Thursday 12 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

3125877073

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The displacement, x m, of a particle at time t s is given by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+2\frac{\mathrm{d}x}{\mathrm{d}t}+2x=30\cos 2t.$$

(i) Find the general solution. **[9]**

The particle is initially at the origin, travelling with velocity $10\,\mathrm{m\,s^{-1}}$.

(ii) Find the particular solution. **[4]**

(iii) Find the amplitude of the oscillations of the particle for large values of t. **[2]**

Consider now the differential equation

$$\frac{\mathrm{d}^3x}{\mathrm{d}t^3}+2\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+2\frac{\mathrm{d}x}{\mathrm{d}t}=0.$$

(iv) Show that 0 is a root of the auxiliary equation and write down the other roots**.** **[2]**

(v) Find the particular solution of this differential equation subject to the initial conditions $x=0$, $\frac{\mathrm{d}x}{\mathrm{d}t}=10$ and $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=4$ when $t=0$. **[5]**

(vi) Sketch the graph of this solution. **[2]**

2 The population, P, of a species at time t hours is to be modelled by a differential equation. The initial population is 100.

At first, the model $\frac{\mathrm{d}P}{\mathrm{d}t}-0.25P=0$ is used.

(i) Find P in terms of t and comment on the suitability of this model. **[4]**

To allow for certain environmental effects, the model is refined to

$$\frac{\mathrm{d}P}{\mathrm{d}t}-0.25P=-18\mathrm{e}^{-0.5t}.$$

(ii) Write down the complementary function for this differential equation. Find a particular integral and hence state the general solution. **[6]**

(iii) Find the solution subject to the given initial condition and comment on the suitability of this refined model. **[3]**

The following mathematical model for the population is now used.

$$\frac{\mathrm{d}P}{\mathrm{d}t}=6\times 10^{-4}P(400-P)$$

(iv) Solve this differential equation subject to the given initial condition, expressing P in terms of t. **[8]**

(v) Show that the time T hours at which $P=200$ is given by

$$T=\frac{25}{6}\ln 3.$$ **[1]**

(vi) What does this model predict for the population of the species in the long term? **[2]**

3 **(a)** The equation of a curve in the x-y plane satisfies the differential equation

$$(x+1)\frac{\mathrm{d}y}{\mathrm{d}x} - xy = \mathrm{e}^{2x}$$

for $x > -1$.

(i) Show that an integrating factor for this differential equation is $\mathrm{e}^{-x}(1+x)$ and hence find the general solution for y in terms of x. **[11]**

The curve passes through the point $(0, -2)$.

(ii) Find the equation of this curve. **[2]**

(b) The differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x^2 + y^2}$$

is to be solved approximately, first by using a tangent field and then by Euler's method.

(i) Show that the isocline for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ is a circle and state its centre and radius. **[2]**

(ii) Sketch the isoclines for the cases $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$. Use these isoclines to draw a tangent field. **[3]**

(iii) Sketch the solution curve through $(0, 1)$. **[2]**

Euler's method is now used, starting at $x = 0$, $y = 1$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$.

(iv) Use a step length of 0.05 to estimate y when $x = 0.15$. **[4]**

Question 4 begins on page 4

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 4y + \mathrm{e}^{-2t}$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} + 3x = 5y + 2\mathrm{e}^{-2t}$$

are to be solved.

(i) Obtain a second order differential equation for x in terms of t. Hence find the general solution for x. **[12]**

(ii) Find the corresponding general solution for y. **[4]**

When $t = 0$, $y = -\frac{2}{3}$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$.

