

OCR

Oxford Cambridge and RSA

Tuesday 9 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

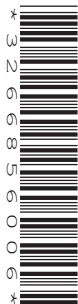
OCR supplied materials:

- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 The displacement, x m, of a particle at time t s is given by the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 25x = 0.$$

Initially the particle is at the origin and has a velocity of $\frac{1}{4}\text{ms}^{-1}$.

- (i) Find the particular solution for x . [8]

- (ii) Find the maximum displacement of the particle from its initial position, giving your answer correct to 3 significant figures. [4]

- (iii) Describe the behaviour of your solution for large values of t . [1]

In a different situation, an additional force is applied to the particle and the differential equation satisfied by x is

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 25x = 5\sin 5t.$$

- (iv) Using the same initial conditions as in part (i), find the new particular solution for x . [10]

- (v) Describe the behaviour of your new solution for large values of t . [1]

- 2 The differential equation

$$x\frac{dy}{dx} - ny = 2x - 1,$$

where n is a non-zero constant, is to be solved for $x > 0$.

Firstly consider the case $n \neq 1$.

- (i) Find the general solution for y in terms of x and n . [8]

- (ii) For $n = -1$, find the equation of the solution curve that passes through the point $(2, 0)$ and sketch the curve for $x > 0$. [4]

Now consider the case $n = 1$.

- (iii) Find the general solution for y in terms of x . [5]

- (iv) Show that the solution curve for which $y = 0$ when $x = 1$ has exactly one stationary point. [3]

Now consider the differential equation

$$x\frac{dy}{dx} - y = \frac{1}{\sqrt{2x-1}}.$$

- (v) Use Euler's method, with a step length of 0.1 and initial conditions $y = 0$ when $x = 1$, to estimate y when $x = 1.3$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$. [4]

- 3 The resistance to motion of a small test car of mass 20 kg is modelled differently according to the aerodynamic features of the bodywork being tested. The motion of the test car is studied as it moves in a horizontal straight line. In each trial, the car is initially at rest at A; at time t s its velocity is v m s⁻¹ and its distance from A is x m. The only horizontal forces acting on the car are a driving force of 100 N and a varying resistance force of magnitude R N.

In the first trial, the resistance to motion is modelled by $R = 4v^2$.

- (i) Write down and solve a differential equation to show that

$$v^2 = 25\left(1 - e^{-\frac{2}{5}x}\right).$$

Find the value of v when $x = 10$. [9]

- (ii) Find the value of t when $x = 10$. [7]

In the second trial, the resistance to motion is modelled by $R = 2v$.

- (iii) Write down and solve a differential equation to find v in terms of t . State the terminal velocity of the car. [7]

- (iv) Find the value of t in the second trial when the car's speed is equal to the value of v found in part (i). [1]

- 4 Two species of small rodent, X and Y, compete for survival in the same environment. The populations of the species, at time t years, are x and y respectively and they are modelled by the simultaneous differential equations

$$\begin{aligned}\frac{dx}{dt} &= 2(x - y), \\ \frac{dy}{dt} &= \frac{3}{8}(x - 80e^{-\frac{1}{2}t}).\end{aligned}$$

- (i) Show that

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + \frac{3}{4}x = 60e^{-\frac{1}{2}t}.$$

Find the general solution for x . [10]

- (ii) Find the corresponding general solution for y . [3]

When $t = 0$, $x = 40$ and $y = 50$.

- (iii) Find the particular solutions for x and y . [4]

- (iv) Find the time T at which the model predicts that the rodents of species X will die out. Find the population of species Y predicted at this time. [6]

- (v) Comment on the suitability of the model for times greater than T . [1]

END OF QUESTION PAPER

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Tuesday 9 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:

- Question Paper 4758/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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1 (i)	(continued)
1 (ii)	

1 (iii)	
1 (iv)	
(answer space continued on next page)	

1 (iv)	(continued)
1 (v)	

2 (ii)	
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2 (iii)	
2 (iv)	
(answer space continued on next page)	

2 (iv) (continued)	
2 (v)	

3 (iii)	
3 (iv)	

4 (ii)	
4 (iii)	

(answer space continued on next page)

4 (iv)	(continued)
4 (v)	



