

Wednesday 23 January 2013 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 Transformation A is represented by matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and transformation B is represented by matrix $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.
- (i) Describe transformations A and B. [3]
- (ii) Find the matrix for the composite transformation A followed by B. [2]
- 2 Given that $z = a + bj$, find $\operatorname{Re}\left(\frac{z}{z^*}\right)$ and $\operatorname{Im}\left(\frac{z}{z^*}\right)$. [4]
- 3 You are given that $z = 2 + j$ is a root of the cubic equation $2z^3 + pz^2 + 22z - 15 = 0$, where p is real. Find the other roots and the value of p . [6]
- 4 (i) Show that $x^2 - x + 2 > 0$ for all real x . [2]
- (ii) Solve the inequality $\frac{2x}{x^2 - x + 2} > x$. [5]
- 5 You are given that $\frac{3}{(5 + 3x)(2 + 3x)} \equiv \frac{1}{2 + 3x} - \frac{1}{5 + 3x}$.
- (i) Use this result to find $\sum_{r=1}^{100} \frac{1}{(5 + 3r)(2 + 3r)}$, giving your answer as an exact fraction. [5]
- (ii) Write down the limit to which $\sum_{r=1}^n \frac{1}{(5 + 3r)(2 + 3r)}$ converges as n tends to infinity. [1]
- 6 Prove by induction that $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$. [8]

Section B (36 marks)

- 7 Fig. 7 shows a sketch of $y = \frac{x-4}{(x-5)(x-8)}$.

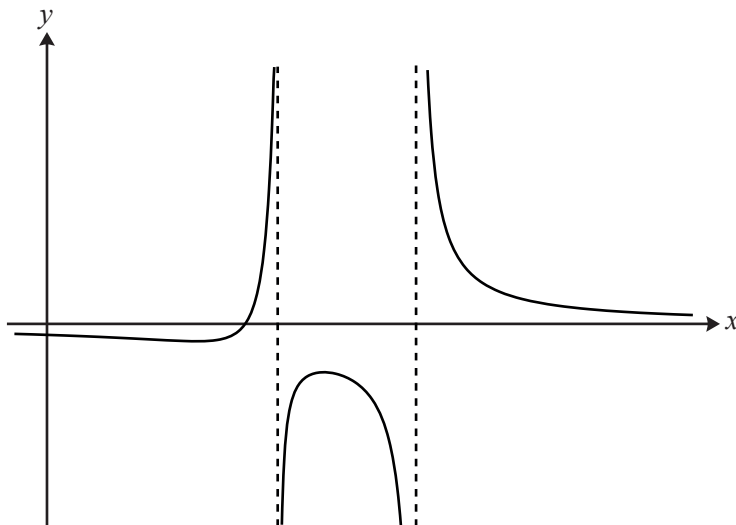


Fig. 7

- (i) Write down the equations of the three asymptotes and the coordinates of the points where the curve crosses the axes. Hence write down the solution of the inequality $\frac{x-4}{(x-5)(x-8)} > 0$. [6]
- (ii) The equation $\frac{x-4}{(x-5)(x-8)} = k$ has no real solutions. Show that $-1 < k < -\frac{1}{9}$. Relate this result to the graph of $y = \frac{x-4}{(x-5)(x-8)}$. [7]
- 8 (i) Indicate on an Argand diagram the set of points z for which $|z - (-8 + 15j)| < 10$. [4]
- (ii) Using the diagram, show that $7 < |z| < 27$. [3]
- (iii) Mark on your Argand diagram the point, P , at which $|z - (-8 + 15j)| = 10$ and $\arg z$ takes its maximum value. Find the modulus and argument of z at P . [4]

9 You are given that $\mathbf{A} = \begin{pmatrix} 8 & -7 & -12 \\ -10 & 5 & 15 \\ -9 & 6 & 6 \end{pmatrix}$ and $\mathbf{A}^{-1} = k \begin{pmatrix} 4 & 2 & 3 \\ 5 & 4 & 0 \\ 1 & -1 & 2 \end{pmatrix}$.

(i) Find the exact value of k . [2]

(ii) Using your answer to part (i), solve the following simultaneous equations.

$$\begin{aligned} 8x - 7y - 12z &= 14 \\ -10x + 5y + 15z &= -25 \\ -9x + 6y + 6z &= 3 \end{aligned} \quad [4]$$

You are also given that $\mathbf{B} = \begin{pmatrix} -7 & 5 & 15 \\ a & -8 & -21 \\ 2 & -1 & -3 \end{pmatrix}$ and $\mathbf{B}^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 0 & 5 \\ -4 & -3 & 1 \\ 2 & 1 & b \end{pmatrix}$.

