	B1
	3 0.6 2	B1
(ii)	Many approaches possible, but all must allow for 3 applications of part (i)	B2
	Offspring from generation 0	B1
	Conditional offspring from generation 1(s)	B1 M1A1 M1A1
	Output	B1
(iii)	Theoretical probabilities (Galton-Watson branching):	
	0 1 2 3 4	M1
	0.154 0.29 0.332 0.16 0.064	A1
(iv)	Two independent runs.	B1
	Sum the numbers in the two second generations.	B1
	(or nested “IF”s)	(M1 A1)
	0, 1, 2, 3, 4, 5, 6, 7, 8	B1

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

4773 Decision Mathematics Computation

General comments
Again, there were fewer candidates than the paper deserves.

Performances were very similar to those seen last year.

The paper really does demand modelling skills, sometimes in a "real world" context (see questions 1, 3 and 4) and sometimes within mathematics (question 2).

Comments on individual questions

1) *(Recurrence Relations)*
The three musketeers, with three different investment plans ... the first simple enough to model using a recurrence relation, and the other two needing Excel.
Athos The recurrence relation was straightforward – linear first order. Not many candidates succeeded in manipulating it correctly.
Porthos This scheme should have been very easy to model in Excel, but many mistakes were made, usually with the timing out of kilter.
Aramis The modelling here was more difficult, with the mean annual balance being needed for computing the interest. Few candidates were able to manage this.

2) *(LP Modelling)*
Candidates were often prepared to invest a very great deal of writing and effort for the 1 mark in part (i). It really was easy!
It was expected that virtually all candidates would be able to succeed with part (ii), but regrettably that was not the case. Those that did succeed could run the LP, but few could draw the picture to interpret the result.
The remaining parts repeated the work, but with the modelling required from the candidate instead of being supplied. Not all candidates succeeded with this.

3) *(Networks)*
Parts (i) and (ii) were very straightforward, and a good number of candidates succeeded with them.
In parts (iii) and (iv) there was an added complication in that the answer to part (ii) was required in the formulation. This tripped up several candidates.
Finally, the modelling in the last two parts was more difficult, in that equations were required linking the flows into and out of the depots. Only some candidates succeeded with this. These candidates invariably noticed that the sub-optimisation in the first two problems did not together deliver the global optimum found by the final model.

4) *(Simulation)*
This question involved the simulation modelling of a Galton-Watson branching process. Most candidates were able to deal with part (i).
It was pleasing that many candidates were also able to produce satisfactory answers to parts (ii) and (iii), although it was often very difficult to disentangle what they were doing either from their attempted explanations, or from their code.
Most candidates had run out of steam by part (iv).

4776 Numerical Methods (Written Examination)

General comments
There was a lot of good work seen, but as ever there were some candidates who appeared to be unready for the examination. Routine numerical calculations were generally carried out accurately, though it is yet again disappointing that so many candidates set their work out badly. This is an algorithmic subject and good work will reflect that. A poor layout is difficult to follow for the examiner – and difficult for the candidate to check.

Interpretation is still a weak area, with quite a number of candidates simply omitting such parts of questions – or writing vaguely and at length in the hope that they will produce something worthy of a mark.

Comments on individual questions

1) *(Fixed point iteration to solve an equation)*
This proved an easy starter for most candidates. There were many solutions gaining full marks, though a significant minority failed to find a convergent iteration in part (ii).

2) *(Numerical differentiation)*
The first part of this question proved easy for most candidates. The second part, however, was a little more challenging for some. A curious error, seen quite a few times, involved saying that if 0.9996 is correct to 4 decimal places then its maximum possible value is 0.99964. Presumably the reasoning was that 0.99965 would round to 0.9997. This is of course incorrect.

3) *(Relative error)*
The relationship $X = x(1 + r)$ proved troublesome once again. Candidates are just not happy with errors analysed this way. For certain sorts of problem – such as the one in this question – it is by far the easiest approach. In part (ii), candidates were asked to 'state, in similar terms, a relationship …'. An algebraic result without a suitable form of words did not gain full marks.

4) *(Numerical approximation)*
The first four marks were obtained easily by most candidates with only a few making errors in the numerical work or the signs. (There are two conventions for the meaning of the word 'absolute' in the term 'absolute error'. Some books take 'absolute' to be a contrast with 'relative'; others take it to mean the positive value. Either interpretation, used consistently, is acceptable.) Part (ii) was not done well. In quite a number of cases the idea was understood well enough, but the calculation of k involved algebraic errors. Many candidates appeared to ignore the information that k is an integer.

5) (*Lagrange's interpolation formula)*
This was an easy source of marks for many, but as usual some got the x and f(x) values muddled. There were some algebraic errors in the simplification, but perhaps fewer than usual.

6) *(Numerical integration)*

Part (i) was an easy source of marks for most, though there was a lot of inefficient work poorly set out. In part (ii), the correct approach is to compare the Simpson's rule estimates and, noting how small the change is between the second and third values, to conclude that 1.56895 is justified. In part (iii), candidates were mostly able to calculate the errors in the mid-point and trapezium rules. The interpretation of those errors was less well done however, with a lot of rather vague statements being made.

7) *(Newton-Raphson and secant methods)*

In part (i), the root was generally found successfully using the Newton-Raphson method. Candidates were then required to find differences and ratios of differences to assess the rate of convergence. Quite a number of candidates said that because the ratios of differences are not constant the process is faster than first order. This was not enough: they needed to say that ratios of differences are decreasing (fast).

In part (ii), some candidates seemed less secure in their use of the secant method. The conclusion about the rate of convergence was handled much as in part (i).