(iii) Find the particular solutions for x and y. **[5]**

(iv) Find the set of values of t for which $y > x$. **[3]**

END OF QUESTION PAPER

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)	
	(answer space continued on next page)

1 (i)	**(continued)**
1 (ii)	

1 (iii)	
1 (iv)	

1 (v)	
1 (vi)	

2 (i)	
2 (ii)	

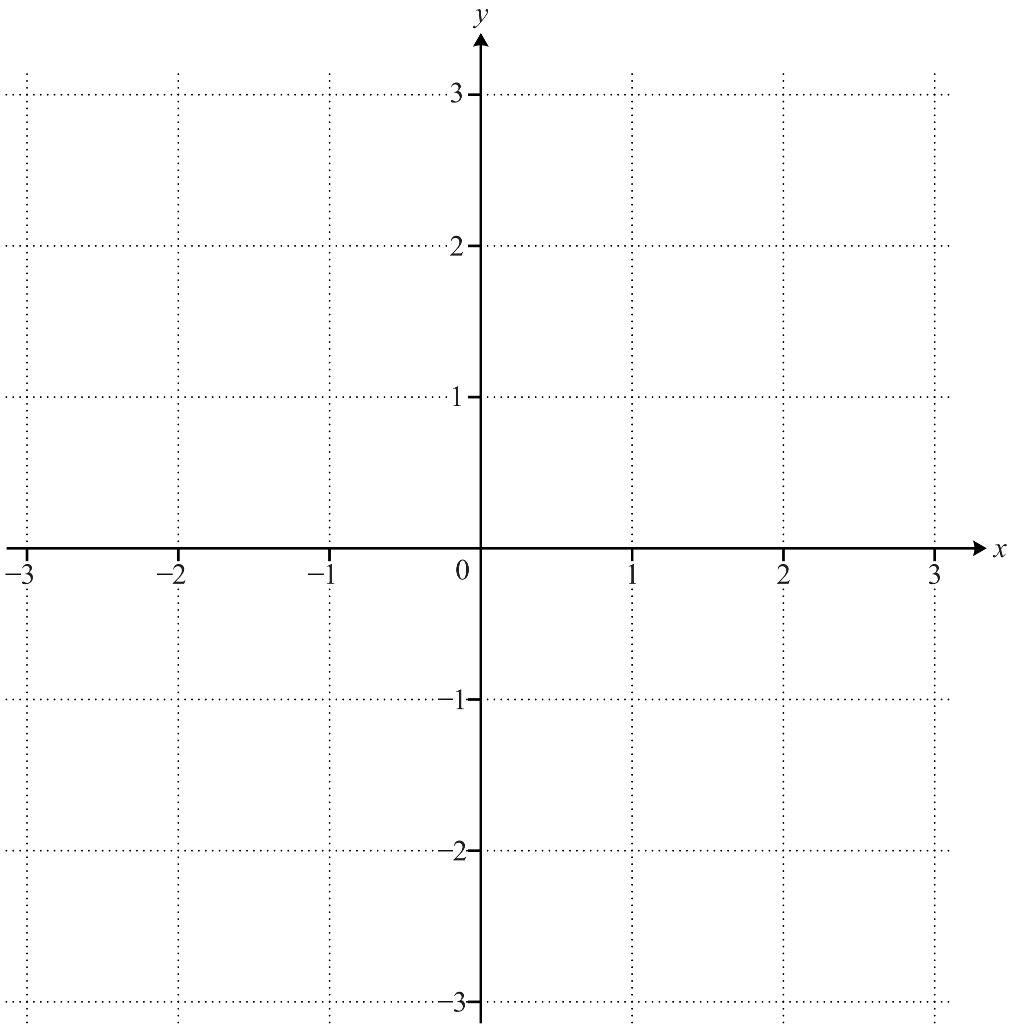
2 (iii)	
2 (iv)	
	(answer space continued on next page)

Turn over

2 (iv)	**(continued)**
2 (v)	
2 (vi)	

3 (a) (i)

3 (a) (ii)	
3 (b) (i)	

3 (b) (ii) & (iii)	 **A spare copy of this diagram can be found on page 16.**
3 (b) (iv)	

4 (i)	
	(answer space continued on next page)