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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for g . E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	Guidance
1	(i)	Auxiliary equation: $m^2 + 8m + 25 = 0$ $m = -4 \pm 3i$ CF: $x = e^{-4t} (A \cos 3t + B \sin 3t)$ $A = 0$ $\dot{x} = e^{-4t} (3B \cos 3t) - 4e^{-4t} \cdot B \sin 3t$ $0.25 = 3B : B = \frac{1}{12}$ $x = \frac{1}{12} e^{-4t} \sin 3t$	M1 A1 F1 M1 M1 M1 A1 F1 [8]	From their roots Use condition Differentiate, product rule Use condition (must use 0.25)
	(ii)	$\frac{dx}{dt} = 0 : e^{-4t} (3 \cos 3t - 4 \sin 3t) = 0$ $\tan 3t = \frac{3}{4} : t = 0.2145$ $x = 0.0212$	M1 M1 A1 A1 [4]	Equate their \dot{x} to zero Obtain expression for tan and attempt to solve Dependent on using correct answer to (i) cao
	(iii)	Oscillations with decreasing amplitude	B1 [1]	Accept amplitude $\rightarrow 0$; accept $x \rightarrow 0$. accept "oscillations with small amplitude" Do not accept "oscillations"

Question	Answer	Marks	Guidance
(iv)	CF: $x = e^{-4t} (C \cos 3t + D \sin 3t)$ PI: $x = P \sin 5t + Q \cos 5t$ $\dot{x} = 5P \cos 5t - 5Q \sin 5t$ $\ddot{x} = -25P \sin 5t - 25Q \cos 5t$ $-25P - 40Q + 25P = 5$ $-25Q + 40P + 25Q = 0:$ $P = 0, Q = -\frac{1}{8}$ $x = e^{-4t} (C \cos 3t + D \sin 3t) - \frac{1}{8} \cos 5t$ $x = 0, y = 0: \quad C = \frac{1}{8}$ $\dot{x} = e^{-4t} (-3C \sin 3t + 3D \cos 3t) -$ $4e^{-4t} (C \cos 3t + D \sin 3t) + \frac{5}{8} \sin 5t$ $\dot{x} = 0.25, t = 0: \quad 0.25 = 3D - 4C$ $(D = \frac{1}{4})$ $x = e^{-4t} \left(\frac{1}{8} \cos 3t + \frac{1}{4} \sin 3t \right) - \frac{1}{8} \cos 5t$	F1 B1 M1 M1 A1 F1 M1 M1 M1 A1 [10]	Correct form Differentiate twice and substitute Compare coefficients and solve Their CF with 2 arbitrary constants + their PI Use condition Differentiate using product rule Use condition (must use 0.25) cao
(v)	Oscillations of approximately constant amplitude $\frac{1}{8}$	B1 [1]	FT their amplitude

Question	Answer	Marks	Guidance
2 (i)	$y' - \frac{n}{x}y = \frac{2x-1}{x} = 2 - \frac{1}{x}$ $\text{IF} = e^{\int -\frac{n}{x} dx}$ $(\text{IF} = e^{-n \ln x}) = x^{-n}$ $\frac{d}{dx}(yx^{-n}) = 2x^{-n} - x^{-n-1}$ $yx^{-n} = \frac{2x^{1-n}}{1-n} + \frac{x^{-n}}{n} + A$ $y = \frac{2x}{1-n} + \frac{1}{n} + Ax^n$	B1 B1 B1 M1 M1 A1 M1 A1 [8]	Divide through by x Multiply both sides by their IF Integrate both sides Must include arbitrary constant Divide both sides, including an arbitrary constant, by their IF cao
(ii)	$y = 0, x = 2, n = -1: \quad (A = -2)$ $y = x - 1 - \frac{2}{x}$ Curve in 1st and 4th quadrants through (2,0) Correct behaviour as $x \rightarrow 0$ and $x \rightarrow \infty$	M1 A1 B1 B1 [4]	Use condition to find a value for A Must use correct form of solution. Sketch going through (2, 0) with positive gradient at (2, 0) Ignore curve for $x < 0$
(iii)	$\frac{d}{dx}(yx^{-1}) = 2x^{-1} - x^{-2}$ $yx^{-1} = 2 \ln x + x^{-1} + B$ $y = 2x \ln x + 1 + Bx$	M1 A1 M1 A1 A1 [5]	Find and multiply by IF Integrate both sides Must include arbitrary constant cao

