OCR
Oxford Cambridge and RSA

Wednesday 7 June 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **24** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 Four points have coordinates A(0, 1, −4), B(5, 6, 1), C(6, 1, 4) and D(−6, 1, 1).

(i) Find the shortest distance from A to the line CD. **[5]**

(ii) Show that the shortest distance between the lines AB and CD is $\sqrt{26}$. **[5]**

(iii) Find points P and Q, lying on AB and CD respectively, such that PQ is of length $\sqrt{26}$. **[6]**

(iv) Explain why the lengths found in parts **(i)** and **(ii)** are not the same. **[1]**

(v) Find the equation of the plane Π that contains the line CD and is parallel to the line AB. **[3]**

(vi) You are given that the point E (0, −1, 1) lies on Π. Find the volume of the tetrahedron PECD. **[4]**

Option 2: Multi-variable calculus

2 A surface has equation $z = (x^2 + y^2)(x + 1)$.

(i) (*A*) Show that there is a stationary point at the origin. **[4]**

(*B*) By considering small values of x and y find the nature of this stationary point. **[3]**

(ii) (*A*) Show that there is exactly one other stationary point. Find its coordinates. **[3]**

(*B*) By considering sections of the surface at this stationary point, show that this point is neither a maximum nor a minimum. **[6]**

(iii) The point P(1, 1, 4) lies on the surface and the point Q(1 + h, 1 + h, 4 + k) is a point on the surface close to P.

Find an approximate expression for k in terms of h. **[4]**

(iv) Find the equation of the tangent plane to the surface at point P. **[4]**

Option 3: Differential geometry

3 A curve has parametric equations $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, for $0 \leqslant \theta \leqslant 2\pi$ where a is a positive constant. The point A on the curve has parameter $\theta = \frac{1}{2}\pi$.

(i) Show that

(A) the curve passes through the origin, **[1]**

(B) the arc length, s, from the origin to the point A is $2a\sqrt{2}$. **[6]**

(ii) The curve from O to A is rotated through 2π about the x-axis. Find the area of the curved surface generated. **[5]**

(iii) Find

(A) the intrinsic equation of the curve, **[4]**

(B) the centre of curvature for point A. **[8]**

Option 4: Groups

4 **(a)** The composition table for a group G of order 6 is given below.

	a	**b**	**c**	**d**	**e**	**f**
a	c	f	e	b	a	d
b	f	a	d	e	b	c
c	e	d	a	f	c	b
d	b	e	f	c	d	a
e	a	b	c	d	e	f
f	d	c	b	a	f	e

(i) State the identity element. **[1]**

(ii) State the order of each element. **[3]**

(iii) Write down the inverse of each element. **[3]**

(iv) Determine whether G is cyclic. **[2]**

(v) List all the proper subgroups. Comment on the order of these groups in relation to Lagrange's theorem. **[3]**

(vi) Specify an isomorphism between G and the group F consisting of $\{1,2,3,4,5,6\}$ under multiplication modulo 7. **[4]**

(b) A group H is commutative and has e as its identity element. Three elements of the group, a, b, and c have order 2, 3, and 5 respectively. The order of H is the minimum value consistent with these properties.

(i) State the order of H. **[1]**

(ii) Prove that the order of ab is 6. **[4]**

(iii) Prove that H is cyclic. **[3]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Two bags, A and B, have a total of 3 balls in them. An event is to choose a ball at random, find the bag it is currently in, and transfer the ball to the other bag.