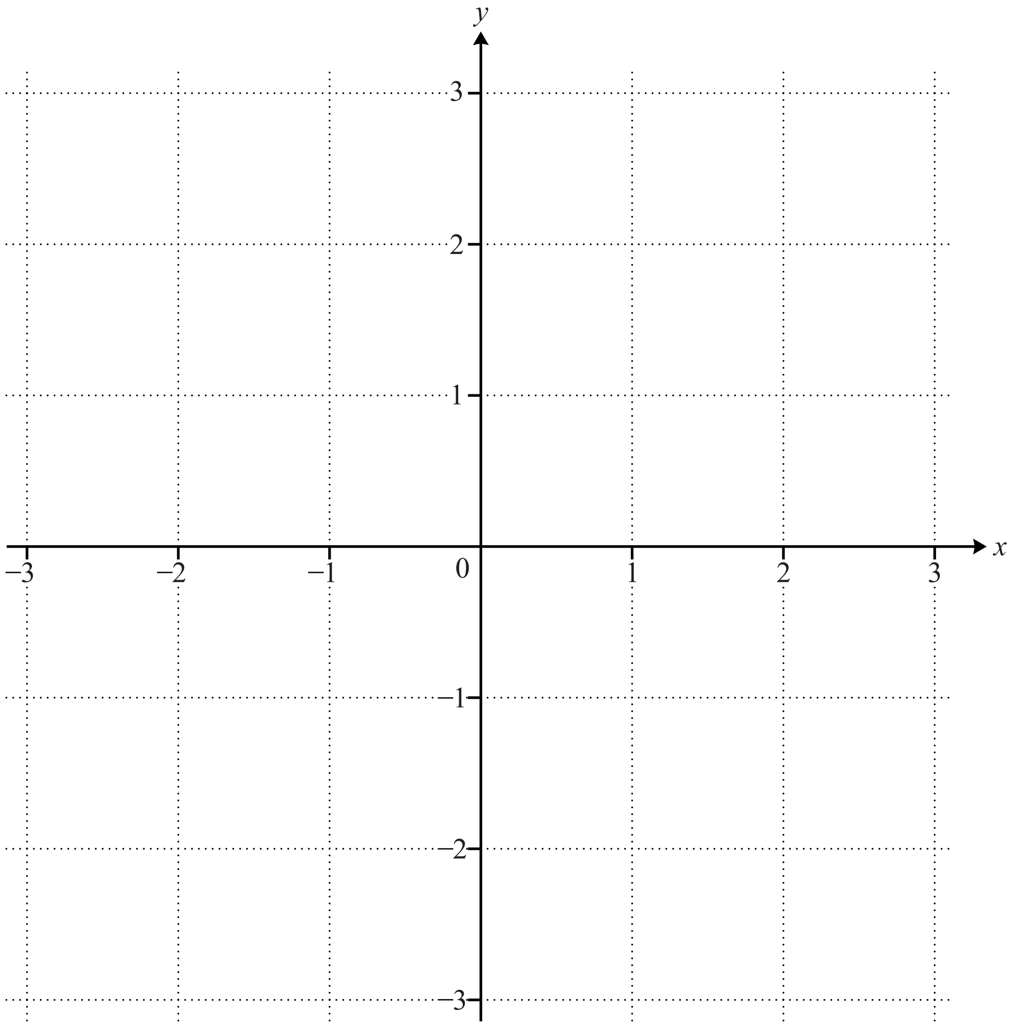
4 (i)	**(continued)**
4 (ii)	

4 (iii)	

4 (iv)

3 (b) (ii) & (iii)

Spare copy of diagram for questions 3(b)(ii) and 3(b)(iii)



GCE

Mathematics (MEI)