(iii) Find the values of a and b . [2]

(iv) Write down an expression for $(\mathbf{AB})^{-1}$ in terms of \mathbf{A}^{-1} and \mathbf{B}^{-1} . Hence find $(\mathbf{AB})^{-1}$. [4]

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Wednesday 23 January 2013 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:

- Question Paper 4755/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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This paper has been pre modified for carrier language

Section A (36 marks)

1 (i)	
1 (ii)	
2	

4(i)	
4(ii)	

5(i)	(continued)

5 (ii)	

PLEASE DO NOT WRITE IN THIS SPACE.

Section B (36 marks)

7(i)	

8(i)	
8(ii)	

9(i)	
9(ii)	

9 (iii)	
9 (iv)	

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Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4755**: Further Concepts for Advanced Mathematics

Mark Scheme for January 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

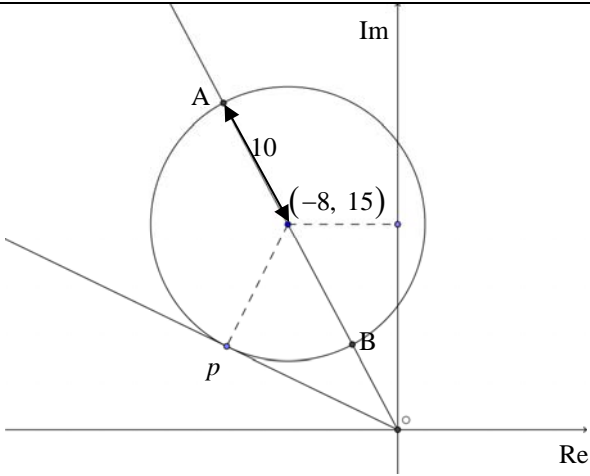
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	A is a reflection in the line $y = x$ B is a two way stretch, (scale) factor 2 in the x -direction and (scale) factor 3 in the y -direction	B1 B1 B1 [3]	Stretch, with attempt at details. Details correct.
1	(ii)	$\mathbf{BA} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$	M1 A1 [2]	Attempt to multiply in correct order
2		$\frac{z}{z^*} = \frac{a+bj}{a-bj} = \frac{(a+bj)^2}{(a-bj)(a+bj)}$ $= \frac{a^2 + 2abj - b^2}{a^2 + b^2}$ $\Rightarrow \operatorname{Re}\left(\frac{z}{z^*}\right) = \frac{a^2 - b^2}{a^2 + b^2}$ and $\operatorname{Im}\left(\frac{z}{z^*}\right) = \frac{2ab}{a^2 + b^2}$	M1 M1 A1 A1 [4]	Multiply top and bottom by $a + bj$ and attempt to simplify Using $j^2 = -1$ cao correctly labelled cao correctly labelled
3		$z = 2 - j$ is also a root $\alpha\beta\gamma = \frac{15}{2}$, or $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{22}{2}$, with $\alpha\beta = (2 + j)(2 - j) = 5$ used. OR $(az + b)(z - 2 + j)(z - 2 - j) = 2z^3 + pz^2 + 22z - 15$ $\Rightarrow (az + b)(z^2 - 4z + 5) = 2z^3 + pz^2 + 22z - 15$ OR $2(2 + 11j) + p(3 + 4j) + 22(2 + j) - 15 = 0$ Complete valid method for then obtaining the other unknown. real root $= \frac{3}{2}$, $p = -11$	B1 M1 A1 M1 A1 M1 A1 A1 [6]	Stated, not just used. Attempt to use roots in a relationship Correct equation obtained for γ . Attempt use of complex factors. Correct complex factors; one pair of factors correctly multiplied Substitution correct equation Root relation, obtaining linear factor, equating real and imaginary parts FT one value Allow incorrect signs Allow incorrect signs ($z + \dots$) Allow an incorrect sign Signs correct