The transition matrix, **M**, for the number of balls in bag A is shown below.

$$\mathbf{M} = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 \\ 1 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 0 & 1 \\ 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

(i) Explain carefully the contents of the second column. **[3]**

(ii) Explain what is meant by a reflecting barrier in a Markov chain.
Identify any reflecting barriers in this situation. **[3]**

(iii) Find $\mathbf{M}^4$ and $\mathbf{M}^5$. **[4]**

(iv) Find the limiting values of $\mathbf{M}^{2n}$ and $\mathbf{M}^{2n+1}$ as n tends to infinity. **[4]**

(v) Find the equilibrium probabilities that bag A contains 0, 1, 2 or 3 balls. Comment on your answer in relation to the results of part **(iv)**. **[4]**

(vi) Find the probability that bag A contains the same number of balls after 15 stages that it did after 12 stages. Show clearly how you obtain your answer. **[3]**

(vii) Now suppose that the situation is as before but with a total of 4 balls in the two bags. Write down the 5×5 transition matrix for bag A. **[3]**

END OF QUESTION PAPER

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4757/01 Further Applications of Advanced Mathematics (FP3)

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Centre number						Candidate number				

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1 (i)

1 (ii)	

1 (iii)

1 (iv)	
1 (v)	
1 (vi)	

2 (i)(*A*)	
2 (i)(*B*)	
2 (ii)(*A*)	
	continued on next page

2 (ii)(*A*)	**Continued**
2 (ii)(*B*)	

2 (iii)	
2 (iv)	

3 (i)(*A*)	
3 (i)(*B*)	

3 (ii)	

3 (iii)(*A*)	

3 (iii)(*B*)

continued on next page

3(iii)(*B*)	**Continued**

4 (a)(i)	
4 (a)(ii)	
4 (a)(iii)	

4 (a)(iv)	
4 (a)(v)	
4 (a)(vi)	

4 (b)(i)	
4 (b)(ii)	
4 (b)(iii)	

5 (i)	
5 (ii)	

5 (iii)

5 (iv)	

5(v)

5 (vi)

5 (vii)

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Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4757**: Further Applications of Advanced Mathematics

Advanced GCE

Mark Scheme for June 2017

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the

establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Mark scheme 4757 June 2017

Question			Answer	Marks	Guidance
1	(i)		$\mathbf{c}-\mathbf{a}=\begin{pmatrix}6\\0\\8\end{pmatrix}$, $\overrightarrow{CD}=\begin{pmatrix}-12\\0\\-3\end{pmatrix}$, $(\mathbf{c}-\mathbf{a})\times\overrightarrow{CD}=\begin{pmatrix}0\\-78\\0\end{pmatrix}$	**M1** **B1**	Finding **c-a** oe Direction CD (any multiple) Length CD
			$\Rightarrow \lvert(\mathbf{c}-\mathbf{a})\times\overrightarrow{CD}\rvert=78$, $\lvert\overrightarrow{CD}\rvert=\sqrt{153}=3\sqrt{17}$ Distance $=\dfrac{78}{3\sqrt{17}}=\dfrac{26\sqrt{17}}{17}$	**M1** **B1** ft **A1**	Method for finding distance For length CD oe (e.g. $78/\sqrt{153}$) *Accept 6.31*
		OR	$\overrightarrow{AQ}=(6+4\lambda,0,8+\lambda)$ $\mathbf{d}_2=(4,0,1)$ $\overrightarrow{AQ}.\mathbf{d}_2=0$ $(\Rightarrow\lambda=-32/17)$ $\overrightarrow{AQ}=(26/17)(-1,0,4)$ $AQ=26\sqrt{17}/17$		**M1 B1** **M1** **A1 A1**
				[5]	
	(ii)		$\mathbf{d}_1=\begin{pmatrix}1\\1\\1\end{pmatrix}$, $\mathbf{d}_2=\begin{pmatrix}4\\0\\1\end{pmatrix}$, $\mathbf{d}=\mathbf{d}_1\times\mathbf{d}_2=\begin{pmatrix}1\\3\\-4\end{pmatrix}$	**M1** **A1**	Vector product of directions Vector product correct
			For e.g. $\mathbf{c}-\mathbf{a}=\begin{pmatrix}6\\0\\8\end{pmatrix}$	**M1** **A1** ft	Method for finding shortest distance Appropriate use of $\mathbf{c}-\mathbf{a}$ oe
			Distance $=\left\lvert\dfrac{(\mathbf{c}-\mathbf{a}).\mathbf{d}}{\lvert\mathbf{d}\rvert}\right\rvert=\dfrac{26}{\sqrt{26}}=\sqrt{26}$ **AG**	**A1**	
				[5]	