Unit **4758**: Differential Equations

Advanced GCE

Mark Scheme for June 2014

1. Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

Question			Answer	Marks	Guidance	
1	(i)		Auxiliary equation $m^2+2m+2=0$	M1		
			$m=-1\pm i$	A1		
			CF $e^{-t}\ (A\cos t+B\sin t)$	F1	Their roots	
			PI $x=P\sin 2t+Q\cos 2t$	B1	cao	
				M1	Differentiate twice	
			$(-4Ps-4Qc)+2(2Pc-2Qs)+2(Ps+Qc)=30c$	M1	Substitute	
			$-4P-4Q+2P=0$ $-4Q+4P+2Q=30$	M1	Compare coefficients and solve	
			$x=6\sin 2t-3\cos 2t$	A1	Implied by P = 6 and Q = – 3 cwo	
			GS: $x=e^{-t}\ (A\cos t+B\sin t)\ +6\sin 2t-3\cos 2t$	F1	Their PI + their CF with 2 arbitrary constants. Must be t on RHS	
				[9]		
1	(ii)		$x=0,\ t=0\quad A=3$	M1	Use given condition	
			$x'=e^{-t}\ (-A\sin t+B\cos t)\ -e^{-t}\ (A\cos t+B\sin t)$ $+12\cos 2t+6\sin 2t$	M1	Differentiate, product rule	
			$x'=10,\ t=0\qquad B=1$	M1	Use given condition	
			$x=e^{-t}\ (3\cos t+\sin t)\ +6\sin 2t-3\cos 2t$	A1	cao	
				[4]		
1	(iii)		$x\to 6\sin 2t-3\cos 2t$	B1 FT	Use large t	
			Amplitude = $\sqrt{6^2+3^2}$ = $3\sqrt{5}$ $=6.71$	B1 FT		
				[2]		
1	(iv)		Auxiliary equation $m^3+2m^2+2m=0$:	B1	Some working must be shown	
			m = 0 is a root $m=-1\pm i$	B1	FT roots from **(i)**	
				[2]		

Question			Answer	Marks	Guidance	
1	(v)		$x = e^{-t}(P\cos t + Q\sin t) + C$ $x=0,\ t=0 \quad 0 = P + C$ $x'=10,\ t=0 \quad 10 = Q - P$ $x''=4,\ t=0 \quad 4 = -P - 2Q + P$ $Q=-2, P=-12, C=12$ $x = e^{-t}(-12\cos t - 2\sin t) + 12$	B1 M1 M1 M1 A1 **[5]**	Follow CF from **(i),** or new CF if different roots in (iv), plus a non-zero constant. Use condition, dependent on CF including the non-zero constant Use condition and use product rule Use condition and use product rule cao	
1	(vi)		Starts at origin with positive gradient Tends to 12 for large values of t	B1 B1 **[2]**	Ignore negative t FT their value of C if correct form of solution in (v)	
2	(i)		$\ln P = 0.25t + A$ or $P = Be^{0.25t}$ $t=0,\ P=100 \quad A = \ln 100$ or $B = 100$ $P = 100e^{0.25t}$ Not suitable as tends to infinity as t increases Alternatives for first M1: Auxiliary eqn gives $m = 0.25$: $P = Be^{0.25t}$ Integrating factor $e^{-0.25t}$: $Pe^{-0.25t} = B$: $P = Be^{0.25t}$	M1 M1 A1 B1 **[4]**	Separate and integrate (need ln on LHS) Use condition cao Allow if constant of integration not found 'Increases for ever/always' gets B1; 'exponential growth' gets B1 'Suitable' gets B0; 'Suitable for small t ' gets B1	
2	(ii)		CF: $Ae^{0.25t}$ PI: $P = ke^{-0.5t}$ $P' = -0.5ke^{-0.5t} = 0.25ke^{-0.5t} - 18e^{-0.5t}$ $k = 24$ GS: $P = Ae^{0.25t} + 24e^{-0.5t}$	B1 B1 M1 M1 A1 F1 **[6]**	 Correct form Differentiate and substitute Compare coefficients and solve Their PI + their CF with one arbitrary constant	

Question			Answer	Marks	Guidance	
2	(iii)		$t=0,\ P=100 \qquad A=76$	M1	Use condition	
			$P = 76e^{0.25t} + 24e^{-0.5t}$	A1	cao	
			Unsuitable, not bounded (or equivalent)	F1 **[3]**	'Increases for ever/always' gets B1; 'exponential growth' gets B1 'Suitable for small t ' gets B1' 'Better/more suitable than previous one' gets F0;	
2	(iv)			M1	Separate variables	
			$\frac{1}{P}+\frac{1}{400-P}$	M1	Use partial fractions with correct denominators	
			$\frac{1}{400}\left(\frac{1}{P}+\frac{1}{400-P}\right)dP = 6\times10^{-4}\,dt$	A1		
			$\frac{1}{400}\ln\left(\frac{P}{400-P}\right)$	M1	Integrate LHS (dependent on previous M1)	
			$\frac{1}{400}\ln\left(\frac{P}{400-P}\right)=6\times10^{-4}t(+B)$	A1		
			$t=0,\ P=100 \qquad B=\frac{1}{400}\ln\frac{1}{3}$	M1	Use condition (not dependent on partial fractions)	
			[$t=\frac{25}{6}\ln\left(\frac{3P}{400-P}\right)$]	M1	Rearrange (dependent on partial fractions)	
			$P=\frac{400}{1+3e^{-\frac{6t}{25}}}$ or $P=\frac{400e^{\frac{6t}{25}}}{3+e^{\frac{6t}{25}}}$	A1 **[8]**	cao aef	
2	(v)		$t=T,\ P=200 \qquad T=\frac{25}{6}\ln\frac{600}{200}=\frac{25}{6}\ln 3$	E1 **[1]**	AG	
2	(vi)		Tends to a limit limit of 400	B1 B1 **[2]**	Dependent on correct form of solution in (iv) cao (Note that $P\to 400$ gets B1B1)	