Question	Answer	Marks	Guidance	
4 (i)	<p>$x^2 - x + 2$ has discriminant -7, so $x^2 - x + 2 \neq 0$ and when e.g. $x = 0$, $x^2 - x + 2 > 0$ so positive for all x</p> <p>OR</p> <p>$x^2 - x + 2 = (x - \frac{1}{2})^2 + \frac{7}{4} \geq \frac{7}{4} > 0$ for all x.</p> <p>OR using $y = x^2 - x + 2$</p> <p>$\frac{dy}{dx} = 2x - 1 = 0$ when $x = \frac{1}{2}$ and $y = \frac{7}{4}$; $\frac{d^2y}{dx^2} = 2 > 0$</p> <p>Hence y has minimum value, and $y \geq \frac{7}{4} > 0$ for all x.</p>	<p>E2,1,0</p> <p>E2,1,0</p> <p>E2,1,0</p>	<p>Discriminant < 0 shown and sign of $x^2 - x + 2$ or curve position discussed.</p> <p>Completing square and minimum value discussed</p> <p>Calculus, showing minimum value > 0.</p>	<p>Allow complex roots found, with discussion</p>
4 (ii)	<p>$\frac{2x}{x^2 - x + 2} > x$</p> <p>$\Rightarrow 2x > x^3 - x^2 + 2x$</p> <p>$\Rightarrow 0 > x^3 - x^2 \Rightarrow 0 > x^2(x - 1)$</p> <p>0, 1 critical values</p> <p>$x < 1$</p> <p>$\Rightarrow x < 0$ or $0 < x < 1$ or $x < 1, x \neq 0$</p> <p>OR</p> <p>Graphical approach by sketching</p> <p>$y = \frac{2x}{x^2 - x + 2}$ and $y = x$ or $y = \frac{2x}{x^2 - x + 2} - x$</p> <p>Critical values 0 and 1</p> <p>$x < 1$</p> <p>$\Rightarrow x < 0$ or $0 < x < 1$ or $x < 1, x \neq 0$</p>	<p>[2]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[5]</p> <p>M2,1,0</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>Valid attempt to eliminate fraction</p> <p>Simplification and factors</p> <p>Both, no other values given.</p> <p>cao</p> <p>Accuracy of sketch</p> <p>Both</p> <p>cao</p>	<p>Or combine to one fraction $>$ or < 0</p> <p>In numerator</p>

Question	Answer	Marks	Guidance
5 (i)	$\sum_{r=1}^{100} \frac{1}{(5+3r)(2+3r)} = k \sum_{r=1}^{100} \left[\frac{1}{2+3r} - \frac{1}{5+3r} \right]$ $= k \left[\left(\frac{1}{5} - \frac{1}{8} \right) + \left(\frac{1}{8} - \frac{1}{11} \right) + \dots \right.$ $\left. + \left(\frac{1}{302} - \frac{1}{305} \right) \right]$ $= k \left(\frac{1}{5} - \frac{1}{305} \right)$ $= \frac{20}{305} = \frac{4}{61}, \text{ oe}$	M1 M1 A1 M1 A1 [5]	Write out terms (at least first and last terms in full) Cancelling inner terms cao
(ii)	$\frac{1}{15}$	B1 [1]	
6	<p>When $n = 1$, $(-1)^0 \frac{1 \times 2}{2} = 1$ and $1^2 = 1$, so true for $n = 1$ Assume true for $n = k$</p> $\Rightarrow 1^2 - 2^2 + 3^2 - \dots + (-1)^{k-1} k^2 = (-1)^{k-1} \frac{k(k+1)}{2}$ $\Rightarrow 1^2 - 2^2 + 3^2 - \dots + (-1)^{k-1} k^2 + (-1)^{k+1-1} (k+1)^2$ $= (-1)^{k-1} \frac{k(k+1)}{2} + (-1)^{k+1-1} (k+1)^2$ $= (-1)^k \left[\frac{-k(k+1)}{2} + (k+1)^2 \right]$ $= (-1)^k (k+1) \left(\frac{-k}{2} + k+1 \right)$	B1 E1 M1* M1 Dep* A1	Assuming true result for some n . Adding $(k+1)$ th term to both sides. Attempt to factorise (at least one valid factor) Correct factorisation Accept $(-1)^{k \pm m}$ provided expression correct. Condone series shown incomplete

Question		Answer	Marks	Guidance
		$= (-1)^k (k+1) \left(\frac{k+2}{2} \right)$ $= (-1)^{[n-1]} \frac{n(n+1)}{2}, n = k+1$ <p>Therefore if true for $n = k$ it is also true for $n = k+1$ Since it is true for $n = 1$, it is true for all positive integers, n.</p>	A1 E1 E1 [8]	Valid simplification with $(-1)^k$ Or target seen Dependent on A1 and previous E1 Dependent on B1 and previous E1
7	(i)	Asymptotes $y = 0,$ $x = 5, x = 8$ Crosses axes at $(4, 0), (0, -\frac{1}{10})$ $\frac{x-4}{(x-5)(x-8)} > 0 \Rightarrow x > 8 \text{ or } 4 < x < 5$	B1 B1 B1 B1 B1 B1 [6]	both
7	(ii)	$\frac{x-4}{(x-5)(x-8)} = k \Rightarrow x-4 = kx^2 - 13kx + 40k$ $\Rightarrow kx^2 - (13k+1)x + 40k + 4 = 0$ $b^2 - 4ac = (13k+1)^2 - 4k(40k+4)$ $= 9k^2 + 10k + 1$ Critical values $-1, -1/9$ For no solutions to exist, $9k^2 + 10k + 1 < 0$ $\Rightarrow -1 < k < -\frac{1}{9}$ No point on the graph has a y coordinate in the range $\Rightarrow -1 < y < -\frac{1}{9}$	M1 A1 M1 A1 A1 E1 E1 [7]	Attempt to remove fraction and simplify 3 term quadratic ($= 0$) Attempt to use discriminant Correct 3-term quadratic Roots found or factors shown Accept equivalent statement