Question			Answer	Marks	Guidance
1	(iii)		Any point on AB is $P(\lambda, 1+\lambda, -4+\lambda)$ Any point on CD is $Q(6+4\mu, 1, 4+\mu)$	B1	Both points
			$\overrightarrow{QP} = \begin{pmatrix} \lambda-6-4\mu \\ \lambda \\ -8+\lambda-\mu \end{pmatrix}$	B1	
			Perpendicular to both lines gives 2 equations: $3\lambda - 5\mu = 14$	M1	Dot product on at least one **OR** QP = (±) (1, 3, −4)
			$5\lambda - 17\mu = 32$	M1	Solving to obtain λ and μ [**OR M2** for $QP^2 = 26$ and valid method for solving]
			Solve simultaneously: $\lambda = 3,\ \mu = -1$	A1	
			$\Rightarrow P(3, 4, -1),\ Q(2, 1, 3)$	A1	For both
		OR	Shortest distance from P to CD is $\lvert (\lambda-6, \lambda, \lambda-8) \times (4, 0, 1) \rvert / \sqrt{17} = \sqrt{26}$		**M1**
			$\lambda^2 + (3\lambda-26)^2 + (-4\lambda)^2 = 442;\quad \lambda = 3$		**A1**
			P is (3, 4, −1)		**A1**
			Similarly for Q(2, 1, 3)		**M1A1A1**
				[6]	
	(iv)		Distance PQ is the shortest, so the distance from A to CD will be longer.	B1	
				[1]	

Question			Answer	Marks	Guidance
1	(v)		Eqn of plane has PQ as its normal: $\overrightarrow{QP} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} \Rightarrow x+3y-4z=k$ Substitute any point: $\Rightarrow x+3y-4z=-7$	**M1** **A1** **A1**	
				[3]	
	(vi)		$\lvert \text{EC} \times \text{ED} \rvert = \left\lvert \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \right\rvert = \left\lvert \begin{pmatrix} -6 \\ -18 \\ 24 \end{pmatrix} \right\rvert$ $\Rightarrow$ Area triangle CED = $\frac{1}{2}\sqrt{6^2+18^2+24^2} = 3\sqrt{26}$ $\Rightarrow$ Volume PCED $= \frac{1}{3}$ Area triangle CED $\times$ PQ $\Rightarrow V = \frac{1}{3} 3\sqrt{26}\sqrt{26} = 26$	**M1** **A1** **M1** **A1** cao	Vector product of two sides Vector product correct (ft from wrong P) Method for finding volume e.g. EP . (EC × ED) / 6 Allow 26 obtained from wrong P
				[4]	

Question			Answer	Mark	Guidance
2	(i)	(*A*)	$\frac{\partial z}{\partial x} = 3x^2 + 2x + y^2$ $\frac{\partial z}{\partial y} = 2y + 2xy \quad [= 2y(x+1)]$ When $x = y = 0$, $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0$ and show that $z = 0$	**M1** **A1** **A1** **B1**	Diffn both Both correct Allow from wrong partial derivatives
				[4]	
	(i)	(*B*)	x^2 and $y^2 > 0$ and for small $x, 1 + x > 0$. So for all small values of x and y, $z > 0$ So minimum.	**B1** **B1** **B1**	Both Adequate explanation Dependent on B2
				[3]	