Question			Answer	Marks	Guidance	
3	**(a)**	**(i)**	$y' - \frac{x}{x+1}y = \frac{e^{2x}}{x+1}$	B1	Divide by $x + 1$	
			IF $e^{\int -\frac{x}{x+1}dx} = e^{-\int\left(1-\frac{1}{x+1}\right)dx}$	M1	Splitting into $-\ 1-\frac{1}{x+1}$	
				A1		
			$e^{-x+\ln(x+1)}$	B1	Seen	
			$e^{-x} \times e^{\ln(1+x)} = e^{-x}\ 1+x$	E1	Show answer	
			$\frac{d}{dx}\ ye^{-x}(1+x)\ = e^{x}$	M1	Multiply both sides by given IF	
				A1	RHS must be simplified	
			$ye^{-x}(1+x)\ = e^{x} + C$	M1	Integrate both sides, including + C	
				A1		
			$y = \frac{e^{x}\ \ e^{x} + C}{1+x}$	M1	Make y the subject (need to divide all terms by coeff of y on LHS)	
				A1	cao aef	
				[11]		
3	**(a)**	**(ii)**	$x = 0,\ y = -2$ $C = -3$	M1		
			$y = \frac{e^{x}\ \ e^{x} - 3}{1+x}$	A1	cao	
				[2]		

Question			Answer	Marks	Guidance	
3	**(b)**	**(i)**	Circle, centre *O* Radius $\sqrt{\frac{1}{4}} = \frac{1}{2}$	B1 B1 **[2]**		
3	**(b)**	**(ii)**	One correct isocline All 3 correct isoclines Correct direction indicators	B1 B1 B1 **[3]**	 Ratios of radii must be correct At least four on each of the three circles	
3	**(b)**	**(iii)**	Attempt at a solution curve Correct solution curve	B1 B1 **[2]**		
3	**(b)**	**(iv)**	One correct use of algorithm 1.05 1.09525 1.137	M1 A1 A1 A1 **[4]**	 1.095 or better 1.14 or better. Do not penalise lack of accuracy in intermediate values if final answer correct	