Question		Answer	Marks	Guidance
	(iv)	$B = -1$ $y' = 2 \ln x + 2 - 1 = 0$ $\ln x = -\frac{1}{2}$: One solution	M1 M1 E1 [3]	Use condition $x = 1, y = 0$ Differentiate, equate to zero
	(v)	y' values: 1 0.92079 or 0.192079 0.86436 y value: 0.2785	M1 A1 A1 A1 [4]	NB the DE is used in the form $\frac{dy}{dx} = \frac{1}{x} \left(y + \frac{1}{\sqrt{2x-1}} \right)$ Agree to 3 s.f. Agree to 3 s.f. 0.279 (or better)
3	(i)	$20v \frac{dv}{dx} = 100 - 4v^2$ $\frac{5v dv}{(25 - v^2)} = dx$ $-\frac{5}{2} \ln(25 - v^2) = x + A$ $A = -\frac{5}{2} \ln 25$ $25 - v^2 = 25e^{-\frac{2}{5}x}$ $v^2 = 25 \left(1 - e^{-\frac{2}{5}x} \right)$ When $x = 10, v = 4.95(4)$	M1* M1dep* M1dep* A1 A1 M1dep* M1dep* E1 B1 [9]	Use N2L with accn in terms of v and x Separate variables Integrate both sides lhs rhs, including +A Use condition rearrange cao

Question	Answer	Marks	Guidance
(ii)	$20 \frac{dv}{dt} = 100 - 4v^2$ $\frac{5dv}{(25 - v^2)} = dt$ $\frac{1}{2} \ln \left(\frac{5+v}{5-v} \right) = t + B$ $B = 0$ $t = \frac{1}{2} \ln \left(\frac{5+v}{5-v} \right)$ When $v = 4.954$, $t = 2.689$	M1* M1dep* M1dep* A1 M1dep* A1 M1 [7]	Use N2L with accn in terms of v and t Separate variables Integrate both sides. The integral may be quoted from MF2 or PF $\frac{1}{2} \left(\frac{1}{5-v} + \frac{1}{5+v} \right)$ used. Correct expression Use condition aef Use answer from (i)
(iii)	$20 \frac{dv}{dt} = 100 - 2v$ $-10 \ln(50 - v) = t + C$ $C = -10 \ln 50$ $t = 10 \ln \left(\frac{50}{50 - v} \right)$ $v = 50 \left(1 - e^{-\frac{1}{10}t} \right)$ Terminal velocity = 50ms^{-1}	M1 M1 M1 A1 M1 A1 B1 [7]	Use N2L with accn in terms of v and t Separate and integrate Use condition Make v subject Correct expression

Question	Answer	Marks	Guidance
	<p>OR: $20 \frac{dv}{dt} = 100 - 2v$</p> <p>[IF: $e^{-0.1t}$: $\frac{d}{dt}(ve^{-0.1t}) = 5e^{-0.1t}$]</p> <p>$ve^{-0.1t} = -50e^{-0.1t} + A$</p> <p>$A = 50$</p> <p>$v = 50 \left(1 - e^{-\frac{1}{10}t} \right)$</p> <p>Terminal velocity = 50ms^{-1}</p> <p>OR: $20 \frac{dv}{dt} = 100 - 2v$</p> <p>Auxiliary eqn $10m + 1 = 0$: CF: $v = Ae^{-0.1t}$</p> <p>PI: $v = B$: $B = 50$</p> <p>GS: $v = 50 + Ae^{-0.1t}$</p> <p>$A = -50$</p> <p>$v = 50 \left(1 - e^{-\frac{1}{10}t} \right)$</p> <p>Terminal velocity = 50ms^{-1}</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>[7]</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>[7]</p>	<p>Use N2L with accn in terms of v and t</p> <p>Multiply through by IF and integrate</p> <p>Use condition</p> <p>Make v subject</p> <p>Correct expression</p> <p>Use N2L with accn in terms of v and t</p> <p>Use condition</p> <p>Correct expression</p>
(iv)	When $v = 4.954$, $t = 10 \ln 1.11 = 1.04$ s	B1 [1]	

Question	Answer	Marks	Guidance
4 (i)	$\frac{d^2x}{dt^2} = 2\frac{dx}{dt} - 2\frac{dy}{dt}$ $= 2\frac{dx}{dt} - \frac{3}{4}x + 60e^{-\frac{1}{2}t}$ $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + \frac{3}{4}x = 60e^{-\frac{1}{2}t}$ <p>Auxiliary equation $m^2 - 2m + \frac{3}{4} = 0$</p> $m = \frac{1}{2}, \frac{3}{2}$ <p>CF: $x = Ae^{\frac{3}{2}t} + Be^{\frac{1}{2}t}$</p> <p>PI: $x = Pe^{-\frac{1}{2}t}$</p> $x' = -\frac{1}{2}Pe^{-\frac{1}{2}t}, x'' = \frac{1}{4}Pe^{-\frac{1}{2}t}$ $P = 30$ $x = Ae^{\frac{3}{2}t} + Be^{\frac{1}{2}t} + 30e^{-\frac{1}{2}t}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>F1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>F1</p> <p>[10]</p>	<p>Differentiate</p> <p>Substitute for $\frac{dy}{dt}$</p> <p>AG Rearrange</p> <p></p> <p>Correct form for their CF</p> <p>Differentiate and substitute</p> <p>Solve</p> <p>PI + CF with 2 arb const</p>
(ii)	$y = x - \frac{1}{2}\frac{dx}{dt}$ <p>Substitute for x and $\frac{dx}{dt}$</p> $y = \frac{1}{4}Ae^{\frac{3}{2}t} + \frac{3}{4}Be^{\frac{1}{2}t} + \frac{75}{2}e^{-\frac{1}{2}t}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Rearrange</p> <p></p> <p>cao. As final answer</p>