Question			Answer	Mark	Guidance
2	(ii)	(*A*)	Either $y=0$ and $3x^2+2x=0$ $x=0,-\frac{2}{3}$ so one further stationary point is $\left(-\frac{2}{3},0,\frac{4}{27}\right)$ Or $x=-1\Rightarrow 3-2+y^2=0$: No roots So there are no other stationary points	M1 A1 A1	
				[3]	
	(ii)	(*B*)	Taking $x=-\frac{2}{3}\Rightarrow z=\frac{1}{3}y^2+\frac{4}{27}$ This section has a minimum at $y=0$ Taking $y=0\Rightarrow z=x^2(x+1)$ This section has a maximum at $x=-2/3$ $\Rightarrow$ Neither maximum nor minimum	M1 A1 M1 A1 A1 A1	Correct sketch; or correct work with derivatives; or $x=-2/3+h\Rightarrow z=4/27-h^2(1-h)$ Final A1 is dependent on all previous marks *Numerical work earns no credit unless it relates to sections of the surface*
				[6]	

Question			Answer	Marks	Guidance
2	(iii)		At P, $\frac{\partial z}{\partial x}=6,\ \frac{\partial z}{\partial y}=4$ $\Rightarrow \delta z=\frac{\partial z}{\partial x}\delta x+\frac{\partial z}{\partial y}\delta y$ $\Rightarrow k\approx 6h+2h=10h$	**B1** ft **B1** ft **M1** **A1**	Allow evaluation at Q instead of P
		OR	$4+k=((1+h)^2+(1+h)^2)(1+h+1)$ $k=10h+8h^2+2h^3$		**M1** **A3, 2, 1**
				[4]	
	(iv)		At $(1,1,4), \frac{\partial z}{\partial x}=6,\ \frac{\partial z}{\partial y}=4$ Equation of tangent is $\begin{pmatrix}x-1\\y-1\\z-4\end{pmatrix}.\begin{pmatrix}6\\4\\-1\end{pmatrix}=0$ $\Rightarrow 6x+4y-z=6$	**B1** ft **M1** **A1** ft **A1** cao	Use of (6, 4, -1) M0 for (6, 4, 1) or $6x+4y-z=c$
				[4]	

Question			Answer	Mark	Guidance
3	**(i)**	(*A*)	Substituting $\theta = 0$ seen	**B1**	
				[1]	
	(i)	(*B*)	$\frac{dx}{d\theta} = a(1+\cos\theta), \quad \frac{dy}{d\theta} = a\sin\theta$	**M1**	Obtaining $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$ in terms of θ
			$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = a^2\left(1+2\cos\theta+\cos^2\theta+\sin^2\theta\right)$		
			$= a^2\left(2+2\cos\theta\right) = 4a^2\cos^2\frac{\theta}{2}$	**M1** **A1**	Use of double angle formula
			$\Rightarrow s = \int_0^{\pi/2} 2a\cos\frac{\theta}{2}\,d\theta = \left[4a\sin\frac{\theta}{2}\right]_0^{\pi/2}$	**M1** **A1**	Integration (ignore limits)
			$= 4a\left(\sin\frac{\pi}{4} - \sin 0\right) = 4a\frac{1}{\sqrt{2}} = 2a\sqrt{2}$ **AG**	**A1**	
				[6]	
	(ii)		$s = 4a\sin\frac{\theta}{2} \Rightarrow \frac{ds}{d\theta} = 2a\cos\frac{\theta}{2}$	**M1**	Integral for finding area
			$\Rightarrow A = \int_0^{\pi/2} 2\pi y\,ds = 4a^2\pi\int_0^{\pi/2}(1-\cos\theta)\cos\frac{\theta}{2}\,d\theta$	**A1**	(ignore limits)
				M1	Use of double angle formula and integrate
			$= 8a^2\pi\int_0^{\pi/2}(\sin^2\frac{\theta}{2})\cos\frac{\theta}{2}\,d\theta = 8a^2\pi\left[\frac{2}{3}\sin^3\frac{\theta}{2}\right]_0^{\pi/2}$	**A1**	Function integrated, ignore limits
			$= \frac{16a^2\pi}{3}\left(\frac{1}{2\sqrt{2}}\right) = \frac{4\sqrt{2}a^2\pi}{3}$	**A1**	
				[5]	