Question		Answer	Marks	Guidance
4	(i)	$\ddot{x}=4\dot{y}-2\dot{x}-2\mathrm{e}^{-2t}$	M1	Differentiate with respect to t
		$y=\frac{1}{4}\ \dot{x}+2x-\mathrm{e}^{-2t}$	M1	Rearrange
		$\ddot{x}=4\ \ 5y+2\mathrm{e}^{-2t}-3x\ \ -2\dot{x}-2\mathrm{e}^{-2t}$	M1	Substitute for $\dot{y}$
		$\ddot{x}+2\dot{x}=5\ \ \dot{x}+2x-\mathrm{e}^{-2t}\ \ +8\mathrm{e}^{-2t}-12x-2\mathrm{e}^{-2t}$	M1	Substitute for y
		$\ddot{x}-3\dot{x}+2x=\mathrm{e}^{-2t}$	A1	
		Auxiliary equation $\lambda^2-3\lambda+2=0$	M1	
		$\lambda=1,2$	A1	
		CF $A\mathrm{e}^{t}+B\mathrm{e}^{2t}$	F1	
		PI $x=a\mathrm{e}^{-2t}$	B1	Correct form for their RHS
		$\dot{x}=-2a\mathrm{e}^{-2t};\ddot{x}=4a\mathrm{e}^{-2t}$: $12a=1$	M1	Differentiate and equate
		$a=\frac{1}{12}$	A1	
		$x=A\mathrm{e}^{t}+B\mathrm{e}^{2t}+\frac{1}{12}\mathrm{e}^{-2t}$	F1	Their PI + their CF with two arbitrary constants
			[12]	
4	(ii)	$y=\frac{1}{4}\ \dot{x}+2x-2\mathrm{e}^{-2t}$	M1	Rearrange first given equation (all terms must be present)
		$y=\frac{1}{4}(A\mathrm{e}^{t}+2B\mathrm{e}^{2t}-\frac{1}{6}\mathrm{e}^{-2t})+\frac{1}{2}x-\frac{1}{4}\mathrm{e}^{-2t}$	M1	Substitute for $\dot{x}$
			M1	Substitute for x
		$y=\frac{3}{4}A\mathrm{e}^{t}+B\mathrm{e}^{2t}-\frac{1}{4}\mathrm{e}^{-2t}$	A1	cwo in (i) and (ii)
			[4]	

Question			Answer	Marks	Guidance	
4	(iii)		$\frac{3}{4}A+B-\frac{1}{4}=-\frac{2}{3}$ $3A+4B+\frac{5}{3}=0$	M1	Use given condition	
			$\frac{3}{4}A+2B+\frac{1}{2}=0$ $3A+8B+2=0$	M1	Use given condition	
			$A=-\frac{4}{9};\ B=-\frac{1}{12}$	M1	Solve	
			$x=-\frac{4}{9}e^{t}-\frac{1}{12}e^{2t}+\frac{1}{12}e^{-2t}$	A1	cao	
			$y=-\frac{1}{3}e^{t}-\frac{1}{12}e^{2t}-\frac{1}{4}e^{-2t}$	A1	cao	
				[5]		
4	(iv)		$-\frac{1}{3}e^{t}-\frac{1}{12}e^{2t}-\frac{1}{4}e^{-2t} > -\frac{4}{9}e^{t}-\frac{1}{12}e^{2t}+\frac{1}{12}e^{-2t}$ $\frac{1}{9}e^{t}>\frac{1}{3}e^{-2t}$	M1	Use condition and simplify	
			$e^{3t}>3$	M1	Take logs	
			$t>\frac{1}{3}\ln 3$	A1	cao	
				[3]		

4758 Differential Equations (Written Examination)

General Comments:

The standard of the responses on this paper were of a pleasingly high standard, and many candidates scored full marks on some or all of the questions. The methods required to solve the second order differential equations in Questions 1 and 4 were known by almost all candidates and these two questions were attempted by the majority of the candidates. Question 2 was more popular than Question 3, with part (b) of Question 3 proving to be the most challenging to the candidates. It appeared that the syllabus item of tangent fields is less well-known by the candidates.

Comments on Individual Questions:

Question No. 1

Second order linear differential equations

(i) All candidates were familiar with the method of solution required in this part. Any marks lost were because of arithmetical errors in solving either the simultaneous equations to find the particular integral or the quadratic equation to find the roots of the auxiliary equation.

(ii) All candidates applied initial conditions to their general solution from part (i). A minority of candidates used the incorrect initial condition $\frac{dx}{dt}=0$ instead of the given condition $\frac{dx}{dt}=10$ a few candidates did not use the product rule when differentiating.

(iii) The majority of candidates considered the behaviour of their particular solution from part (ii) for large values of t and obtained an expression of the form $x \text{->} p\sin 2t + q\cos 2t$. A significant number of candidates, however, did not seem to know how to find the amplitude of these oscillations, with p or q or $p + q$ being common incorrect answers.

(iv) Almost all candidates scored both marks.

(v) The majority of candidates scored full marks in this part. The most common error was to omit the constant term in the general solution that arose from the zero root of the auxiliary equation. Some candidates used incorrect initial conditions or did not use the product rule when differentiating.