Question	Answer	Marks	Guidance
(iii)	$x = 40, t = 0 \Rightarrow 40 = A + B + 30$ $y = 50, t = 0 \Rightarrow 50 = \frac{1}{4}A + \frac{3}{4}B + \frac{75}{2}$ $A = -10, B = 20$ $x = -10e^{\frac{3}{2}t} + 20e^{\frac{1}{2}t} + 30e^{-\frac{1}{2}t}$ $y = -\frac{5}{2}e^{\frac{3}{2}t} + 15e^{\frac{1}{2}t} + \frac{75}{2}e^{-\frac{1}{2}t}$	M1 M1 A1 A1 [4]	Use condition Use condition cao cao
(iv)	When $x = 0$ $20e^T - 10e^{2T} + 30 = 0$ $e^{2T} - 2e^T - 3 = 0$ $e^T = 3$ (or -1) $T = \ln 3 (= 1.10)$ $y = 34.64$	M1 M1 A1 A1 M1 A1 [6]	Multiply by $e^{\frac{1}{2}T}$ Attempt to solve as a quadratic cao Substitute T in expression for y cao
(v)	Unsuitable, X is negative	B1 [1]	

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GCE Mathematics (MEI)			Max Mark	a	b	c	d	e	u
4751	01 C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	48	43	0
		UMS	100	80	70	60	50	40	0
4752	01 C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	50	44	39	34	0
		UMS	100	80	70	60	50	40	0
4753	01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	56	51	46	41	36	0
4753	02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
		UMS	100	80	70	60	50	40	0
4754	01 C4 – MEI Applications of advanced mathematics (A2)	Raw	90	74	67	60	54	48	0
		UMS	100	80	70	60	50	40	0
4755	01 FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	62	57	53	49	45	0
		UMS	100	80	70	60	50	40	0
4756	01 FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
		UMS	100	80	70	60	50	40	0
4757	01 FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	59	52	46	40	34	0
		UMS	100	80	70	60	50	40	0
4758	01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	38	0
4758	02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
		UMS	100	80	70	60	50	40	0
4761	01 M1 – MEI Mechanics 1 (AS)	Raw	72	62	54	46	39	32	0
		UMS	100	80	70	60	50	40	0
4762	01 M2 – MEI Mechanics 2 (A2)	Raw	72	54	47	40	33	27	0
		UMS	100	80	70	60	50	40	0
4763	01 M3 – MEI Mechanics 3 (A2)	Raw	72	64	56	48	41	34	0
		UMS	100	80	70	60	50	40	0
4764	01 M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
		UMS	100	80	70	60	50	40	0
4766	01 S1 – MEI Statistics 1 (AS)	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
4767	01 S2 – MEI Statistics 2 (A2)	Raw	72	65	60	55	50	46	0
		UMS	100	80	70	60	50	40	0
4768	01 S3 – MEI Statistics 3 (A2)	Raw	72	64	58	52	47	42	0
		UMS	100	80	70	60	50	40	0
4769	01 S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771	01 D1 – MEI Decision mathematics 1 (AS)	Raw	72	56	51	46	41	37	0
		UMS	100	80	70	60	50	40	0
4772	01 D2 – MEI Decision mathematics 2 (A2)	Raw	72	54	49	44	39	34	0
		UMS	100	80	70	60	50	40	0
4773	01 DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776	01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	45	40	34	0
4776	02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
4777	01 NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)										
			Max Mark	a	b	c	d	e	u	
G241	01	Statistics 1 MEI (Z1)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
G242	01	Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
G243	01	Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
GCE Quantitative Methods (MEI)										
			Max Mark	a	b	c	d	e	u	
G244	01	Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02	Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1 MEI	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision 1 MEI	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0