Question			Answer	Mark	Guidance
3	(iii)	(*A*)	$\frac{dx}{d\theta} = a(1+\cos\theta)$, $\frac{dy}{d\theta} = a\sin\theta$ $\Rightarrow \frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$ $\Rightarrow \psi = \frac{\theta}{2} \quad \Rightarrow s = 4a\sin\frac{\theta}{2} = 4a\sin\psi$	**M1** **M1** **A1** **A1**	Finding $\frac{dy}{dx}$ Use of double angles
				[4]	
	(iii)	(*B*)	$\rho = \frac{ds}{d\psi} = 4a\cos\psi$ At A, $\theta = \frac{\pi}{2}$, $\psi = \frac{\pi}{4}$ $\Rightarrow \rho = 4a\cos\frac{\pi}{4} = \frac{4a}{\sqrt{2}} = 2a\sqrt{2}$ $x = a\left(1+\frac{\pi}{2}\right)$, $y = a$ $\hat{\mathbf{n}} = \begin{pmatrix} -\sin\psi \\ \cos\psi \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\Rightarrow$ Centre has coordinates $\left(a\left(1+\frac{\pi}{2}\right) - \frac{a2\sqrt{2}}{\sqrt{2}}, a + \frac{a2\sqrt{2}}{\sqrt{2}}\right)$ i.e. $\left(a\left(\frac{\pi}{2}-1\right), 3a\right)$	**M1** **A1** **A1** **A1** **B1** **M1** **M1** **A1**	Differentiate intrinsic eqn; or use other formula for ρ For coordinates of A To obtain normal vector *(not necessarily unit vector)* Method for finding centre of curvature
				[8]	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Mark</th><th>Guidance</th></tr>
<tr><td>4</td><td>(a)</td><td>(i)</td><td>Identity element is e</td><td>B1</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td></tr>
<tr><td></td><td>(a)</td><td>(ii)</td><td><table><tr><td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr><tr><td>3</td><td>6</td><td>3</td><td>6</td><td>1</td><td>2</td></tr></table></td><td>B3</td><td>All correct.
B2 one error
B1 2 errors</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[3]</td><td></td></tr>
<tr><td></td><td>(a)</td><td>(iii)</td><td><table><tr><td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr><tr><td>c</td><td>d</td><td>a</td><td>b</td><td>e</td><td>f</td></tr></table></td><td>B3</td><td>All correct.
B2 one error
B1 2 errors</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[3]</td><td></td></tr>
<tr><td></td><td>(a)</td><td>(iv)</td><td>Yes, the group is cyclic. For this to be so at least one element has to have an order the same as the group.
This so for two elements (b and d) as they have order 6 and therefore generate the set.</td><td>B1

B1</td><td>Do not award this mark without an attempt at a reason

Correct reason</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[2]</td><td></td></tr>
<tr><td></td><td>(a)</td><td>(v)</td><td>{e,a,c}
{e,f}
Order of these subgroups are 2 and 3. The order of the group is 6. 2 and 3 are factors of 6.</td><td>B1
B1

B1</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[3]</td><td></td></tr>
<tr><td></td><td>(a)</td><td>(vi)

OR</td><td><table><tr><td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr><tr><td>2</td><td>3</td><td>4</td><td>5</td><td>1</td><td>6</td></tr></table>
<table><tr><td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr><tr><td>4</td><td>5</td><td>2</td><td>3</td><td>1</td><td>6</td></tr></table></td><td>B4</td><td>B2 one pair interchanged</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[4]</td><td></td></tr>
</table>