Question	Answer	Marks	Guidance
8 (i)	 <p data-bbox="342 754 1014 882">The set of points for which $z - (-8 + 15j) < 10$ is all points inside the circle, radius 10, centre $(-8, 15)$, excluding the points on the circumference.</p>	<p data-bbox="1088 746 1128 775">B4</p> <p data-bbox="1088 895 1128 924">[4]</p>	<p data-bbox="1648 276 1995 336">The circle should be reasonably circular.</p> <p data-bbox="1648 371 2051 461">The radius should be shown to be 10 by annotation as in the diagram or by other positions marked.</p> <p data-bbox="1648 496 2051 557">The centre point should be indicated and correct.</p> <p data-bbox="1648 592 2051 681">The region should be shown by a key or by description. Accept a “dotty” outline to a shaded interior.</p> <p data-bbox="1648 748 2051 837">Correctly placed: the “circle” must lie above the Re axis and intersect the Im axis twice as in the diagram.</p>

Question	Answer	Marks	Guidance
8 (ii)	Origin to centre of circle = $\sqrt{(-8)^2 + 15^2} = 17$. Origin to centre of the circle ± 10 Point A is the point on the circle furthest from the origin. Since the radius of the circle is 10, OA = 27. Point B is the point on the circle closest to the origin. Since the radius of the circle is 10, OB=7. Hence for z in the circle $7 < z < 27$	M1 M1 E1 [3]	Allow centre at $\pm 8 \pm 15j$ and FT Use of radius of circle Correct explanation for both
8 (iii)	P is the point where a line from the origin is a tangent to the circle giving the greatest argument θ , $-\pi < \theta \leq \pi$ $ p = \sqrt{17^2 - 10^2} = \sqrt{189} = 13.7$ (3 s.f.) $\arg p = \frac{\pi}{2} + \arcsin \frac{8}{17} + \arcsin \frac{10}{17}$ $= 2.69$ (3 s.f.)	B1 B1 M1 A1 [4]	Correctly positioned on circle Accept $\sqrt{189}$ or $3\sqrt{21}$ or 13.7 Attempt to calculate the correct angle. cao Accept 154° Allow circles centred as in (ii) Correct circle only
9 (i)	$(8 \times 4) - (7 \times 5) - (12 \times 1) = -15$ $\Rightarrow k = -\frac{1}{15}$	M1 A1 [2]	Any valid method soi No working or wrong working SC B1
9 (ii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{15} \begin{pmatrix} 4 & 2 & 3 \\ 5 & 4 & 0 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 14 \\ -25 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$ $x = -1, y = 2, z = -3$	B1 M1 A2 [4]	Use of \mathbf{A}^{-1} in correct position(s) Attempt to multiply matrices to obtain column vector -1 each error Condone missing k
9 (iii)	$(1 \times a) + (-8 \times -4) + (-21 \times 2) = 0 \Rightarrow a = 10$ $(-7 \times 5) + (5 \times 1) + (15 \times b) = 0 \Rightarrow b = 2$	M1 A1 [2]	Attempt to multiply \mathbf{BB}^{-1} matrices to find a or b soi For both

Question		Answer	Marks	Guidance
9	(iv)	$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ $= \frac{1}{3} \begin{pmatrix} 1 & 0 & 5 \\ -4 & -3 & 1 \\ 2 & 1 & 2 \end{pmatrix} \times -\frac{1}{15} \begin{pmatrix} 4 & 2 & 3 \\ 5 & 4 & 0 \\ 1 & -1 & 2 \end{pmatrix}$ $= -\frac{1}{45} \begin{pmatrix} 9 & -3 & 13 \\ -30 & -21 & -10 \\ 15 & 6 & 10 \end{pmatrix}$	<p>B1</p> <p>M1</p> <p>A2</p> <p>[4]</p>	<p>By notation or explicitly</p> <p>Attempt to multiply in correct sequence, may be implied by the answer (at least 7 elements correct)</p> <p>-1 each error FT their value of b.</p> <p>Must include k</p>