(vi) The first mark in this part was awarded for a sketch in which the solution curve had a positive gradient (given in the question as 10) at the origin. The second mark was awarded for a curve that tended, for large values of t, to the constant value in the particular solution from part (v). The majority of candidates earned both marks.

Question No. 2

First order differential equations

(i) Almost all candidates were able to obtain the correct general solution to this differential equation, but a significant minority did not apply the initial condition. To gain the final mark, comments on the suitability of the model needed to include the idea that it predicted infinite growth for large values of t. About half of the candidates earned this final mark, with many of the other candidates not making a comment.

(ii) This part was answered well by almost all of the candidates. The most common mistake was a sign error when calculating the particular integral.

(iii) A significant number of candidates were unable to give a meaningful Interpretation of the suitability of the solution obtained in part (ii). Many candidates offered the observation that it was more suitable than that in part (i), with no further comment. Others suggested that it was more suitable because it took environmental factors into account. Since this model was introduced in the question as a refined model taking account of environmental factors, neither of these comments earned any marks.

(iv) Attempts at this part of the question tended to be polarised into those that were totally correct and those that gained only one or two marks. A significant number of candidates appeared to struggle to make any real progress and this often led them to abandon their attempts at Question 2 and instead try Question 3. It was very pleasing, however, to see a considerable number of excellent solutions, with candidates showing confidence in the method of solution and an ability to apply it accurately.

(v) The single mark in this part was only available to those candidates who had made progress in part (v) and it was earned by the vast majority of them.

(vi) Again, those candidates who had made progress in part (iv) usually scored both marks in this part.

Question No. 3

First order linear differential equation and tangent fields

This was by far the least popular question on the paper, with a significant number of attempts resulting in little progress.

(a)

(i) Only a small minority of candidates were able to complete the integration involved in finding the integrating factor. The stumbling block was in trying to find $\int \frac{-x}{x+1}\,dx$. Most, however, used the given result to attempt the general solution for y. A common error was to omit the $(x+1)$ when multiplying through by the integrating factor.

(ii) The method mark was earned by almost all candidates, but accuracy errors abounded, either because of an incorrect solution in part (i) or because of an arithmetical slip.

(b)

(i) Almost all candidates who attempted this part earned both marks.

(ii) Although there were a few excellent solutions to this part, the majority of candidates seemed confused by what was required. It was surprising that having been told, and indeed having shown in part (i), that one of the isoclines was a circle, many candidates did not then sketch this circle. Some hinted at it by drawing a few direction indicators roughly around what would have been the circumference of the circle. Attempts at drawing the tangent field, by sketching direction indicators, were generally weak. Many candidates simply put indicators, in seemingly random directions, at the corners of the squares of the grid.

(iii) Many of the candidates who had made a good attempt at the previous part also sketched an appropriate solution curve. Most candidates, however, made no progress.

(iv) Almost invariably, solutions to this request were very good.

Question No. 4

Simultaneous second order linear differential equations

(i) There were many excellent responses to this part and the majority of candidates scored full marks. The most common errors were made in obtaining the second order differential equation for x, resulting in an incorrect coefficient of e^{-2t} on the right hand side. This usually led to the loss of two accuracy marks.

(ii) Almost all candidates gained the three method marks.

(iii) All candidates made a good attempt at this part. For many candidates, application of the correct method failed to lead to success because of accuracy errors. Some of these followed on from errors in the earlier parts of the question, but many arose when solving the pair of simultaneous equations produced by applying the initial conditions.

(iv) Most candidates made good progress applying the correct method.

Unit level raw mark and UMS grade boundaries June 2014 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	61	56	51	46	42	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	57	51	45	39	33	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	68	61	54	47	41	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	60	54	48	42	36	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	57	51	45	39	34	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	57	49	41	34	27	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	57	49	41	34	27	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	55	48	42	36	30	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	48	41	34	28	22	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	61	53	46	39	32	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	60	53	46	40	34	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	51	46	41	36	31	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	46	41	36	31	26	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	54	48	43	38	32	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0
4798/01	(FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241/01	(Z1) Statistics 1	Raw	72	61	53	46	39	32	0
		UMS	100	80	70	60	50	40	0
G242/01	(Z2) Statistics 2	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243/01	(Z3) Statistics 3	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0