Question			Answer	Mark	Guidance
4	**(b)**	**(i)**	Order is 30.	**B1**	
				[1]	
	(b)	**(ii)**	$(ab)^6 = a^6b^6 = e$ because H is commutative $ab \neq e$ so not of order 1 and must be a factor of 6 $(ab)^2 = a^2b^2 = b^2 \neq e$ so not of order 2 $(ab)^3 = a^3b^3 = a \neq e$ so not of order 3 So order must be 6 **AG**	**M1** **M1** **A1** **A1**	Using $(ab)^n = a^n b^n$ Considering all powers relevant to their argument All relevant powers correctly evaluated
				[4]	
	(b)	**(iii)**	Consider *abc* *abc* has order 30 So at least one element has order the same as the group and so the group is cyclic. **AG**	**M1** **A1** **A1**	
				[3]	

Question			Answer	Mark	Guidance
5	(i)		If there is one ball in A then it has a chance of $\frac{1}{3}$ of being chosen. If it is then there will be no balls in A. If one of the balls in B is chosen, with probability $\frac{2}{3}$ Then the number of balls in A will increase by 1 to 2. There is no chance of there being 1 ball still in bag A and no chance of there being 3.	B1 B1 B1	Explaining the 1/3 Explaining the 2/3 Explaining the zeros
				[3]	
	(ii)		Any reasonable definition 0 balls is a reflecting barrier 3 balls is a reflecting barrier	B1 B1 B1	
				[3]	
	(iii)		$\mathbf{M}^4 = \begin{pmatrix} .259 & 0 & .247 & 0 \\ 0 & .753 & 0 & .741 \\ .741 & 0 & .753 & 0 \\ 0 & .247 & 0 & .259 \end{pmatrix}$, $\mathbf{M}^5 = \begin{pmatrix} 0 & .251 & 0 & .247 \\ .753 & 0 & .749 & 0 \\ 0 & .749 & 0 & .753 \\ .247 & 0 & .251 & 0 \end{pmatrix}$	B2 B2	For $\mathbf{M}^4$, B1 one error. For $\mathbf{M}^5$, B1 one error.
				[4]	
	(iv)		$\mathbf{M}^{2n} \to \begin{pmatrix} .25 & 0 & .25 & 0 \\ 0 & .75 & 0 & .75 \\ .75 & 0 & .75 & 0 \\ 0 & .25 & 0 & .25 \end{pmatrix}$, $\mathbf{M}^{2n+1} \to \begin{pmatrix} 0 & .25 & 0 & .25 \\ .75 & 0 & .75 & 0 \\ 0 & .75 & 0 & .75 \\ .25 & 0 & .25 & 0 \end{pmatrix}$	B2 B2	For $\mathbf{M}^{2n}$ B1 one error For $\mathbf{M}^{2n+1}$ B1 one error
				[4]	

Question			Answer	Mark	Guidance
5	(v)		$\begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 \\ 1 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 0 & 1 \\ 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$	M1	
			$\Rightarrow \frac{1}{3}b = a,\ a + \frac{2}{3}c = b,\ \frac{2}{3}b + d = c,\ \frac{1}{3}c = d$	A1	
			$\Rightarrow b = 3a, c = 3a, d = a$		
			$\Rightarrow a = 0.125, b = 0.375, c = 0.375.d = 0.125$	A1	
			$a = \frac{0.25+0}{2} = 0.125$ etc.	A1	
				[4]	
	(vi)		Considering diagonal elements of $\mathbf{M}^3$ Diagonal elements of $\mathbf{M}^3$ are all zero Hence the probability is 0	M1 A1 A1	
				[3]	
	(vii)		$\begin{pmatrix} 0 & 0.25 & 0 & 0 & 0 \\ 1 & 0 & 0.5 & 0 & 0 \\ 0 & 0.75 & 0 & 0.75 & 0 \\ 0 & 0 & 0.5 & 0 & 1 \\ 0 & 0 & 0 & 0.25 & 0 \end{pmatrix}$	B1 B1 B1	1st and 5th columns 2nd and 4th columns 3rd column
				[3]	