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Mathematics (MEI)

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4755 Further Concepts for Advanced Mathematics

General Comments

Some of the questions in this paper were unusual, and presented a challenge to which not all managed to rise. Many candidates demonstrated a lack of basic algebraic skills, which marred their progress. There were often different approaches to the questions and candidates sometimes confused themselves by changing between methods, and if not confused, still using up time. There was evidence that some found the paper too long. Many were however successful in showing not only knowledge but also excellent understanding.

Comments on Individual Questions

- 1 This straightforward question was well answered by most. The transformation A was sometimes described as a rotation while B, more often, was written down as enlargement, even though the different scale factors were correctly described. A few candidates favoured “shear”. In (ii) there were very few multiplications performed in the wrong sequence.
- 2 This was the first question to reveal poor algebraic skill. Most candidates knew that the complex number fraction had to be multiplied by $(a+bj)/(a+bj)$ but could not accurately carry out the work in the numerator. The term in j was frequently seen as $2bj$ and the term in j^2 became $-b$. The candidates who successfully multiplied and defined $\text{Re}(z/z^*)$ then in many cases wrote down $\text{Im}(z/z^*)$ to include j . Many candidates did not know how to start, and tried to cancel terms within the fraction.
- 3 A well answered question that was tackled in a variety of ways. Probably the swiftest solution was through the root relationships, but finding the quadratic and linear factors was also a useful route to take. Those candidates that chose a “pick and mix” approach took up time. There were a number of candidates who were confused between a “factor” and a “root”.
- 4 Part (i) was surprisingly badly done. Trials of different values for x were popular, as well as sweeping generalisations about x^2 being greater than x . Partial explanations were frequently seen. Solutions by completing the square or by elementary calculus were usually well explained. Part (ii) was not well done. Many candidates failed to factorise their cubic expression, prematurely dividing by x or x^2 thereby losing essential roots. Careless algebraic work initially failed to obtain the simple cubic expression and a few candidates became enmeshed in multiplying both sides of the initial inequality by $(x^2-x+2)^2$ which frequently led to errors.
- 5 Part (i) was well done, most candidates scored full marks but the factor $1/3$ was fairly commonly forgotten and sometimes seen misapplied as multiplication by 3. In (ii) very many made the mistake of believing the limit was zero.
- 6 Most candidates knew what to do and were meticulous in presenting their argument. The factor $(-1)^k$ was successfully dealt with by many although it caused a problem for some. The precise wording needed to round off the argument was in the main well expressed.

- 7** In part (i) few candidates encountered any difficulty, but it would be good to see coordinates presented, as requested. Some candidates used one or more inclusive inequalities, which lost a mark. Part (ii) was not well answered, apart from a few who not only saw a clear method to follow but also possessed the algebraic precision required and a good understanding of the relationship between the initial analysis and the graph, which they expressed with clarity. The bulk of the work did not need calculus (which second guessed the final answer), although a calculus method was of course given full credit where the argument covered all the detail satisfactorily. The straightforward method of finding a negative discriminant was most often attempted but was bedevilled by careless work with signs and brackets. Substitution of the given values $-1/9$ and -1 for k was chosen as a method by some but then needed a thorough explanation of the nature of the resulting points to earn all the marks.
- 8** Very few fully correct solutions to this question were seen. In part (i) there were many very poor circles. In an Argand diagram a sketch should show the nature of the object first and any scales shown on the axes need little beyond what might be used to establish the centre and radius. Candidates on the whole placed the object in roughly the right place. A key remains the best way of explaining both region and boundary as there is no accepted convention on how to show this. Part (ii) was not well explained in many cases, $|z|$ was frequently used to refer to the centre of the circle, not to a general point. The diagram was often ignored. In part (iii) P was often placed wrongly and even when roughly correctly placed (quality of diagram permitting) it was given assumed co-ordinates which were incorrect.
- 9** This question was often very well answered, although there was also evidence that some candidates lacked time by this point. In part (i) the correct k was usually found, but some were confused between k and $1/k$. Part (ii) was usually correctly done, and the right value of k frequently recovered. Algebraic error was common in part (iii), which could be avoided by writing down complete equations in a row by column calculation. In part (iv) many candidates did not know that $(AB)^{-1}$ was equal to $B^{-1}A^{-1}$ and the subsequent calculation had to show this.