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4757 Further Applications of Advanced Mathematics (FP3)

General Comments:

The work on this paper was generally of a high standard, with most candidates producing substantial attempts at all three of their chosen questions. Q.1 (on vectors) was the most popular question, chosen by over three quarters of the candidates. Q.2 (on multi-variable calculus) and Q.4 (on groups) were each chosen by about 70% of candidates; and Q.5 (on Markov chains) was chosen by about 60% of the candidates. The least popular question was Q.3 (on differential geometry), which was chosen by fewer than a quarter of the candidates.

Comments on Individual Questions:

Q.1(i) This was very well answered, with most candidates using the standard formula involving the magnitude of a vector product. Some used the alternative method of finding when the vector from A to a general point on CD was perpendicular to CD.

Q.1(ii) Most candidates knew how to find the shortest distance between the two lines, almost always using a scalar triple product.

Q.1(iii) Candidates used a variety of methods to find the points where the shortest distance occurred. Some applied scalar products of the general chord with the directions of the two lines to obtain two simultaneous equations. Some put the general chord parallel to the common perpendicular, which had already been found in part (ii). Another approach was to take the general point on one line and put its shortest distance from the other line equal to $\sqrt{26}$, applying the formula from part (i). The proportion of candidates who obtained the correct two points P and Q was very high. Some candidates tried putting the length of the general chord equal to $\sqrt{26}$, but the resulting equation proved much too difficult to solve.

Q.1(iv) Most candidates were able to explain this satisfactorily, for example by stating that A and P are different points, so the distance from A is not the shortest distance.

Q.1(v) This was very well answered. Most candidates gave the Cartesian equation of the plane, but of course any standard equation (such as the vector equation involving two parameters) was equally acceptable.

Q.1(vi) Almost all candidates quoted a correct formula for the volume of a tetrahedron, as one sixth of the scalar triple product of three edge vectors, and this was very often evaluated accurately. Candidates who had not found P in part (iii) usually gave up without doing any calculations, although they could have earned 2 marks by finding the vector product of any two edges; there was sufficient information in the question to find the volume without knowing the coordinates of P (the answer is the same wherever P is on the line AB).

Q.2(i)(A) The partial derivatives were almost always found correctly, and shown to be zero at the origin. Many candidates omitted to verify that the origin was on the surface.

Q.2(i)(B) Here candidates were expected to explain why $(x^2 + y^2)(x + 1)$ is positive for all sufficiently small values of x and y, and deduce that the origin is a minimum point on the surface. Many considered only the sections given by $x = 0$ and $y = 0$, which is not enough to establish that it is a minimum. Some simply substituted in a few numerical values.

Q.2(ii)(A) This was well answered, with most candidates finding the additional stationary point and showing that there are no others.

Q.2(ii)(B) There were very many good solutions here, in which the sections corresponding to $x = -2/3$ and $y = 0$ were each found and suitably investigated. Some candidates used other valid methods (such as numerical investigation) to show that the point is neither a maximum nor a minimum, but unless these referred explicitly to sections of the surface they did not earn any marks.

Q.2(iii) Most candidates answered this correctly.

Q.2(iv) Most candidates found the equation of the tangent plane correctly; although some gave the equation of the normal line instead.

Q.3(i)(A) Almost every candidate showed that $\theta = 0$ gave the origin.

Q.3(i)(B) The formula for finding the arc length was well known, and the given result was very often obtained correctly and efficiently. However, a significant number did not make any attempt to use half-angle formulae to simplify the integral, and they were unable to earn more than one mark.

Q.3(ii) Most candidates wrote down a correct integral expression for the curved surface area. Further progress required the use of half-angle formulae to obtain an integrable form; this was often completed elegantly, but sometimes the integral was split into two terms, followed by very lengthy, though ultimately successful, manipulation.

Q.3(iii)(A) This was often answered correctly. Success again depended on the use of half-angle formulae, in this case to obtain the result $\mathrm{d}y/\mathrm{d}x = \tan(\frac{1}{2}\theta)$, leading to $\psi = \frac{1}{2}\theta$.

Q.3(iii)(B) The radius of curvature was usually found correctly, either by differentiating the intrinsic equation, or by using the formula involving first and second derivatives of the parametric equations. Finding the unit normal vector, and obtaining the centre of curvature, were also well understood and quite often completed accurately.

Q.4(a)(i) Almost every candidate gave the correct identity element.

Q.4(a)(ii) The orders of the elements were almost always given correctly.

Q.4(a)(iii) The inverses were usually given correctly, although some candidates stated that the identity element does not have an inverse.

Q.4(a)(iv) Most candidates could explain why the group *G* was cyclic.

Q.4(a)(v) Most candidates gave the two non-trivial subgroups correctly, although quite a few included an extra 'subgroup' consisting of *e, b* and *d* (where in fact *b* and *d* were generators of the whole group *G*). Some candidates just quoted Lagrange's theorem; this was not sufficient to earn the mark for a comment unless it was applied to the group *G*.

Q.4(a)(vi) Most candidates gave a correct isomorphism. Those who considered powers of generators of the two groups, or wrote out the composition table for *F*, could be confident that their answer was correct. Many candidates just considered the orders of the elements, but this by itself does not guarantee to pair up the elements correctly.

Q.4(b)(i) Most candidates correctly gave the order of the group *H* as 30. The most common incorrect answer given was 15.

Q.4(b)(ii) Most candidates considered powers of *ab* and correctly showed that this element had order 6. Some thought that it was sufficient just to show that $(ab)^6 = e$.

Q.4(b)(iii) A good proportion of candidates identified *abc* as an element of order 30, which therefore generated the cyclic group *H*.

Q.5(i) Most candidates explained how the 1/3 and 2/3 referred to the probabilities of losing or gaining a ball, but many did not explain what the zeros meant. Most assumed that the columns represented the states of having 0, 1, 2, 3 balls in A from left to right; and others assumed that it was 3, 2, 1, 0. Both these interpretations worked equally well.

Q.5(ii) The expected definition was a state from which the next state is certain (and not the same). Variations which applied to this example (such as a state from which the system always returns to its previous state) were also accepted. The states of having 0 balls and 3 balls were usually correctly identified as the reflecting barriers.

Q.5(iii) The two powers of **M** were almost always given correctly.

Q.5(iv) The limiting values were usually given correctly.

Q.5(v) A variety of methods were used to find the equilibrium probabilities. Some solved **Mp** = **p**, others applied the limiting matrices found in part (iv) to an initial condition in which all four states had probability 1/4, and others found the ‘average’ of the two limiting matrices. Some candidates gave probabilities (usually 0.25, 0.75, 0.75, 0.25) which did not add up to one.

Q.5(vi) Most candidates stated that it was impossible for the system to be in the same state. This was usually explained by the zeros in the main diagonal of $\mathbf{M}^3$, but some gave a convincing argument based on the context (such as whether the number of balls in A was odd or even).

Q.5(vii) This was well understood, with most candidates giving the correct transition matrix.

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Unit level raw mark and UMS grade boundaries June 2017 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	49	45	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	67	61	55	49	43	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	57	52	47	42	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	0

		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

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Level 3 Certificate Mathematics for Engineering

				Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering		This unit has no entries in June 2017							
H860	02	Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

				Max Mark	a*	a	b	c	d	e	u
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H866	02	Critical maths	Raw	60*	48	42	36	30	24	0
		*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H867	02	Statistical problem solving	Raw	60*	41	36	31	27	23	0
		*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	72	63	55	47	39	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0